Section 9.5

Definition: A **first-order linear differential equation** is an equation that can be put into the form

$$y' + P(x)y = Q(x),$$

where P and Q are continuous functions.

Exercise 1. Is $xy' + y = 2x$ a linear equation?

Class Exercise 1. Determine whether the differential equation is linear:
(a) $y' + xy^2 = \sqrt{x}$
(b) $y \sin x = x^2 y' - x$

Theorem: The first-order linear differential equation $y' + P(x)y = Q(x)$ may be transformed into a separable differential equation by multiplying both sides by the integrating factor $e^{\int P(x)\,dx}$.

Exercise 2. Solve the differential equation.
(a) $\frac{dy}{dx} - 3x^2 y = x^2$
(b) $x^2 y' + 5xy + 3x^5 = 0$, where $x \neq 0$.

Exercise 3. Solve the differential equation:

$$y' + y \tan x = \sec x + 2x \cos x.$$

Class Exercise 2. Solve the differential equation.
(a) $y' - y = e^x$
(b) $4x^3 y + x^4 y' = \sin^3 x$
(c) $y' + y = \sin(e^x)$
(d) $x\frac{dy}{dx} - 4y = x^4 e^x$
(e) $t \ln t \, \frac{dr}{dt} + r = te^t$

Exercise 4. Solve the initial-value problem: $t^3 \frac{dy}{dt} + 3t^2 y = \cos t$, $y(\pi) = 0$.

Class Exercise 3. Solve the initial-value problem.
(a) $2xy' + y = 6x$, $x > 0$, $y(4) = 20$
(b) $(x^2 + 1)\frac{dy}{dx} + 3x(y - 1) = 0$, $y(0) = 2$.

Exercise 5. An object of mass m is released from a hot-air balloon. Find the distance it falls in t seconds if the force of resistance due to the air is directly proportional to the speed of the object.

Class Exercise 4. A simple electrical circuit consists of a resistance R and an inductance L connected in series, with a constant electromotive force V. If the switch S is closed at $t = 0$, then it follows from one of Kirchhoff's rules for electrical circuits that if $t > 0$, the current I satisfies the differential equation

$$L\frac{dI}{dt} + RI = V.$$

Express I as a function of t.

Homework: 1-21 (every 4th), 27-39 (every 4th)