Section 4.1

I am sure you have heard phrases such as "What is the probability of ... ?" and "The probability of ... is ...". The word *probability* is not a term that is easily defined. In considering probability, we deal with procedures that produce outcomes.

Definition: **Probability** is a numerical measure between 0 and 1 that describes the likelihood that an event will occur. Probabilities closer to 1 indicate that the event is most likely to occur. Probabilities closer to 0 indicate that the event is less likely to occur.

Definition: An **event** is a collection of one or more outcomes of a statistical experiment or observation.

Definition: A **simple event** is one particular outcome of a statistical experiment.

Definition: The **sample space** of an experiment is the set of all simple events.

Example: Let's suppose we toss a coin. The sample space is {heads, tails} and a simple event would be a tail.

Class Exercise 1. Suppose we toss a coin three times. What is the sample space? Is the event of "2 heads" a simple event?

Class Exercise 2. Suppose you roll two dice. How many different outcomes are there?

Exercise 1. Suppose you roll a die. Is the event of "even number" a simple event? Is the event of '5' a simple event?

Here is some notation for probability:

Notation: P denotes a probability.

Notation: A, B, and C denote specific events.

Notation: $P(A)$ denotes the probability of event A occurring.

There are three different methods of finding a probability.

1. Relative Frequency Approximation of Probability
Conduct (or observe) a procedure, and count the number of times that event A actually occurs. Based on these actual results, $P(A)$ is *approximated* as follows:

$$P(A) = (\text{number of times } A \text{ occurred})/(\text{number of times the procedure was repeated}).$$

Exercise 2. Last year, at a small college, 50 out of 76 students passed a statistics class. Find the probability that a student at the college this year will pass statistics.

Class Exercise 3. Mr. V rolled a six-sided die 100 times (he was very bored!) and correctly guessed the number 20 times. Using the relative frequency approach, what is the probability he will correctly guess the number when he rolls the die next?

Expressing Probability Results

The probability of event A is a number between 0 and 1. There are several ways to write the numerical value of $P(A)$.

(a) As a reduced fraction

(b) In decimal form. Rounding to two or three digits after the decimal is appropriate for most applications.

(c) As a percent

(d) As an unreduced fraction. This representation displays the number of distinct outcomes of the sample space in the denominator and the number of distinct outcomes favorable to event A in the numerator.

2. Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has n different simple events and that *each of those simple events has an equal chance of occurring.* If event A can occur in s of these n ways, then

$$P(A) = \text{(number of ways } A \text{ can occur)}/\text{(number of different simple events)} = \frac{s}{n}.$$

Exercise 3. What is the probability that a randomly chosen Major League Baseball (MLB) team is from the state of Pennsylvania?

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Remark: Except for simplistic situations such as simple random sampling or flipping balanced coins or rolling fair dice, different outcomes are not usually equally likely.

Example: The chance of winning a lottery - especially lotteries with very large payoffs - is s-mall. Regardless, people continue to buy tickets. In an attempt to understand why, an interviewer asked someone who had just purchased a lottery ticket, "What do you think your chances are of winning the lottery?" The reply was "Oh, about 50-50". The shocked interviewer asked, "How do you get that?" to which the response was, "Well, the way I figure it, either I win or I don't!" The moral of this story is that outcomes are *not always* equally likely.

Class Exercise 4. What is the probability that a randomly chosen person in our class is a girl?

Class Exercise 5. What is the probability that a randomly chosen person in our class graduated from an EUHSD high school?

Class Exercise 6. A family has four children. If each child is equally likely to be a girl or a boy, find the probability that all four children are female. What's wrong with the following logic? A family with 4 children can have 0, 1, 2, 3, or 4 girls. Since there are five possibilities, the probability of 4 girls is 1/5.

3. Subjective Probabilities $P(A)$, the probability of event A, is *estimated* by using knowledge of the relevant circumstances.

Exercise 4. What is the probability that Pete Buttigieg will pick Ted Cruz as his running mate if he wins the 2024 Democratic primary?

Class Exercise 7. What is the probability that Justin Bieber will be elected as the president of the United States in 2024?

Class Exercise 8. What is the probability that the San Francisco 49ers will win the Super Bowl?

Class Exercise 9. Before the first human heart transplant, Dr. Christian Barnard of South Africa was asked to assess the probability that the operation would be successful. Did he need to rely on the relative frequency definition or the subjective definition of probability? Explain.

Class Exercise 10. Is there intelligent life on other planets in the universe? If you are asked to state the probability that there is, would you need to rely on the relative frequency or the subjective definition of probability?

Class Exercise 11. Suppose you toss a coin three times. What is the probability of getting exactly two heads?

Probability Rules:

1. The probability of an impossible event is 0.
2. The probability of an event that is certain to occur is 1.
3. For any event A, the probability of A is between 0 and 1 inclusive. That is $0 \leq P(A) \leq 1$.

Exercise 5. Which of the following values cannot be probabilities: -0.12, 0.93, 0.23, 1.17, and 0.65?

Class Exercise 12. Which of the following values cannot be probabilities: 0.79, 2.87, 0.28, 0.21, and -0.54?

Definition: The **complement** of event A, denoted by A^c, consists of all outcomes in which event A does not occur.

Exercise 6. Suppose you roll a die and let E = the event that "the number is at least 5". What is the complement of E?

Rule of Complementary Events:

$P(A) + P(A^c) = 1$
$P(A^c) = 1 - P(A)$
$P(A) = 1 - P(A^c)$.

Exercise 7. Suppose you have a probability of 0.65 of passing a class. What is the probability of not passing the class?

Class Exercise 13. Suppose you roll a die. What is the probability of not rolling a '5'?

Student Feedback

My teaching methods are (I hope) continually subject to improvement. If you have any comments, suggestions, or ideas, please email them to me at Sithparran.Vanniasegaram@evc.edu.

Homework

C problems

Section 4.1: 15, 17, 19

B problems

Section 4.1: 1, 3

A problems

Section 4.1: 5-13 ODD