Section 5.1

Definition: A variable x is a **random variable** if the value that x takes on in a given experiment or observation is a chance or random outcome.

Definition: A **discrete random variable** can take on only a finite number of values or a countable number of values.

Definition: A **continuous random variable** can take on any of the countless number of values in a line interval.

Class Exercise 1. Identify the given random variable as being discrete or continuous:
(a) The total amount (in ounces) of soft drinks that you consumed in the past year.
(b) The number of cans of soft drinks that you consumed in the past year.
(c) The number of movies currently playing in U.S. theaters.

In Section 5.1, we will only be dealing with discrete random variables.

Definition: A **probability distribution** is an assignment of probabilities to each distinct value of a discrete random variable or to each interval of values of a continuous random variable.

Features of the Probability Distribution of a Discrete Random Variable
1. The probability distribution has probability assigned to *each* distinct value of the random variable.
2. The sum of all the assigned probabilities must be 1.

Exercise 1. If you draw an M & M candy at random from a bag of the candies, the candy you draw will have one of six colors. The probability of drawing each color depends on the proportion of each color among all candies made. The table below gives the probability of each color for a randomly chosen plain M & M:

Color	Brown	Red	Yellow	Green	Orange	Blue
Probability	0.3	0.2	0.2	0.1	0.1	?

What must be the probability of drawing a blue candy?

Class Exercise 2. The probabilities for peanut M& M's are a bit different. Here they are:

Color	Brown	Red	Yellow	Green	Orange	Blue
Probability	0.2	0.2	0.1	0.1	0.1	?

What must be the probability of drawing a blue candy?

Class Exercise 3. All human blood can be typed as one of O, A, B or AB, but the distribution of the type varies a bit with race. Here is the distribution of the blood type of a randomly chosen black American:

Blood type	O	A	B	AB
Probability	0.49	0.27	0.20	?

What is the probability of type AB blood?

Exercise 2. Here is the distribution for the number of children for 100 families in the Evergreen Valley area:

Number of Children	0	1	2	3	4	5	6	7	8
Probability	0.20	0.25	0.35	0.07	0.05	0.03	0.02	0.01	0.02

(a) Find the probability that a randomly chosen family will have seven children.

(b) Find the probability that a randomly chosen family will have at least seven children.

(c) Construct a probability histogram for the distribution.

Class Exercise 4. Here is the distribution for the number of movies watched for 100 San Jose high school students during the last month:

Number of Movies	0	1	2	3	4	5	6
Probability	0.29	0.20	0.15	0.12	0.10	0.08	0.06

(a) Find the probability that a randomly chosen student will have watched five movies.
(b) Find the probability that a randomly chosen student will have watched at least five movies.
(c) Construct a probability histogram for the distribution.

Class Exercise 5. Here is the distribution for the number of glasses of water for 100 Evergreen Valley College students yesterday:

Number of Glasses	0	1	2	3	4	5	6
Probability	0.03	0.01	0.40	0.30	0.20	0.05	0.01

(a) Find the probability that a randomly chosen student has drank one glass of water.
(b) Find the probability that a randomly chosen student has drank at most one glass of water.
(c) Construct a probability histogram for the distribution.

Class Exercise 6. A jury of 12 people is chosen for a trial. The defense attorney claims it must have been chosen in a biased manner, because 50% of the city's adults residents are female yet the jury contains no women.
If the jury were randomly chosen from the population, what is the probability the jury would have
(a) no females?
(b) at least one female?

In an earlier chapter, we discussed measures of center and spread for data. We now discuss measures of center and spread for probability distributions.

Mean for a probability distribution: $\mu = \Sigma\ [x \cdot P(x)]$.

Standard deviation for a probability distribution: $\sigma = \sqrt{\Sigma\ [(x-\mu)^2 \cdot P(x)]}$.

Exercise 3. The probability model below describes the number of repair calls that an appliance repair shop may receive during an hour.

Repair Calls	0	1	2	3
Probability	0.1	0.3	0.4	0.2

(a) What is the mean number of calls?

(b) What is the standard deviation?

Class Exercise 7. A commuter must pass through five traffic lights on her way to work and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown below.

x = number of red	0	1	2	3	4	5
$P(x)$	0.05	0.25	0.35	0.15	0.15	0.05

(a) What is the mean number of red lights?
(b) What's the standard deviation?

Class Exercise 8. In an experiment on the behavior of young children, each subject is placed in an area with five toys. The response of interest is the number of toys that the child plays with. Past experiments with many subjects have shown the probability distribution of the number x of toys played with is as follows:

x = number of toys	0	1	2	3	4	5
$P(x)$	0.03	0.16	0.30	0.23	0.17	0.11

Calculate the mean and the standard deviation.

Exercise 4. Let the random variable x represent the number of heads in three tosses of a coin.

(a) Construct a table describing the probability distribution.
The sample space consists of the following outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

<u>**Definition**</u>: The range of a random variable x is the possible values that x can take on.

The range of X =

For $x = 0$, the outcome is

For $x = 1$, the outcomes are

For $x = 2$, the outcomes are

For $x = 3$, the outcome is

x = number of heads	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

(b) Find the mean and standard deviation.

The mean =

The standard deviation ≈

Class Exercise 9. Let the random variable x represent the number of heads in four tosses of a coin.
(a) Construct a table describing the probability distribution.
(b) Find the mean and standard deviation.
(c) It is unusual to get four tails?

Class Exercise 10. Let the random variable x represent the number of sixes in two rolls a die.
(a) Construct a table describing the probability distribution.
(b) Find the mean and standard deviation.
(c) It is unusual to get two sixes?

Exercise 5. Suppose you are walking to class one day and a random stranger with a die in his hand walks up to you and makes an offer. If he rolls a die and it lands on a 6, you will receive 9 dollars. If the die lands on a different number, you lose 2 dollars. Would you take that offer?

<u>Definition</u>: The <u>**expected value**</u> of a discrete random variable is denoted by μ, and it represents the mean value of the outcomes. It is obtained by finding the value of $\Sigma\ [x \cdot P(x)]$.

$$\mu = \Sigma\ [x \cdot P(x)].$$

<u>Remark</u>: Intuitively, the expected value of a random variable can be thought of as the long term limiting average of its values in independent repeated experiments.

Exercise 6. Let's go back to the stranger and the die situation. What is μ, the expected value?

To find μ, we need to use the formula for expected value above. We must establish the possible values and their probabilities:

Using the intuitive definition of expected value from the remark preceding this exercise, if you play with the stranger for a long enough period of time, you would expect to
on average.

Exercise 7. Would you rather have a probability of 1/2 of getting 2.2 million dollars (versus 0 dollars) or a probability of 1 of getting 1 million dollars?

We will find μ for both offers. Let's start with first offer.

Now, let's find the expected value for the second offer.

Class Exercise 11. The casino game of American Roulette consists of a circular wheel with 38 spots. Thirty six of the spots are numbered 1, 2, 3,......., 36. Eighteen of these numbers are colored black and the other eighteen are colored red. The remaining two spots are colored green with no number assignment. If you play a color (either red or black), the casino will pay you even odds. That is, you play black and wager \$1, you will profit \$1 if black comes up on that particular spin of the wheel. How much is money is expected to be won or loss after one spin?

Class Exercise 12. You roll a die. If it comes up a 6, you win \$100. If not, you get to roll again. If you get a 6 the second time, you win \$50. If not, you lose. Find the expected amount you'll win.

Class Exercise 13. An insurance policy costs \$100 and will pay policyholders \$10,000 if they suffer a major injury (resulting in hospitalization) or \$3,000 if they suffer a minor injury (resulting in lost time from work). The company estimates that each year 1 in every 2000 policyholders may have a major injury, and 1 in 500 a minor injury. What's the expected profit on this policy?

Homework

C problems

Section 5.1: 7, 11-17 ODD

B problems

Section 5.1: 1-5 ODD

A problems

Section 5.1: 9

Section 5.2

In the last class, we learned about discrete probability distributions. Today, we learn about a special type of discrete probability distribution: the binomial probability distribution.

Definition: A **binomial probability distribution** results from a procedure that meets all the following requirements:

1. The procedure has a *fixed number of trials.*

2. The trials must be *independent.* (The outcome of any individual trial doesn't affect the probabilities in the other trials.)

3. Each trial must have all outcomes classified into two categories (commonly referred to as *success* or *failure*).

4. The probability of a success remains the same in all trials.

Independence Requirement: If calculations are cumbersome and if a sample size is no more than 5% of the size of the population, treat the selections as being *independent* (even if the selections are made without replacement, so that they are technically dependent).

Exercise 8. Determine whether or not the given procedure results in a binomial distribution. For those that are not binomial, identify at least one requirement that is not satisfied.

(a) Treating 152 couples with the YSORT gender selection method developed by the Genetics IVF Institute and recording the ages of the parents.

(b) Five hundred different New York City voters are randomly selected from the population of 2.8 million registered voters, and each is asked if he or she is a Democrat.

Class Exercise 14. Determine whether or not the given procedure results in a binomial distribution.
(a) Treating 863 subjects with Lipitor (Atorvastatin) and asking each subject "How does your head feel?" (based on data from Pfizer, Inc.).
(b) Two hundred statistics students are randomly selected (out of 200,000 students) and each is asked if he or she owns a TI-84 Plus calculator.

You can find binomial probabilities using the binompdf function on your calculator. binompdf(n,p,x) gives the probability of x successes in n trials with p being the probability of getting a success. You can find the binompdf() function on your calculator by hitting 2nd DISTR.

Exercise 9. A certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others.

If she serves 6 times, what's the probability that she gets exactly 4 serves in?

Class Exercise 15. Assume that 4% of people have type AB blood. If we select 9 people at random, find the probability that there are three people with type AB blood.

Exercise 10. Assume that a procedure yields a binomial distribution with a trial repeated 14 times. Use the binompdf function to find the probability of 4 successes given the probability 0.3 of success on a single trial.

Class Exercise 16. Assume that a procedure yields a binomial distribution with a trial repeated 12 times. Use the binompdf function to find the probability of 7 successes given the probability 0.8 of success on a single trial.

Class Exercise 17. Assume that a procedure yields a binomial distribution with a trial repeated 8 times. Use the binompdf function to find the probability of 2 successes given the probability 0.01 of success on a single trial.

Exercise 11. What is the probability of getting at least 6 heads in 9 coin tosses?

First, let's establish the random variable.

Let X = the number of heads in 9 coin tosses.

Remark: $\mathrm{P}(X \geq x) = \mathrm{P}(X = x) + \mathrm{P}(X = x+1) + \cdots + \mathrm{P}(X = n)$.

Remark: If you have not purchased your TI-83+ or TI-84+ calculator, please do so. Also, make sure you bring your calculator to class from now on.

Exercise 12. What is the probability of getting at least 1 head in 9 coin tosses?

There are two methods to doing this problem: the hard way and the easy way.

First, we will approach the problem the hard way:

Doing it this way takes a long time. Now, we'll try the easy way.

You can find binomial cumulative probabilities using the binomcdf function on your calculator.

Exercise 13. Assume that a procedure yields a binomial distribution with a trial repeated 14 times. Use the binomcdf function to find the probability of getting at most 4 successes given the probability 0.3 of success on a single trial.

Class Exercise 18. Assume that a procedure yields a binomial distribution with a trial repeated 12 times. Use the binomcdf function to find the probability of at most 7 successes given the probability 0.8 of success on a single trial.

Class Exercise 19. Assume that a procedure yields a binomial distribution with a trial repeated 8 times. Use the binomcdf function to find the probability of at most 2 successes given the probability 0.01 of success on a single trial.

Class Exercise 20. About 8% of males are colorblind. If we select 8 people at random, find the probability that there are at most six people who are colorblind.

Homework

C problems

Section 5.2: 7, 11-29 ODD

B problems

Section 5.2: 1, 3

A problems

Section 5.2: 5-9 ODD, 29, 31

Section 5.3

Exercise 14. A certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others. If she serves six times, what is the probability that she doesn't get exactly 4 serves in?

By the Complement Rule, P(not getting 4 serves in)

Class Exercise 21. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability that there are exactly three lefties in the group. Also, find the probability that the number of lefties is not three.

Class Exercise 22. An Olympic archer is able to hit the bull's eye 80% of the time. Assume each shot is independent of the others. If she shoots 6 arrows, find the probability that there are exactly 4 bull's-eyes in the group. Also, find the probability that the number of bull's-eyes is not four.

Mean of a Binomial Distribution: $\mu = np$

Variance of a Binomial Distribution: $\sigma^2 = npq$, where q = 1 - p.

Standard Deviation of a Binomial Distribution: $\sigma = \sqrt{npq}$.

Exercise 15. Assume that 10% of the United States residents live in California. Suppose we selected 20 United States residents at random. Find the mean and standard deviation of the number of California residents in the group.

Class Exercise 23. Suppose that 75% of all drivers always wear seatbelts. Suppose we select 30 drivers at random. Find the mean and standard deviation of the number of drivers who wear seatbelts.

Class Exercise 24. Suppose that a computer chip manufacturer rejects 2% of the chips produced because they fail presale testing. We select 80 chips at random. Find the mean and standard deviation of the chips that are rejected.

Homework

C problems

Section 5.3: 9(b), 11(d), 11(e), 13(b), 15(b), 17(b), 21(a), 21(b), 21(c), 23(a), 23(b), 23(c), 25(a), 25(b)

B problems

Section 5.3: 1