Section 7.1

Exercise 1. Let's suppose we wanted to find the average amount of time Evergreen Valley College students spend watching baseball per week. Since it is impractical to ask every single student at EVC for this piece of information, we decide to take a simple random sample of 64 students. The average time for these 64 students is 4 hours. It is also known that the standard deviation for the **entire college** is 1 hour.

(a) How likely is it that the average for the entire college is exactly 4 hours?

It is that the average is **exactly** four hours.

(b) How likely is it that the average is "close" to 4 hours?

This depends on Since the sample size , it is that 4 hours is "close".

Definition: A **confidence interval** (or **interval estimate**) is a range (or an interval) of values used to estimate the true value of a population parameter. A confidence interval is sometimes abbreviated as CI.

Example: An example of a confidence interval for the mean baseball watching time would be (3, 5).

Exercise 2. What is another example of a confidence interval?

Definition: For a confidence level c, the **critical value** z_c is the number such that the area under the standard normal curve between -z_c and z_c equals c.

Exercise 3. Find the critical value z_c that corresponds to a 95% confidence level.

Class Exercise 1. Find the critical value z_c that corresponds to a 90% confidence level. **Answer: 1.645**

Class Exercise 2. Find the critical value z_c that corresponds to a 98% confidence level. **Answer: 2.326**

Definition: A c **confidence interval for** μ is an interval computed from sample data in such a way that c is the probability of generating an interval containing the actual value of μ. In other words, c is the proportion of confidence intervals, based on random samples of size n, that actually contain μ.

A 95% **confidence interval for** μ is:

$$(\bar{x} - 1.959963986 \cdot \sigma/\sqrt{n},\ \bar{x} + 1.959963986 \cdot \sigma/\sqrt{n}).$$

The above interval is typically written the following way:

$$\bar{x} \pm 1.959963986 \cdot \sigma/\sqrt{n} \qquad (*).$$

Remark: By the class exercise above, 95% of all 95% confidence intervals contain the population mean.

Exercise 4. Let's suppose we wanted to find the average amount of time De Anza College students spend watching baseball per week. Since it is impractical to ask every single student at De Anza for this piece of information, we decide to take a simple random sample of 64 students. The average time for these 64 students is 4 hours. It is also known that the standard deviation for the **entire college** is 1 hour. Construct a 95% confidence interval for the population mean.

By (*), the 95% confidence interval is

Substituting $\bar{x} =$, $\sigma =$, and $n =$ yields

$$4 \pm 1.959963986 \cdot 1/\sqrt{64}.$$

The last expression simplifies to The 95% confidence interval for the average amount of time De Anza College students spend watching baseball per week is

Confidence Interval for Population Mean

The confidence interval for estimating a population mean is:

$$\bar{x} \pm z_c \cdot \frac{\sigma}{\sqrt{n}}.$$

The margin of error is $E = z_c \frac{\sigma}{\sqrt{n}}$.

Here are the three requirements of a sample in order to construct a confidence interval:

1) The sample is a simple random sample.
2) The value of the population standard deviation σ is known.
3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Exercise 5. Suppose you wanted to estimate the mean SAT Math score for the more than 385,000 high school seniors in California. Only about 49% of California students take the SAT. These self-selected seniors are planning to attend college and so are not representative of all California seniors. You know better than to make inferences about the population based on the sample data. At considerable effort and expense, you give the test to a simple random sample (SRS) of 500 California high school seniors. The mean for your sample is $\bar{x} = 461$. Assume that $\sigma = 100$. Construct and interpret a 99% confidence interval for the mean score μ in the population of all 385,000 seniors.

Are the three requirements satisfied?

The first step is to find the margin of error. To do that, we need to find z_c.

So, $z_c =$

Applying the formula,

$E = z_c \frac{\sigma}{\sqrt{n}} =$

Again, the confidence interval is ($\bar{x}$ - E, $\bar{x}$ + E).

So, the confidence interval is

We are 99% confident that the population mean is in the interval:

Class Exercise 3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. As part of your study, you randomly select 40 repair costs and find the mean to be $120.00. The population standard deviation is $17.50. Construct and interpret a 95% confidence interval for the population mean repair cost. **Answer: (114.58, 125.42)**

Class Exercise 4. Hoping to lure more shoppers downtown, a city builds a new public parking garage in the central business district. The city plans to pay for the structure through parking fees. During a two-month period (44 weekdays), daily fees collected average $126. Assume σ = $15. Write and interpret a 90% confidence interval for the mean daily income this parking garage will generate. **Answer: (122.28, 129.72)**

Exercise 6. A publisher wants to estimate the mean length of time (in minutes) all adults spend reading newspapers. To determine the estimate, the publisher takes a random sample of 15 people and obtains the following results: 11, 9, 8, 10, 10, 9, 7, 11, 11, 7, 6, 9, 10, 8, and 10. From past studies, the publisher assumes σ is 1.5 minutes and that the population of times is normally distributed. Construct and interpret a 90% confidence interval for the mean length of time.

The first step is to find the margin of error. To do that, we need to find z_c.

So, z_c =

$\bar{x}$ =

Applying the formula,

$E = z_c \frac{\sigma}{\sqrt{n}} =$

Again, the confidence interval is ($\bar{x}$ - E, $\bar{x}$ + E).

So, the confidence interval is

We are 90% confident that the population mean is in the interval: .

Class Exercise 5. A manufacturer of high-resolution video terminals must control the tension on the mesh of fine wires that lies behind the surface of the viewing screen. Too much tension will tear the mesh and too little will allow wrinkles. The tension is measured by an electrical device with output readings in millivolts (mV). Some variation is inherent in the production process. Careful study has shown that when the process is operating properly, the standard deviation of the tension readings is σ = 43 mV. Here are the tensions readings from an SRS of 20 screens from a single day's production: 269.5, 297, 269.6, 283.3, 304.8, 280.4, 233.5, 257.4, 317.5, 327.4, 264.7, 307.7, 310.0, 343.3, 328.1, 342.6, 338.8, 340.1, 374.6, and 336.1. Construct and interpret a 90% confidence interval for the mean tension μ of all the screens produced on this day. **Answer: (290.5, 322.14)**

Formula for Sample Size

$$n = (\frac{z_c \sigma}{E})^2,$$

where
E = specified maximal margin of error
σ = population standard deviation
z_c = critical value from the normal distribution for the desired confidence level c

Exercise 7. Suppose x has a normal distribution with $\sigma = 1.2$. Find the minimal sample size required so that for a 90% confidence interval, the maximal margin of error is $E = 0.5$. (#14)

Class Exercise 6. Overproduction of uric acid in the body can be an indication of cell breakdown. This may be an advance indication of illness such as gout, leukemia, or lymphoma. Over a period of months, an adult male patient has taken eight blood tests for uric acid. The mean concentration was $\bar{x} = 5.35$ mg/dl. The distribution of uric acid in healthy adult males can be assumed to be normal, with $\sigma = 1.85$ mg/dl. Find the sample size necessary for a 95% confidence level with maximal margin of error $E = 1.10$ for the mean concentration of uric acid in this patient's blood. **Answer:** n = **11** (#16)

Class Exercise 7. What price do farmers get for their watermelon crops? In the third week of July, a random sample of 40 farming regions gave a sample mean of $\bar{x} = \$6.88$ per 100 pounds of watermelon. Assume that σ is known to be \$1.92 per 100 pounds. Find the sample size necessary for a 90% confidence level with maximal margin of error $E = 0.3$ for the mean price per 100 pounds of watermelon. **Answer:** n = **111**

Homework

C problems

Section 7.1: 11(a), 11(b), 13, 15(a), 15(b), 17(a), 17(b), 19(a), 19(b), 19(c), 19(d), 21(a), 21(b), 21(c), 21(d)

B problems

Section 7.1: 1-7 ODD

A problems

Section 7.1: 9, 11(c), 15(c), 17(c), 19(e), 21(e)