Section 7.2

Assume that x has a normal distribution with mean μ. For samples of size n with sample mean $\bar{x}$ and sample standard deviation s, the **t variable**

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

has a **Student's t distribution** with **degrees of freedom d.f.** $= n - 1$.

Properties of a Student's t Distribution
1. The distribution is *symmetric* about the mean 0.
2. The distribution depends on the *degrees of freedom, d.f.* (d.f. $= n - 1$ for μ confidence intervals).
3. The distribution is *bell-shaped*, but has thicker tails than the standard normal distribution.
4. As the degrees of freedom increase, the t distribution *approaches* the standard normal distribution.
5. The area under the entire curve is 1.

In Chapter 5, we learned about the normal distribution. In this section, we will learn about the Student's t-distribution. From now on, I will use t-distribution for short. The following is the equation for the t-distribution:

$$f(x) = \frac{\gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1 + \frac{x^2}{n})^{-(\frac{n+1}{2})},$$

where Γ is the Gamma function and n is a positive integer.

The family of t-distribution curves is similar to the normal distribution curve. As n increases, the t_{n-1} curve looks more and more like the normal curve.

We can find the area under a t-distribution curve using the $t$cdf function on the calculator.

Calculator Function: $t$cdf(a, b, n) gives the area under the t_n curve between a and b.

Exercise 1. (a) Suppose $W \sim t_4$. Find P($W > 5$).

Here is an illustration:

P($W > 5$) = the area under the

P($W > 5$) =

(b) Suppose $R \sim t_6$. Find P(-0.43 $< R <$ 0.68).

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Class Exercise 1. Suppose $W \sim t_8$. Find P($W < 2$). **Answer: 0.960**

Class Exercise 2. Suppose $Y \sim t_{25}$. Find P($0.12 < Y < 1.74$). **Answer: 0.406**

Remember how we used the invNorm function to find percentiles for the normal distribution? There is a similar function for the t-distribution, the invT function.

Definition: Let y = invT(a,n). a is the area to the left of y under the t_n curve.

Exercise 2. (a) Find invT(.975,5).

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(b) Suppose $W \sim t_5$. Find the value of a such that P(W < a) = 0.34.

Class Exercise 3. Suppose $X \sim t_7$. For what value of a is $\mathrm{P}(X > a) = 0.34$? **Answer:** $a =$ **0.430**

Exercise 3. A sample of size $n = 5$ is a simple random sample selected from a normally distributed population. Find the critical value t_c corresponding to a 90% confidence level.

Class Exercise 4. A sample of size $n = 8$ is a simple random sample selected from a normally distributed population. Find the critical value t_c corresponding to a 99% confidence level. **Answer: 3.499**

Class Exercise 5. A sample of size $n = 15$ is a simple random sample selected from a normally distributed population. Find the critical value t_c corresponding to a 95% confidence level. **Answer: 2.145**

<u>Formula</u>: Suppose the mean of a population is μ and the sample standard deviation is s. The confidence interval for estimating a population mean is:

$$\bar{x} \pm t_c \cdot \tfrac{s}{\sqrt{n}}.$$

Exercise 4. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. As part of your study, you randomly select 40 repair costs and find the mean to be \$120.00. (Assume that repair costs are normally distributed.) The sample standard deviation is \$17.50. Construct a 95% confidence interval for the population mean repair cost.

Class Exercise 6. In a random sample of 60 refrigerators, the mean repair cost was \$150.00 and the standard deviation was \$15.50. (Assume the repair costs are normally distributed.) Construct a 99% confidence interval for the population mean repair cost. **(144.67, 155.33)**

Class Exercise 7. A random sample of forty-eight 200-meter swims has a mean time of 3.12 minutes. (Assume that $\sigma = 0.09$ and that swim-times are normally distributed.) Construct a 95% confidence interval for the population mean repair cost. **Answer: (3.0945, 3.1455)**

Homework

C problems

Section 7.2: 1, 3, 11(a), 11(b), 13(a), 13(b), 15(a), 15(b), 17(a), 17(b)

B problems

Section 7.2: 5, 11(c), 13(c), 15(c), 17(c)

A problems

Section 7.2: 7, 9, 19