Section 7.4

Definition: Two samples are **independent** if sample data drawn from one population are completely unrelated to the selection of sample data from the other population.

Definition: Two samples are **dependent** if each data value in one sample can be paired with a corresponding data value in other sample.

Formula: Here is the confidence interval for the difference of two means (independent samples and known standard deviations):

$$(\bar{x}_1 - \bar{x}_2) \pm z_c \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

Requirements
1. Both population distributions are normal or $n_1 \geq 30$ and $n_2 \geq 30$.
2. σ_1 and σ_2 are known.

Exercise 1. Consider two independent distributions that are mound-shaped. A random sample of size $n_1 = 36$ from the first distribution showed $\bar{x}_1 = 15$ and a random sample of size $n_2 = 40$ from the second distribution showed $\bar{x}_2 = 14$. Given $\sigma_1 = 3$ and $\sigma_2 = 4$, find a 95% confidence interval for $\mu_1 - \mu_2$.(#8)

Class Exercise 1. "Parental Sensitivity to Infant Cues: Similarities and Differences Between Mothers and Fathers" by M.V. Graham (*Journal of Pediatric Nursing*, Vol. 8, No. 6) reports a study of parental empathy for sensitivity cues and baby temperament (higher scores mean more empathy). Let x_1 be a random variable that represents the score of a mother on an empathy test (as regards her baby). Let x_2 be the empathy score of a father. A random sample of 32 mothers gave a sample mean of $\bar{x}_1 = 69.44$. Another random sample of 32 gave $\bar{x}_2 = 59$. Assume that $\sigma_1 = 11.69$ and $\sigma_2 = 11.60$. Let μ_1 be the population mean of x_1 and let μ_2 be the population mean of x_2. Find a 99% confidence interval for $\mu_1 - \mu_2$. **Answer:** $(2.941, 17.939)$

Formula: Here is the confidence interval for the difference of two means (independent samples and unknown standard deviations):

$$(\bar{x}_1 - \bar{x}_2) \pm t_c \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Requirements

1. Both population distributions are normal or $n_1 \geq 30$ and $n_2 \geq 30$.
2. Sample standard deviations are known.

Exercise 2. The mean daily lodging cost for 35 families traveling in North Carolina is 131 dollars and the standard deviation is 26 dollars. The mean daily lodging cost for 35 families traveling in South Carolina is 136 dollars and the standard deviation is 19 dollars. Construct a 90% confidence interval for the difference between the mean daily lodging cost for North Carolina and the mean daily lodging cost for South Carolina.

Class Exercise 2. The mean ACT score for 43 male high school students is 21.1 and the standard deviation is 5.0. The mean ACT score for 56 female high school students is 20.9 and the standard deviation is 4.7. Construct a 95% confidence interval for the difference in mean ACT score between males and females.**Answer:** $(-1.794, 2.194)$

Formula: Here is the confidence interval estimate of the difference p_1 - p_2 is

$$\hat{p}_1 \text{ - } \hat{p}_2 \pm z_c \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}.$$

Requirements

1. Two independent binomial experiments.
2. $n_1\hat{p}_1 > 5$, $n_1\hat{q}_1 > 5$, $n_2\hat{p}_2 > 5$, $n_2\hat{q}_2 > 5$

Exercise 3. In 1995, 24.8% of 550 white adults surveyed reported that they smoked cigarettes, while 25.7% of the 550 black adults surveyed were smokers. Create a 90% confidence interval for the difference in the percentage of smokers among black and white American adults.

Class Exercise 3. In October 2000 the U.S. Department of Commerce reported the results of a large-scale survey on high school graduation. Researchers contacted more than 25,000 Americans age 24 years to see if they had finished high school; 84.9% of the 12,460 males and 88.1% of the 12,768 females indicated that they had high school diplomas. Create a 99% confidence interval for the difference in graduation rates between males and females. **Answer:** $(-0.0431, -0.0209)$

Homework

C problems

Section 7.4: 7(a)-(e), 9(a), 9(b), 11(a), 11(b), 13(a), 13(b), 15(a), 15(b), 17(a), 17(b), 19(a), 19(b)

B problems

Section 7.4: 1, 7(f), 9(c), 11(c), 13(c), 15(c), 17(c), 19(c)

A problems

Section 7.4: 3, 5, 27