Section 8.2

Requirements to test μ when σ is known

Let x be a random variable appropriate to your application. Obtain a simple random sample (of size n) of x values from which you compute the sample mean $\bar{x}$. The value of σ is already known (perhaps from a previous study). If you can assume that x has a normal distribution, then any sample size n will work. If you cannot assume this, then use a sample size $n \geq 30$.

Exercise 1. A consumer protection group is interested in testing a brand of cereal. They suspect, on average, the cereal boxes contain less than the advertised mean weight of 18 ounces of cereal. A random sample of thirty cereal boxes is weighed producing an average weight of 17.7 ounces. It is known that $\sigma = 0.8$ ounces. Does the consumer protection group have a right to be suspicious?

Let $\mu =$

H_0:

H_1:

Let $X =$

Let's suppose

By the Central Limit Theorem for Means,

In words, the p-value is the probability of obtaining a sample mean as smaller or smaller than 17.7 if the population mean is 18.

Since 0.05, we reject the null hypothesis.

There is to suggest that the consumer protection group has a right to be suspicious.

Exercise 2. Do middle-aged male executives have different average blood pressure than the general population? The National Center for Health Statistics reports that the mean systolic blood pressure for males 35 to 44 years of age is 128. The medical director of a company looks at the medical records of 72 company executives in this age group (chosen at random) and finds that the mean systolic blood pressure in this sample is $\bar{x} = 126.07$. Is this enough evidence that executive blood pressures differ from the national average? Assume that the standard deviation for all middle aged executives is 15.

Let $\mu =$

H_0:

H_1:

Let X =

Let's suppose

By the Central Limit Theorem for Means,

Unlike in the previous exercise, the alternative hypothesis in this case is two-sided. Therefore, we want to find the probability of getting a sample mean as extreme or more extreme as 126.07 if H_o is true. Since 126.07 is 1.93 away from 128, we are looking for the probability of obtaining a sample mean that is at least 1.93 away from 128 if H_o is true. This is equal to the probability of obtaining a sample mean of at most 126.07 or a sample mean of at least 129.93 if H_o is true. In summary,

Since the normal distribution is symmetric,

Here is an illustration of the above fact:

Since 0.05, we reject the null hypothesis.

There is to suggest that the middle-age executives have a different blood pressure than the general population.

Class Exercise 1. The mean yield of corn in the United States is about 120 bushels per acre. A survey of 40 farmers this year gives a sample mean of 123.8 bushels per acre. We want to know whether this is good evidence that the national mean this year is not 120 bushels per acre. Assume that the farmers surveyed are an SRS from the population of all commercial corn growers and that the standard deviation of the yield in this population is $\sigma = 10$ bushels per acre. Is there enough evidence from the sample to suggest that the population mean is not 120 bushels per acre? **Answer: p-value = 0.0162**

Class Exercise 2. A student measured the sitting heights of 36 male classmate friends, and she obtained a mean of 92.8 cm. The population of males has sitting heights with a mean of 91.4 cm. Assume $\sigma = 3.6$ cm. Use a 0.05 significance level to test the claim that males at her college have a mean sitting height different from 91.4 cm. Is there anything about the sample data suggesting that the methods of this section should not be used? **Answer: p-value = 0.0196**

Class Exercise 3. A simple random sample of 40 salaries of NCAA football coaches in the NCAA has a mean of $415,953. The standard deviation of all salaries of NCAA football coaches is $463,364. Use a 0.05 significance level to test the claim that the mean salary of a football coach in the NCAA is less than $500,000. **Answer: p-value =0.126**

Requirements to test μ when σ is unknown

Let x be a random variable appropriate to your application. Obtain a simple rand sample (of size n) of x values from which you compute the sample mean $\bar{x}$ and the sample standard deviation s. If you can assume that x has a normal distribution or simply a mound-shaped and symmetric distribution, then any sample size n will work. If you cannot assume this, use a sample size $n \geq 30$.

Exercise 3. A consumer protection group is interested in testing a brand of cereal. It is known that the distribution of the brand of cereal follows the normal distribution. They suspect that, on average, the cereal boxes contain less than the advertised mean weight of 18 ounces of cereal. A random sample of twenty cereal boxes is weighed producing an average weight of 17.8 ounces and a sample standard deviation of 1 ounce. (We don't know the standard deviation for the population of cereal boxes.) Is the consumer protection group right?

Let $\mu =$

H_0:

H_1:

Let $X =$

Since we don't know , we must use the t-distribution for this problem.

Under H_0, $\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$.

Since 0.05, we reject H_o. There is to suggest that the consumer protection group is right.

Exercise 4. The amount of lead in a certain type of soil, when released by a standard extraction method, averages 86 parts per million (ppm). A new extraction method is tried on 17 specimens of the soil, yielding a mean of 83 ppm lead and a standard deviation of 6 ppm. Is there significant evidence at the 1% level that the new method frees less lead from the soil?

Let μ =

H_0:

H_1:

Let X =

Since 0.01, we reject H_o. There is to suggest that the new method frees less lead from the soil.

Class Exercise 4. A microwave oven repairer says that the mean repair cost for damaged microwave ovens is less than 100 dollars. You work for the repairer and want to test this claim. You find that a random sample of 5 microwave ovens has a mean repair cost of 75 dollars and a standard deviation of \$12.50. At $\alpha = 0.01$, do you have enough evidence to support the repairer's claim? **Answer:** p**-value = 0.00553 or 0.00525**

Class Exercise 5. An environmentalist estimates that the mean waste recycled by adults in the United States is more than 1 pound per person per day. You want to test this claim. You find that the mean waste recycled per person per day for a random sample of 12 adults in the United States is 1.46 pounds and the standard deviation is 0.28 pounds. At $\alpha = 0.05$, can you support the claim? **Answer:** p**-value = 7.002**$\cdot 10^{-5}$

Exercise 5. An employment information service claims the mean annual pay for full-time male workers over age 25 and without a high school diploma is 25,000 dollars. The annual pay (in thousands of dollars) for a random sample of 10 full-time male workers without a high school diploma is listed below:

26.2 23.8 22.4 25.2 26.3 20.8 30.8 29.5 24.6 29.0

At $\alpha = 0.05$, test the claim that the mean salary is 25,000 dollars.

Let μ =

H_0:

H_1:

Let X =

The sample mean is

$$\$1000 \cdot \frac{26.2+23.8+22.4+25.2+26.3+20.8+30.8+29.5+24.6+29}{10} =$$

Since we don't know σ, we will have to calculate the sample standard deviation. Here is the formula for the sample standard deviation:

$$s = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+.....+(x_n-\bar{x})^2}{n-1}}.$$

I already calculated the sample standard deviation, it is

Unlike in the previous two exercises, the alternative hypothesis in this case is two-sided. Therefore, we want to find the probability of getting a sample mean as extreme or more extreme as \$25,860 if H_o is true. This is equal to the probability of obtaining a sample mean of at least \$25,860 or a sample mean of at most \$24,140 if H_o is true.

Since 0.05, we reject H_o. There is to suggest that the average salary is not \$25,000.

Class Exercise 6. A restaurant association says that the typical household in the United States spends a mean of \$2634 per year on food away from home. You are a consumer reporter for a national publication and want to test this claim. You randomly select 8 U.S. households and find out how much each spend on food away from home per year. Here are the results (in dollars): 3013, 1724, 1949, 3516, 2475, 2767, 2231, and 4512. Can you reject the restaurant association's claim at $\alpha = 0.02$? **Answer: p-value = 0.678**

Class Exercise 7. The National Highway Traffic Safety Administration conducted crash tests of child booster seats for cars. Listed below are results from those tests, with the measurements given in hic (standard *head injury condition* units). The safety requirement is that hic measurement should be less than 1000 hic. Use a 0.01 significance level to test the claim that the sample is from a population with a mean less than 1000 hic. Do the result suggest that all of the child booster seats meet the specified requirement?

774	649	1210	546	431	612

Answer: p-value = 0.022

Class Exercise 8. A simple random sample of pages from *Merriam-Webster's Collegiate Dictionary, 11th edition,* is obtained. Listed below are the number of words defined on those pages. Given that this dictionary has 1459 pages with defined words, the claim that there are more than 70,000 defined words is the same as the claim that the mean number of defined words on a page is greater than 48.0. Use a 0.05 significance level to test the claim the mean number of defined words on a page is greater than 48.0. What does the result suggest about the claim that there are more than 70,000 defined words in the dictionary?

51	63	36	43	34	62	73	39	53	79

Answer: p-value = 0.1562

Homework

C problems

Section 8.2: 7-21 ODD

B problems

Section 8.2: 1, 3

A problems

Section 8.2: 5, 23