Math 1A Final Review
Part 1

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The Fundamental Theorems of Calculus

Refer to section 5.3 for the following section

Theorem 1 (FTC Part 1)
Assume that you have a function $f(x) = \int_a^x g(t)\,\mathrm{d}t$*, then* $f'(x) = g(x)$

This is very simple in the case where the upper limit of the integrand is a simple variable (i.e. x). However, it seems like a lot of students get confused when the limit is more involved.

Example 1

Find the derivative of $f(x) = \int_0^x \frac{1+t^3}{t^2-5t}\,\mathrm{d}t$.

In this case it is as simple as substituting an x wherever you see t. Fun, right ...

Example 2

Find the derivative of $f(x) = \int_0^{x^3} \frac{1+t^3}{t^2-5t}\,\mathrm{d}t$.

This is more involved. The first step here is to understand that you want to make your integral look similar to the in the theorem. In particular, you want to make the limit of you integral a variable with a coefficient of one and raised to the first power. The simplest way to do this is to make a substitution; in this case let $u = x^3$.

Now comes the tricky part: You are able to substitute 'u' into the integrand, but because you are now differentiating with respect to a new variable, you must use the chain rule on your substituion.

$$f'(x) = \frac{d}{\mathrm{d}u} \left[\int_0^u \frac{1+t^3}{t^2-5t} \,\mathrm{d}t \right] \frac{\mathrm{d}u}{\mathrm{d}x} \tag{1}$$

Notationally, this might seem confusing, but it is very simple. Applying the first differential operator to the brackets is a matter of repeating the same steps as in Example 1 (subsituting t's directly for u). The only difference is that now you have to make sure that you multiply that result by $\frac{\mathrm{d}u}{\mathrm{d}x}$, which in this case is $3x^2$. If you are having trouble understanding how to calculate $\frac{\mathrm{d}u}{\mathrm{d}x}$, review the section on differentials.

$$f'(x) = \left[\frac{1+u^3}{u^2-5u} \right] \frac{\mathrm{d}u}{\mathrm{d}x} = \left[\frac{1+x^9}{x^6-5x^3} \right] 3x^2$$

Theorem 2 (FTC Part 2)
Let $F(x)$ be the antiderivative of $f(x)$. Then $\int_b^a f(x)\mathrm{d}x$ is $F(b) - F(a)$.

Hopefully this is straightforward. It will be the basis for most of the integration that you encounter in Math 1B.

Hope that helps kiddies. Cheers!