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ScienceDirect

Applied Mathematics and Computation 180 (2006) 559–568

APPLIED MATHEMATICS AND COMPUTATION

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Selecting two different defective coins ☆

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Abstract

In this paper, given a balance scale and the information that there are exactly two different defective coins present, the authors consider the problem of ascertaining the minimum number of testing which suffice to determine the two different defective coins in a set of λ coins in same appearance, and here $\lambda \geqslant 3$. A testing algorithm for all the possible values of λ is constructed, and the testing algorithm needs at most one testing step more than the optimal testing algorithm.

Keywords: Defective coin; Standard coin; Group testing; Information-theoretic lower bound; Information feedback

1. Introduction

Group testing is a problem of optimization and it has strong practical background, it also belong to dynamic programming. Today, group testing has been widely applied in industry application such as printed circuit board test [1], image compression [2], pattern recognition [3], screen sizing [4], etc. A common phenomenon in group testing theory is that while it is often straightforward to find an optimal testing algorithm for m defective coins, it is immensely more difficult to search the optimal testing algorithm for m defective coins. Cairns in [5] and Tošić in [6], studied the group testing problem of identifying two uniform defective coins, and proposed a testing algorithm respectively. But neither of them related to the group testing problem of identifying or selecting two different defective coins. Hwang only studied the basic model of identifying two different defective coins and proposed a testing algorithm in [3], while leaving the group testing problem of the other models of identifying two different defective coins open. In this paper, the authors studied the group testing problem of identifying two different defective coins in a set of λ coins. A testing (weighing) algorithm for all the possible values of λ would be constructed, and here $\lambda \geqslant 3$, $\lambda \in N$. Let $I^{\lambda}_{D_{C(2)}}(p)$ denote the minimum number of testing of identifying two different defective coins and $S^{\lambda}_{D_{C(2)}}(p)$ denote the minimum number of

☆ Project supported by National Natural Science Foundation of China, Grant No. 60574075.
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0096-3003/$ - see front matter © 2006 Published by Elsevier Inc.
doi:10.1016/j.amc.2005.12.043

testing of selecting two different defective coins in a set of λ coins with a balance scale, and p denote a random testing algorithm, $D_C(2)$ denote the group testing model of identifying and selecting two different defective coins in a set of λ coins $øø|øe_1, øe_2, ø\overline{e_1e_2}, e_1e_2|e_1ø, e_2ø, \overline{e_1e_2}ø, e_2e_1$. We will prove: for random $\lambda \geqslant 2$, $\lambda \in N$, a testing algorithm p is existed, and inequality $\left\lceil \log_3 \frac{P_\lambda^2}{2!} \right\rceil \leqslant S^\lambda_{D_C(2)}(p) \leqslant \left\lceil \log_3 \frac{P_\lambda^2}{2!} \right\rceil + 1$ holds, here $\lceil z \rceil$ denotes the smallest integer not less than the number z, symbol P_λ^m denotes the permutation number of selecting m different elements out of λ different elements. This inequality indicates that the testing algorithm of selecting two different defective coins p need at most one testing step more than the optimal testing algorithm of model $D_C(2)$.

2. Some symbols, definitions and lemmas

The symbol, definitions and lemmas used in the paper will be introduced as follows. Symbol and terminology can be refereed in monograph [9] except that presented by this paper.

Let $X(h, \lambda - h)$ be a set consisting of λ coins, and H is the subset of the set X, $H \neq \emptyset$, the set H contains $h \leqslant \lambda$ defective coins, and the other $\lambda - h$ coins are standard. The symbol, definitions and lemmas used in the paper will be introduced as follows. Symbol and terminology can be refereed in monograph [9] except that presented by this paper.

Definition 2.1 [6]. Let $X(h, \lambda - h)$ be a set consisting of λ coins and H is the subset of the set X, $H \neq \emptyset$. Each element in set H is called a defective coin. A is a set consisting of n-dimensional order group $(A_1, A_2, \ldots, A_n)$, $A_i \in P(H)$, $i = 1, 2, \ldots, n$, $A_i \cap A_j = \emptyset$, $i \neq j$, $i, j = 1, 2, \ldots, n$, where $P(H)$ denotes the power set of H. If $F = \{F_1, F_2, \ldots, F_m\}$ is a partition of set A, then the partition F is called the model of identifying the defective coins with n set parallel devices, model for short. We refer to m as the resolution number of the partition F. Each of the F_j $(j = 1, 2, \ldots, m)$ is called a feedback. The model F is symbolized with $F_1|F_2|\cdots|F_m$ for short.

Definition 2.2 [6]. Let $F = \{F_1, F_2, \ldots, F_m\}$ be a partition of the set A. If each feedback F_j $(j = 1, 2, \ldots, m)$ contains only one n-dimensional order group, then feedback F_j $(j = 1, 2, \ldots, m)$ is usually called basic and the model $F = \{F_1, F_2, \ldots, F_m\}$ is known as basic model.

For example, if there are two set parallel devices for the set $X(2, \lambda - 2)$, its subset $H = \{e_1, e_2\}$, then the model $øø|øe_1|øe_2|ø\overline{e_1e_2}|e_1ø|e_1e_2|e_2ø|e_2e_1|\overline{e_1e_2}ø$ is a basic model.

Definition 2.3 [7]. Let $X(h, \lambda - h)$ be the set consisting of λ coins and H is the subset of set X, $H \neq \emptyset$. Each element in set H is called a defective coin. When the defective coins in the set H are distinguishable, the set $P = \{(x_1, x_2, \ldots, x_h) \,|\, x_i \in X(h, \lambda - h),\ i = 1, 2, \ldots, h\}$ is call the original solution space of the set $X(h, \lambda - h)$, where $(x_1, x_2, \ldots, x_h)$ is an h-dimensional order group. When the defective coins in the set H are not distinguishable, the set $P = \{(x_1, x_2, \ldots, x_h) \,|\, x_i \in X(h, \lambda - h), i = 1, 2, \ldots, h\}$ is call the original solution space of the set $X(h, \lambda - h)$, where $(x_1, x_2, \ldots, x_h)$ is an h-dimensional unorder group.

Definition 2.4 [6]. Suppose that the cardinality of the original solution space of the set $X(h, \lambda - h)$ is s, and the resolution number of model F is m. This $\lceil \log_m s \rceil$ is usually referred as the information-theoretic lower bound of model F. Here $\lceil z \rceil$ denotes the smallest integer not less than the number z.

Lemma 2.5 [8]. *Suppose that the cardinality of the original solution space of the set $X(h, \lambda - h)$ is s, and the resolution number of the model F is m, let p denote a testing algorithm of identifying all the h defective coins among λ coins, and $I_F^\lambda(p)$ denote the maximum number of testing under testing algorithm p, then $I_F^\lambda(p) \geqslant \lceil \log_m s \rceil$.*

We can get following corollary directly by Lemma 2.5.

Corollary 2.6. *Suppose that the cardinality of the original solution space of the set $X(h, \lambda - h)$ is s, and the resolution number of the model F is m, let p denote a testing algorithm of selecting all the h defective coins among λ coins, and $S_F^\lambda(p)$ denote the maximum number of testing under testing algorithm p, then $S_F^\lambda(p) \geqslant \left\lceil \log_m \frac{s}{h!} \right\rceil$, here $h! = 1 \cdot 2 \cdot \cdots \cdot h$, $h \in N$.*

Lemma 2.7 [8]. *The information-theoretic lower bound of model F: øø|øe$_1$|e$_1$ø is accessible. Namely if $\lceil \log_m s \rceil$ is the information-theoretic lower bound of the model, when $3^{i-1} < \lambda \leqslant 3^i$, $i \in N$, then there is a group testing procedure p to identify all the h defective coins among λ coins, such that $I_F^{\lambda}(p) = \lceil \log_m s \rceil = \lceil \log_3 \lambda \rceil = i$.*

The model $øø|øe_1,\ øe_2,\ ø\overline{e_1e_2},\ e_1e_2|e_1ø,\ e_2ø,\ \overline{e_1e_2}ø,\ e_2e_1$ can be symbolized by $D_{\mathrm{C}}(2)$ for short. The three testing feedbacks of model $D_{\mathrm{C}}(2)$ can be symbolized by F_1, F_2, F_3 respectively.

Lemma 2.8. *For the model $D_{\mathrm{C}}(2)$, then for a selection testing algorithm p existed, $S^3_{D_{\mathrm{C}}(2)}(p) \geqslant \lceil \log_3 \binom{3}{2} \rceil + 1 = 2$ holds.*

Lemma 2.9. *For the model $D_{\mathrm{C}}(2)$, a selection testing algorithm p is existed such that $S^{\lambda}_{D_{\mathrm{C}}(2)}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil$, and testing algorithm p is optimal, here $\lambda \in [4,6] \cap N$.*

The conclusions of Lemmas 2.8 and 2.9 are true apparently, and the proof is omitted.

Lemma 2.10. *For the model $D_{\mathrm{C}}(2)$, then there is a selection testing algorithm p such that $S^7_{D_{\mathrm{C}}(2)}(p) \geqslant \lceil \log_3 \binom{7}{2} \rceil + 1 = 4$.*

Proof. Apparently, $\lceil \log_3 \binom{7}{2} \rceil = 3$. The testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2,5)$, the first step is (A,B), here $|A| = |B| = 1$.

Case 1: If $F(A,B) = øø$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $|R| = 5$, by Lemma 2.9, three additional steps are needed to select out e_1, e_2.
Case 2: If $F(A,B) = F_2$, then the second step is (C,D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 1$.
Case 2.1: If $F(C,D) = F_1$, then the third step is (E,F), here $E, F \subseteq X - A - B - C - D$, $E \cap F = \emptyset$, $|E| = |F| = 1$.
Case 2.1.1: If $F(E,F) = F_1$, then the forth step is (A,R), here $R = X - A - B - C - D - E - F$, $|R| = 1$.
Case 2.1.1.1: Apparently, the case of $F(A,R) = F_1$ is not existed.
Case 2.1.1.2: If $F(A,R) = F_2$, then $e_1, e_2 \in B \cup R$. Because $|B \cup R| = 2$, e_1, e_2 can be selected.
Case 2.1.1.3: If $F(A,R) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| = 1$, e_1, e_2 can be selected.
Case 2.1.2: If $F(E,F) = F_2$, then $e_1, e_2 \in B \cup F$. Because $|B \cup F| = 2$, e_1, e_2 can be selected.
Case 2.1.3: If $F(E,F) = F_3$, then the case is quite similar to the case of $F(E,F) = F_2$.
Case 2.2: If $F(C,D) = F_2$, then $e_1, e_2 \in B \cup D$. Because $|B \cup D| = 2$, e_1, e_2 can be selected.
Case 2.3: If $F(C,D) = F_3$, then the case is quite similar to the case of $F(C,D) = F_2$.
Case 3: If $F(A,B) = F_3$, then the case is quite similar to the case of $F(A,B) = F_2$.

So four steps at most are needed to select out e_1, e_2, the conclusion is proved. □

Lemma 2.11. *For the model $D_{\mathrm{C}}(2)$, a selection testing algorithm p is existed such that $S^{\lambda}_{D_{\mathrm{C}}(2)}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil = 4$, and testing algorithm p is optimal, here $\lambda \in [8,10] \cap N$.*

Proof. Apparently, $\lceil \log_3 \binom{\lambda}{2} \rceil = 4$, here $\lambda \in [8,10] \cap N$. The testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2, \lambda - 2)$, the first step is (A,B), here $|A| = |B| = \lfloor \frac{\lambda}{3} \rfloor$, $\lambda \in [8,10] \cap N$, and $\lfloor z \rfloor$ denotes the greatest integer $\leqslant z$.

Case 1: If $F(A,B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $3 \leqslant |R| \leqslant 4$, by Lemmas 2.8 and 2.9, two additional steps at most are needed to select out e_1, e_2.
Case 2: If $F(A,B) = F_2$, then the second step is (A,C), here $C \subseteq X - A - B$, $|C| = \lfloor \frac{\lambda}{3} \rfloor$.
Case 2.1: If $F(A,C) = F_1$, then $e_1, e_2 \in B \cup R$, $R = X - A - B - C$. Because $3 \leqslant |B \cup R| \leqslant 4$, by Lemmas 2.8 and 2.9, two additional steps at most are needed to select out e_1, e_2.
Case 2.2: If $F(A,C) = F_2$, then $e_1 \in B$, $e_2 \in C$ or $e_2 \in B$, $e_1 \in C$. Because $|B| = |C| = 3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(A,C) = F_3$, then the case is quite similar to the case of $F(C,D) = F_2$.
Case 3: If $F(A,B) = F_3$, then the case is quite similar to the case of $F(A,B) = F_2$.

So four steps at most are needed to select out e_1, e_2. By Corollary 2.6, $S^{\lambda}_{D_C(2)}(p) \geqslant \left\lceil \log_3 \frac{P^2_\lambda}{2!} \right\rceil = \lceil \log_3 \binom{\lambda}{2} \rceil = 4$, here $\lambda \in [8, 10] \cap N$. So testing algorithm p is the optimal testing algorithm, the conclusion is proved. □

Lemma 2.12. *For the model $D_C(2)$, then for a selection testing algorithm p existed, $S^{\lambda}_{D_C(2)}(p) \geqslant \lceil \log_3 \binom{\lambda}{2} \rceil + 1 =$ 5, here $\lambda \in [11, 13] \cap N$.*

Proof. Apparently, $\lceil \log_3 \binom{\lambda}{2} \rceil = 4$, here $\lambda \in [11, 13] \cap N$.

I. When $\lambda = 11$, the testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, 9)$, the first step is (A, B), here $|A| = |B| = 3$.
Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $|R| = 5$, by Lemma 2.9, three additional steps are needed to select out e_1, e_2.
Case 2: If $F(A, B) = F_2$, then the second step is (A, C), here $C \subseteq X - A - B$, $|C| = 3$.
Case 2.1: If $F(A, C) = F_1$, then $e_1, e_2 \in B \cup R$. Because $|B \cup R| = 5$, by Lemma 2.9, three additional steps are needed to select out e_1, e_2.
Case 2.2: If $F(A, C) = F_2$, then $e_1 \in B$, $e_2 \in C$ or $e_2 \in B$, $e_1 \in C$. Because $|B| = |C| = 3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(A, C) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| = 3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

So five steps at most are needed to select out e_1, e_2, the conclusion holds.

II. When $\lambda = 12, 13$, the testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, \lambda - 2)$, the first step is (A, B), here $|A| = |B| = 3$, $\lambda = 12, 13$.
Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $6 \leqslant |R| \leqslant 7$, by Lemmas 2.9 and 2.10, we can select out e_1, e_2 using four additional steps at most.
Case 2: If $F(A, B) = F_2$, then the second step is (C, D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 3$.
Case 2.1: If $F(C, D) = F_1$, then the third step is (A, C).
Case 2.1.1: If $F(A, C) = F_1$, then $e_1, e_2 \in B \cup R$, $R = X - A - B - C - D$. Because $3 \leqslant |B \cup R| \leqslant 4$, by Lemmas 2.8 and 2.9, two additional steps are needed to select out e_1, e_2.
Case 2.1.2: Apparently, the case of $F(A, C) = F_2$ is not existed.
Case 2.1.3: If $F(A, C) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| = 3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.2: If $F(C, D) = F_2$, then $e_1 \in B$, $e_2 \in D$ or $e_2 \in B$, $e_1 \in D$. Because $|B| = |D| = 3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(C, D) = F_3$, then the case is quite similar to the case of $F(C, D) = F_2$.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

So five steps at most are needed to select out e_1, e_2, the conclusion is proved. □

Lemma 2.13. *For the model $D_C(2)$, then for a selection testing algorithm p existed, $S^{\lambda}_{D_C(2)}(p) \geqslant \lceil \log_3 \binom{\lambda}{2} \rceil = 5$, and testing algorithm p is optimal, here $\lambda \in [14, 16] \cap N$.*

Proof. Apparently, $\lceil \log_3 \binom{\lambda}{2} \rceil = 5$, here $\lambda \in [14, 16] \cap N$.

I. When $\lambda = 14$, the testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, 12)$, the first step is (A, B), here $|A| = |B| = 3$.
Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $|R| = 8$, by Lemma 2.11, four additional steps are needed to select out e_1, e_2.
Case 2: If $F(A, B) = F_2$, then the second step is (C, D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 3$.

Case 2.1: If $F(C,D)=F_1$, then $e_1,e_2 \in B\cup R$. Because $|B\cup R|=5$, by Lemma 2.9, three additional steps are needed to select out e_1, e_2.
Case 2.2: If $F(C,D)=F_2$, then $e_1\in B$, $e_2\in D$ or $e_2\in B$, $e_1\in D$. Because $|B|=|D|=3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(C,D)=F_3$, then the case is quite similar to the case of $F(C,D)=F_2$.
Case 3: If $F(A,B)=F_3$, then the case is quite similar to the case of $F(A,B)=F_2$.

So five steps at most are needed to select out e_1, e_2.

II. When $\lambda=15,16$, the testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2,\lambda-2)$, the first step is (A,B), here $|A|=|B|=3$, $\lambda=15,16$.
Case 1: If $F(A,B)=F_1$, then $e_1,e_2\in R$, $R=X-A-B$. Because $9\leqslant|R|\leqslant 10$, by Lemma 2.11, four additional steps are needed to select out e_1, e_2.
Case 2: If $F(A,B)=F_2$, then the second step is (C,D), here $C,D\subseteq X-A-B$, $C\cap D=\emptyset$, $|C|=|D|=3$.
Case 2.1: If $F(C,D)=F_1$, then the third step is $(A,R\cup C_1)$, here $C_1\subseteq C$, $R=X-A-B-C-D$, $|R\cup C_1|=3$.
Case 2.1.1: If $F(A,R\cup C_1)=F_1$, then $e_1,e_2\in B$. Because $|B|=3$, by Lemma 2.8, two additional steps are needed to select out e_1, e_2.
Case 2.1.2: If $F(A,R\cup C_1)=F_2$, then $e_1\in B$, $e_2\in R\cup C_1$ or $e_2\in B$, $e_1\in R\cup C_1$. Because $|B|=|R\cup C_1|=3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.1.3: If $F(A,R\cup C_1)=F_3$, then $e_1\in A$, $e_2\in B$. Because $|A|=|B|=3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.2: If $F(C,D)=F_2$, then $e_1\in B$, $e_2\in D$ or $e_2\in B$, $e_1\in D$. Because $|B|=|D|=3$, by Lemma 2.7, two additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(C,D)=F_3$, then the case is quite similar to the case of $F(C,D)=F_2$.
Case 3: If $F(A,B)=F_3$, then the case is quite similar to the case of $F(A,B)=F_2$.

So five steps at most are needed to select out e_1, e_2. By Corollary 2.6, $S^{\lambda}_{D_C(2)}(p)\geqslant\left\lceil\log_3\frac{P^2_{\lambda}}{2!}\right\rceil=\left\lceil\log_3\binom{\lambda}{2}\right\rceil=5$, here $\lambda\in[14,16]\cap N$. So testing algorithm p is the optimal testing algorithm, the conclusion is proved. □

Lemma 2.14. *For the model* $D_C(2)$, *then for a selection testing algorithm p existed*, $S^{\lambda}_{D_C(2)}(p)\geqslant\left\lceil\log_3\binom{\lambda}{2}\right\rceil+1=6$, *here* $\lambda\in[17,22]\cap N$.

Proof. Apparently, $\left\lceil\log_3\binom{\lambda}{2}\right\rceil=5$, here $\lambda\in[17,22]\cap N$. The testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2,\lambda-2)$, the first step is (A,B), here $|A|=|B|=\left\lfloor\frac{\lambda}{3}\right\rfloor$, $\lambda\in[17,22]\cap N$.

Case 1: If $F(A,B)=F_1$, then $e_1,e_2\in R$, $R=X-A-B$. Because $6\leqslant|R|\leqslant 8$, by Lemmas 2.9, 2.10 and 2.11, four additional steps at most are needed to select out e_1, e_2.
Case 2: If $F(A,B)=F_2$, then the second step is (A,C), here $C\subseteq X-A-B$, $|C|=\left\lfloor\frac{\lambda}{3}\right\rfloor$.
Case 2.1: If $F(A,C)=F_1$, then $e_1,e_2\in B\cup R$, here $R=X-A-B-C$. Because $6\leqslant|B\cup R|\leqslant 8$, by Lemmas 2.9, 2.10 and 2.11, four additional steps at most are needed to select out e_1, e_2.
Case 2.2: If $F(A,C)=F_2$, then $e_1\in B$, $e_2\in C$ or $e_2\in B$, $e_1\in C$. Because $|B|=|C|=\left\lfloor\frac{\lambda}{3}\right\rfloor\leqslant 8$, by Lemma 2.7, four additional steps at most are needed to select out e_1, e_2.
Case 2.3: If $F(A,C)=F_3$, then $e_1\in A$, $e_2\in B$. Because $|A|=|B|=\left\lfloor\frac{\lambda}{3}\right\rfloor\leqslant 8$, by Lemma 2.7, four additional steps at most are needed to select out e_1, e_2.
Case 3: If $F(A,B)=F_3$, then the case is quite similar to the case of $F(A,B)=F_2$.

So six steps are at most needed to select out e_1, e_2, the conclusion is proved. □

Lemma 2.15. *For the model* $D_C(2)$, *then for a selection testing algorithm p existed*, $S^{\lambda}_{D_C(2)}(p)\geqslant\left\lceil\log_3\binom{\lambda}{2}\right\rceil=6$, *here* $\lambda\in[23,28]\cap N$, *and testing algorithm p is optimal.*

Proof. Apparently, $\lceil \log_3 \binom{\lambda}{2} \rceil = 6$, here $\lambda \in [23, 28] \cap N$. The testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, \lambda - 2)$, the first step is (A, B), here $|A| = |B| = \lfloor \frac{\lambda}{3} \rfloor$, $\lambda \in [23, 28] \cap N$.

Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $8 \leqslant |R| \leqslant 10$, by Lemma 2.11, four additional steps are needed to select out e_1, e_2.
Case 2: If $F(A, B) = F_2$, then the second step is (A, C), here $C \subseteq X - A - B$, $|C| = \lfloor \frac{\lambda}{3} \rfloor$.
Case 2.1: If $F(A, C) = F_1$, then $e_1, e_2 \in B \cup R$, here $R = X - A - B - C$. Because $8 \leqslant |R| \leqslant 10$, by Lemma 2.11, four additional steps are needed to select out e_1, e_2.
Case 2.2: If $F(A, C) = F_2$, then $e_1 \in B$, $e_2 \in C$ or $e_2 \in B$, $e_1 \in C$. Because $|B| = |C| = \lfloor \frac{\lambda}{3} \rfloor \leqslant 9$, by Lemma 2.7, four additional steps at most are needed to select out e_1, e_2.
Case 2.3: If $F(A, C) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| = \lfloor \frac{\lambda}{3} \rfloor \leqslant 9$, by Lemma 2.7, four additional steps at most are needed to select out e_1, e_2.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

So six steps at most are needed to select out e_1, e_2. By Corollary 2.6, $S^{\lambda}_{D_C(2)}(p) \geqslant \left\lceil \log_3 \frac{P^2_{\lambda}}{2!} \right\rceil = \lceil \log_3 \binom{\lambda}{2} \rceil = 5$, here $\lambda \in [23, 28] \cap N$. So testing algorithm p is the optimal testing algorithm, the conclusion is proved. □

3. A testing algorithm of selecting two different defective coins

Next for model $D_C(2)$, a group testing algorithm of selecting two different defective coins will be constructed.

Theorem 3.1. *For the model* $D_C(2)$, *a selection testing algorithm p is existed, and* $S^{\lambda}_{D_C(2)}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil = 2k + 2$, *here* $\lambda \in [m + 1, 3^{k+1} + 1] \cap N$, $k \in N$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 2$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 2$, $k, m \in N$, *and testing algorithm p is optimal.*

Proof. Because $\binom{\lambda}{2} \leqslant \binom{3^{k+1}+1}{2} < 3^{2k+2}$, then $\lceil \log_3 \binom{\lambda}{2} \rceil \leqslant 2k + 2$. While $\lceil \log_3 \binom{\lambda}{2} \rceil \geqslant 2k + 2$, so equality $\lceil \log_3 \binom{\lambda}{2} \rceil = 2k + 2$ holds. Here $\lambda \in [m + 1, 3^{k+1} + 1] \cap N$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 2$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 2$, $k, m \in N$.

Next we use mathematical induction to prove: for random natural number k, a testing algorithm p exists absolutely, and in the testing algorithm p, $2k + 2$ steps at most are needed to select out e_1, e_2.

For $k = 1$, by Lemma 2.11, the conclusion holds. And the testing algorithm p is the testing algorithm in Lemma 2.11. Suppose for $k = l$, the conclusion is true and that corresponding algorithms are constructed. Then for $k = l + 1$, the testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, \lambda - 2)$, here $|A| = |B| = \lfloor \frac{\lambda}{3} \rfloor$, $\lambda \in [m + 1, 3^{k+2} + 1] \cap N$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 2$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 2$, $k, m \in N$. The first step is (A, B).

Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, here $R = X - A - B$. Because $|R| \leqslant 3^{l+1} + 1$, by the inductive assumption, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2: If $F(A, B) = F_2$, then the second step is (A, C), here $C \subseteq X - A - B$, $|C| = \lfloor \frac{\lambda}{3} \rfloor$.
Case 2.1: If $F(A, C) = F_1$, then $e_1, e_2 \in B \cup R$, here $R = X - A - B - C$. Because $|B \cup R| \leqslant 3^{l+1} + 1$, by the inductive assumption, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2.2: If $F(A, C) = F_2$, then $e_1 \in B$, $e_2 \in C$ or $e_2 \in B$, $e_1 \in C$. Because $|B| = |C| = \lfloor \frac{\lambda}{3} \rfloor \leqslant 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2.3: If $F(A, C) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| \leqslant 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

For $k = l + 1$, $2l + 4$ steps at most are needed to select out e_1, e_2, here $\lambda \in [m + 1, 3^{k+2} + 1] \cap N$, $k \in N$. By induction principle, for random natural number k, $2k + 2$ at most steps are needed to select out e_1, e_2, here

$\lambda \in [m+1, 3^{k+2}+1] \cap N$, $k \in N$. By Corollary 2.6, $S^{\lambda}_{D_C(2)}(p) \geqslant \left\lceil \log_3 \frac{P^2_\lambda}{2!} \right\rceil = \left\lceil \log_3 \binom{\lambda}{2} \right\rceil = 2k+2$, here $\lambda \in [m+1, 3^{k+2}+1] \cap N$, $k \in N$. So testing algorithm p is the optimal testing algorithm, the theorem is proved. □

Theorem 3.2. *For the model* $D_C(2)$, *then for a selection testing algorithm p existed*, $S^{\lambda}_{D_C(2)}(p) = \left\lceil \log_3 \binom{\lambda}{2} \right\rceil \leqslant 2k+3$, *here* $\lambda \in [3^{k+1}+2, 5\cdot 3^k+1] \cap N$, $k \in N$.

Proof. Because $\binom{\lambda}{2} \leqslant \binom{5\cdot 3^k+1}{2} < 3^{2k+3}$, then $\left\lceil \log_3 \binom{\lambda}{2} \right\rceil \leqslant 2k+3$, here $\lambda \in [3^{k+1}+2, 5\cdot 3^k+1] \cap N$, $k \in N$. If we can prove the conclusion: for $\lambda \in [3^{k+1}+2, 5\cdot 3^k+1] \cap N$, $k \in N$, a selection testing algorithm p is existed, and the algorithm needs $2k+3$ steps at most to select out two different defective coins e_1, e_2, then the conclusion of the lemma can be proved. Next we use mathematical induction to prove.

For $k=1$, by Lemmas 2.12 and 2.13, the conclusion holds. And the testing algorithm p is the testing algorithm in Lemmas 2.12 and 2.13. Suppose for $k=l$, the conclusion is true, and that corresponding algorithms are constructed, then for $k=l+1$:

I. When $\lambda \in [3^{l+2}, 4\cdot 3^{l+1}-1] \cap N$, $k \in N$, the testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2,\lambda-2)$, here $|A|=|B|=3^{l+1}$. The first step is (A,B).
Case 1: If $F(A,B)=F_1$, then the second step is $(C, R\cup A_1)$, here $C\subseteq X-A-B$, $A_1\subseteq A$, $R=X-A-B-C$, $|C|=|R\cup A_1|=3^{l+1}$.
Case 1.1: Apparently, the case of $F(C,R\cup A_1)=F_1$ is not existed.
Case 1.2: If $F(C,R\cup A_1)=F_2$, then the third step is (C,B).
Case 1.2.1: If $F(C,B)=F_1$, then $e_1,e_2\in R\cup A_1$. Because $|R\cup A_1|=3^{l+1}$, by Theorem 3.1, $2l+2$ additional steps are needed to select out e_1, e_2.
Case 1.2.2: Apparently, the case of $F(C,B)=F_2$ is not existed.
Case 1.2.3: If $F(C,B)=F_3$, then $e_1\in C$, $e_2\in R\cup A_1$. Because $|C|=|R\cup A_1|=3^{l+1}$, by Lemma 2.7, $2l+2$ additional steps are needed to select out e_1, e_2.
Case 1.3: If $F(C,R\cup A_1)=F_3$, then the case is quite similar to the case of $F(C,R\cup A_1)=F_2$.
Case 2: If $F(A,B)=F_2$, then the second step is $(C,R\cup A_1)$, here $C\subseteq X-A-B$, $A_1\subseteq A$, $|C|=|R\cup A_1|=3^{l+1}$.
Case 2.1: If $F(C,R\cup A_1)=F_1$, then the case is quite similar to the case of $F(A,B)=F_1$ and $F(C,R\cup A_1)=F_2$.
Case 2.2: If $F(C,R\cup A_1)=F_2$, then $e_1\in B$, $e_2\in R\cup A_1$ or $e_2\in B$, $e_1\in R\cup A_1$. Because $|B|=|R\cup A_1|=3^{l+1}$, by Lemma 2.7, $2l+2$ additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(C,R\cup A_1)=F_3$, then the case is quite similar to the case of $F(C,R\cup A_1)=F_2$.
Case 3: If $F(A,B)=F_3$, then the case is quite similar to the case of $F(A,B)=F_2$.

So for $k=l+1$, $2l+5$ steps are needed to select out e_1, e_2.

II. When $\lambda \in [4\cdot 3^{l+1}, 5\cdot 3^{l+1}-1] \cap N$, $k \in N$, the testing algorithm p can be constructed according to the following schemes: let (A,B) be the disjoint subset of the set $X(2,\lambda-2)$, here $|A|=|B|=3^{l+1}$. The first step is (A,B).
Case 1: If $F(A,B)=F_1$, then the second step is (C,D), here $C,D\subseteq X-A-B$, $C\cap D=\emptyset$, $|C|=|D|=3^{l+1}$.
Case 1.1: If $F(C,D)=F_1$, then $e_1,e_2\in R$, $R=X-A-B-C-D$. Because $|R|\leqslant 3^{l+1}$, by Theorem 3.1, $2l+2$ additional steps at most are needed to select out e_1, e_2.
Case 1.2: If $F(C,D)=F_2$, then the third step is $(C,R\cup A_1)$, here $C\subseteq X-A-B$, $A_1\subseteq A$, $R=X-A-B-C-D$, $|C|=|R\cup A_1|=3^{l+1}$.
Case 1.2.1: If $F(C,R\cup A_1)=F_1$, then $e_1,e_2\in D$. Because $|D|=3^{l+1}$, by Theorem 3.1, $2l+2$ additional steps are needed to select out e_1, e_2.
Case 1.2.2: If $F(C,R\cup A_1)=F_2$, then $e_1\in D$, $e_2\in R\cup A_1$ or $e_2\in D$, $e_1\in R\cup A_1$. Because $|D|=|R\cup A_1|=3^{l+1}$, by Lemma 2.7, $2l+2$ additional steps at most are needed to select out e_1, e_2.

Case 1.2.3: If $F(C, R \cup A_1) = F_3$, then $e_1 \in C$, $e_2 \in D$. Because $|C| = |D| = 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps are needed to select out e_1, e_2.
Case 1.3: If $F(C, D) = F_3$, then the case is quite similar to the case of $F(C, D) = F_2$.
Case 2: If $F(A, B) = F_2$, then the second step is (C, D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 3^{l+1}$.
Case 2.1: If $F(C, D) = F_1$, then the case is quite similar to the case of $F(A, B) = F_1$ and $F(C, D) = F_2$.
Case 2.2: If $F(C, D) = F_2$, then $e_1 \in B$, $e_2 \in D$ or $e_2 \in B$, $e_1 \in D$. Because $|B| = |D| = 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2.3: If $F(C, D) = F_3$, then the case is quite similar to the case of $F(C, D) = F_2$.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

So for $k = l + 1$, $2l + 5$ steps at most are needed to select out e_1, e_2.

III. When $\lambda \in [5 \cdot 3^{l+1}, 5 \cdot 3^{l+1} + 1] \cap N$, $k \in N$, the testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, \lambda - 2)$, here $|A| = |B| = 3^{l+1}$. The first step is (A, B).
Case 1: If $F(A, B) = F_1$, then the second step is (C, D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 3^{l+1}$.
Case 1.1: If $F(C, D) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B - C - D$. Because $|R| \leqslant 3^{l+1} + 1$, by Theorem 3.1, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 1.2: If $F(C, D) = F_2$, then the third step is (C, E), here $E \subseteq X - A - B - C - D$, $|E| = 3^{l+1}$.
Case 1.2.1: If $F(C, E) = F_1$, then $e_1, e_2 \in D$. Because $|D| = 3^{l+1}$, by Theorem 3.1, $2l + 2$ additional steps are needed to select out e_1, e_2.
Case 1.2.2: If $F(C, E) = F_2$, then $e_1 \in D$, $e_2 \in E$ or $e_2 \in D$, $e_1 \in E$. Because $|D| = |E| = 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps at most are needed to select out e_1, e_2.
Case 1.2.3: If $F(C, E) = F_3$, then $e_1 \in C$, $e_2 \in D$. Because $|C| = |D| = 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps are needed to select out e_1, e_2.
Case 1.3: If $F(C, D) = F_3$, then the case is quite similar to the case of $F(C, D) = F_2$.
Case 2: If $F(A, B) = F_2$, then the second step is (C, D), here $C, D \subseteq X - A - B$, $C \cap D = \emptyset$, $|C| = |D| = 3^{l+1}$.
Case 2.1: If $F(C, D) = F_1$, then the case is quite similar to the case of $F(A, B) = F_1$ and $F(C, D) = F_2$.
Case 2.2: If $F(C, D) = F_2$, then $e_1 \in B$, $e_2 \in D$ or $e_2 \in B$, $e_1 \in D$. Because $|B| = |D| = 3^{l+1}$, by Lemma 2.7, $2l + 2$ additional steps are needed to select out e_1, e_2.
Case 2.3: If $F(C, D) = F_3$, then the case is quite similar to the case of $F(C, D) = F_2$.
Case 3: If $F(A, B) = F_3$, then the case is quite similar to the case of $F(A, B) = F_2$.

So for $k = l + 1$, $2l + 5$ steps at most are needed to select out e_1, e_2. By induction principle, for random natural number k, $2k + 3$ steps at most are needed to select out e_1, e_2, here $\lambda \in [3^{k+1} + 2, 5 \cdot 3^k + 1] \cap N, k \in N$. The conclusion is proved. □

Corollary 3.3. *For the model $D_C(2)$, in the selection testing algorithm p of Theorem 3.2, $S^{\lambda}_{D_{C(2)}}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil = 2k + 3$, $\lambda \in [m + 1, 5 \cdot 3^k + 1] \cap N$, $k \in N$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 3$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 3$, $k, m \in N$, here $\lambda \in [m, 5 \cdot 3^k + 1] \cap N$, $k \in N$, and the testing algorithm p is optimal.*

Proof. By Theorem 3.2, $S^{\lambda}_{D_{C(2)}}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil \leqslant 2k + 3$. And $\lceil \log_3 \binom{\lambda}{2} \rceil \geqslant \lceil \log_3 \binom{m+1}{2} \rceil = 2k + 3$, so $\lceil \log_3 \binom{m}{2} \rceil = 2k + 3$. By Corollary 2.6, $\left\lceil \log_3 \frac{P^2_{\lambda}}{2!} \right\rceil = \lceil \log_3 \binom{\lambda}{2} \rceil = 2k + 3$ steps at least are needed to select out e_1, e_2, here $\lambda \in [m + 1, 5 \cdot 3^k + 1] \cap N$, $k \in N$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 3$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 3$, $k, m \in N$. So, the testing algorithm p is the optimal testing algorithm. The corollary is proved. □

Theorem 3.4. *For the model $D_C(2)$, a selection testing algorithm p is existed, and $S^{\lambda}_{D_{C(2)}}(p) = \lceil \log_3 \binom{\lambda}{2} \rceil + 1 = 2k + 4$, here $\lambda \in [5 \cdot 3^k + 2, m] \cap N$, $k \in N$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 4$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 4$, $k, m \in N$.*

Proof. Apparently, $\lceil \log_3 \binom{\lambda}{2} \rceil + 1 = 2k + 4$. If we can prove the conclusion: for $\lambda \in [5 \cdot 3^k + 2, m] \cap N$, $k \in N$, a testing algorithm p is existed, and the algorithm needs $2k + 4$ steps to select out two different defective coins e_1, e_2, then the conclusion of the lemma can be proved. Because $\lambda \in [5 \cdot 3^k + 2, m] \cap N$, $\lceil \log_3 \binom{m}{2} \rceil + 1 = 2k + 4$, $\lceil \log_3 \binom{m+1}{2} \rceil = 2k + 4$, $k, m \in N$, apparently $\lambda < 3^{k+2} + 1$, $k \in N$. The testing algorithm p can be constructed according to the following schemes: let (A, B) be the disjoint subset of the set $X(2, \lambda - 2)$, the first step is (A, B), here $|A| = |B| = \text{big}\lfloor \frac{\lambda}{3} \rfloor$, $\lambda \in [5 \cdot 3^k + 2, m] \cap N$, $k \in N$.

Case 1: If $F(A, B) = F_1$, then $e_1, e_2 \in R$, $R = X - A - B$. Because $|R| \leqslant 3^{k+1} + 1$, by Theorem 3.1, $2k + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2: If $F(A, B) = F_2$, then the second step is (A, C), here $C \subseteq X - A - B$, $|C| = \text{big}\lfloor \frac{\lambda}{3} \rfloor$.
Case 2.1: If $F(A, C) = F_1$, then $e_1, e_2 \in B \cup R$, here $R = X - A - B - C$. Because $|B \cup R| \leqslant 3^{k+1} + 1$, by Theorem 3.1, $2k + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2.2: If $F(A, C) = F_2$, then $e_1 \in B$, $e_2 \in C$ or $e_2 \in B$, $e_1 \in C$. Because $|B| = |C| = \frac{\lambda}{3} \leqslant 3^{k+1}$, by Lemma 2.7, $2k + 2$ additional steps at most are needed to select out e_1, e_2.
Case 2.3: If $F(A, C) = F_3$, then $e_1 \in A$, $e_2 \in B$. Because $|A| = |B| = \text{big}\lfloor \frac{\lambda}{3} \leqslant 3^{k+1} \rfloor$, by Lemma 2.7, $2k + 2$ additional steps at most are needed to select out e_1, e_2.
Case 3: If $F(A, B) = F_3$, the case is quite similar to the case of $F(A, B) = F_2$.

So $2k + 4$ steps at most are needed to select out e_1, e_2, the theorem is proved. □

Algorithm 1. For the coin set $X(2, \lambda - 2)$ and model $D_C(2)$, an selection group testing algorithm o can be described as follows.

(1) When $\lambda \in [4, 6] \cap N$, the algorithm o is the selection group testing algorithm p in Lemma 2.9.
(2) When $\lambda \in ([m + 1, 3^{k+1} + 1] \cup [m' + 1, 5 \cdot 3^k + 1]) \cap N$, the algorithm o is the selection group testing algorithm p in Theorem 3.1 and Corollary 3.3.
(3) When $\lambda \in ([3^{k+1} + 2, m] \cup [5 \cdot 3^k + 2, m']) \cap N$, the algorithm o is the selection group testing algorithm p in Theorem 3.2 and Theorem 3.3.

Table 3.1
The relationship between sampling of λ and test number of the optimal testing algorithm p

k	Span of λ	Test number	Span of λ	Test number
	$[4,4]^0$	2	$[5,6]^0$	3
	$[m+1, 3^{k+1}+1]^0$	$2k+2$	$[m'+1, 5 \cdot 3^k+1]^0$	$2k+3$
1	$[8,10]^0$	4	$[14,16]^0$	5
2	$[23,28]^0$	6	$[39,46]^0$	7
3	$[67,82]^0$	8	$[116,136]^0$	9
4	$[199,244]^0$	10	$[345,406]^0$	11
5	$[592,730]^0$	12	$[1032,1216]^0$	13
6	$[1787,2188]^0$	14	$[3094,3646]^0$	15
⋮	⋮	⋮	⋮	⋮
	$[3,3]^1$	2	$[7,7]^1$	4
	$[3^{k+1}+2, m']^1$	$2k+3$	$[5 \cdot 3^k+2, m'']^1$	$2k+4$
1	$[11,13]^1$	5	$[17,22]^1$	6
2	$[29,38]^1$	7	$[47,66]^1$	8
3	$[83,115]^1$	9	$[137,198]^1$	10
4	$[245,344]^1$	11	$[407,591]^1$	12
5	$[731,1031]^1$	13	$[1217,1786]^1$	14
6	$[2189,3093]^1$	15	$[3647,5357]^1$	16
⋮	⋮	⋮	⋮	⋮

Note: (1) $[p,q]^x$ denotes the set of all the integers λ such that $p \leqslant \lambda \leqslant q$; (2) $[p,q]^x$ means the testing interval which needs x steps at most more than optimal testing algorithm.

Here $\left\lceil \log_3 \binom{m'+1}{3} \right\rceil = 2k+4$, $\left\lceil \log_3 \binom{m'}{2} \right\rceil = 2k+3$, $\left\lceil \log_3 \binom{m+1}{2} \right\rceil = 2k+3$, $\left\lceil \log_3 \binom{m}{2} \right\rceil = 2k+2$, $k, m, m' \in N$.

Lemma 2.9 indicate that, when $\lambda \in ([4,6] \cup [m+1, 3^{k+1}+1] \cup [m'+1, 5 \cdot 3^k + 1]) \cap N$, Algorithm 1 also is optimal. When $\lambda \in ([3^{k+1}+2, m] \cup [5 \cdot 3^k + 2, m']) \cap N$, Algorithm 1 can differ from an optimal algorithm by at most one.

Table 3.1 shows the relationship between sampling of λ and test number in the optimal selection testing algorithm p of model $D_C(2)$, here λ denotes the cardinal number of a given set of coins including two different defective coins.

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