

# Algorithms and Heuristics

## Lecture 22

# Is a Tomato a Fruit or a Vegetable?

Justice Gray writing for the US Supreme Court

Botanically speaking, tomatoes are the fruit of a vine, just as are cucumbers, squashes, beans, and peas.

But **in the common language of the people...** all these are vegetables... like potatoes, carrots... and lettuce, **usually served at dinner** in, with, or after the soup, fish, or meats which constitute the principal part of the repast, **and not, like fruits, generally as dessert.**



# Algorithmic Reasoning

- Logical, Systematic Rules
- Application Inevitably Solves Problem
- Guaranteed to Reach Correct Answer
- “Recipe” for Problem-Solving
  - Specifies Necessary Ingredients
  - Amounts
  - Order of Combination

*Just Follow the Steps!*

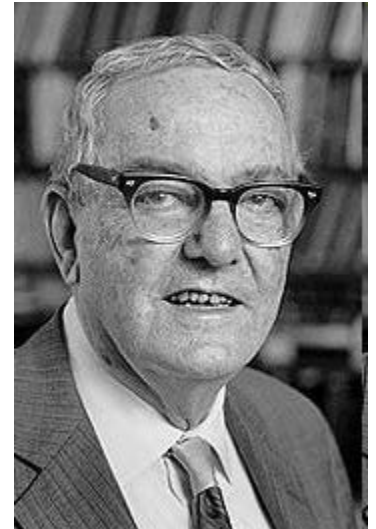
# Algorithm for Estimation

- Approximate Frequency
  - Count
  - Estimation
- Algorithm for Estimation
  - Draw Representative Sample
    - Random
    - Stratified
  - Extrapolate from Sample to Population
    - Statistical Inference
    - Hypothesis Testing

# Means-End Analysis of Problem-Solving

Newell & Simon (1972)

- Elements of Problems
  - Givens
  - Goals
  - Transformations
  - Obstacles
- Means-End Analysis
  - Represent Current State, Goal
  - Calculate Difference
  - Reduce Difference
  - Repeat



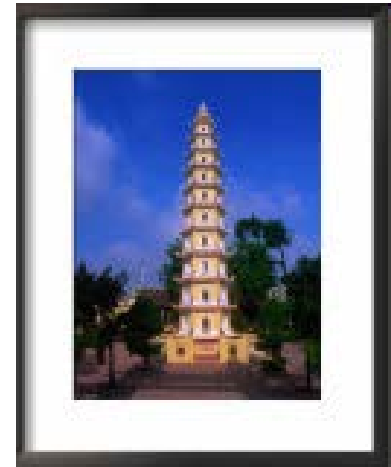
# The Legend of the Tower of Hanoi

Lucas (1883)

In Hanoi there is a temple in which there is a tower of 64 sacred golden disks, trimmed with diamonds. The disks are stacked on top of each other, with the largest on the bottom and the smallest on the top.

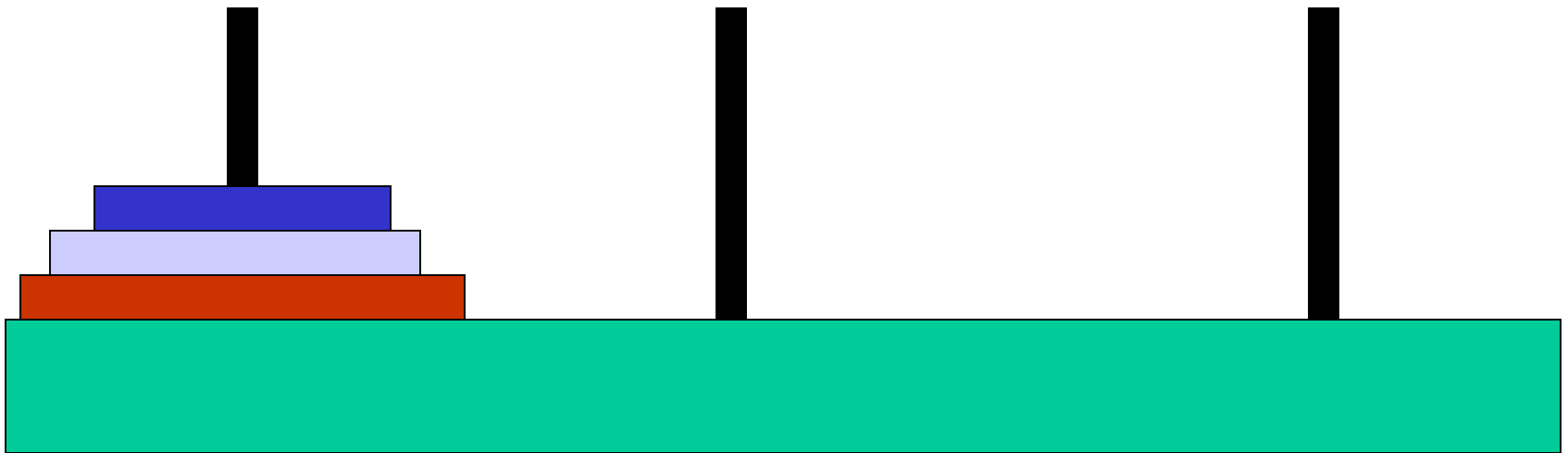
The monks must move the disks from one location to another, one at a time, such that a larger disk is never placed on top of a smaller disk. Besides the original location, and the new location, there is only one other place in the temple sacred enough to hold the disks.

The legend holds that before the monks complete the task, their temple will crumble into dust and the world will end.



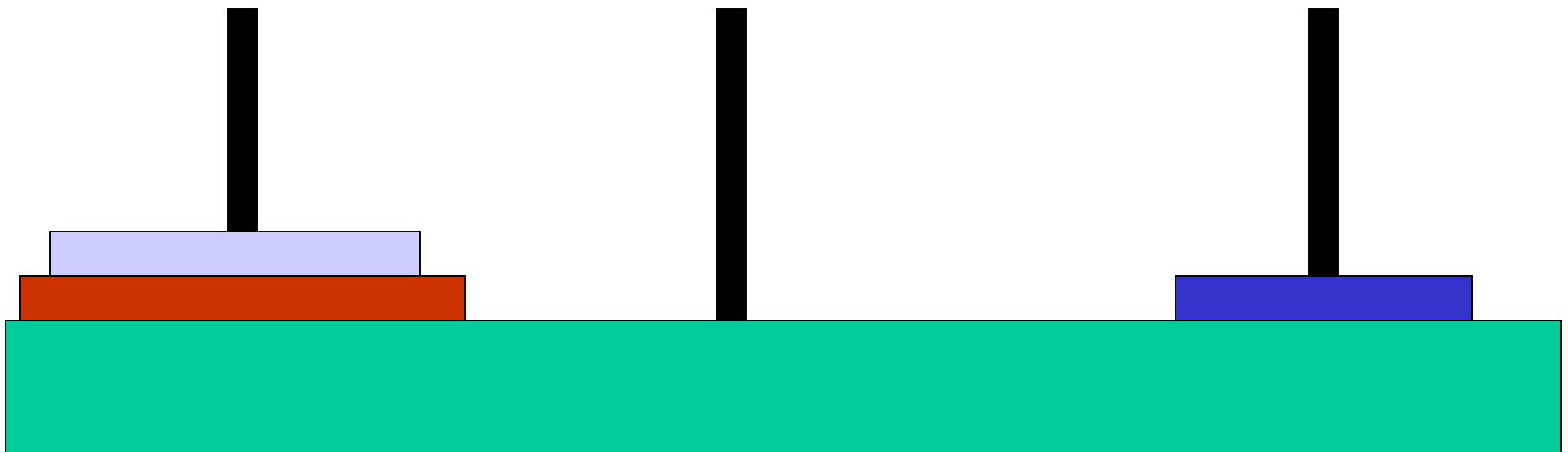
Chua Lien Phai Tower, Hanoi  
Photo by Bill Wassman  
Allposters.com

# Simplification of the Tower of Hanoi



# Means-End Analysis of The Tower of Hanoi

- Represent Current State, Goal
- Calculate Difference
- Reduce Difference
- Repeat





# The Legend of the Tower of Hanoi

Moving **64** disks  
in the manner specified  
will require

**18,446,744,073,709,551,615**

moves.

At a rate of one move per second,  
this would take more than

**5 billion years.**



Chua Lien Phai Tower, Hanoi  
Photo by Bill Wassman  
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# The Hobbits and Orcs Problem

Alcuin (d. 804); aka “Missionaries and Cannibals”

On one side of a river are three hobbits and three orcs.

Orcs eat hobbits when they outnumber them.

The creatures have a boat on their side that is capable of carrying two creatures at a time across the river.

The goal is to transport all six creatures to the other side of the river.

At no point on either side of the river can orcs outnumber hobbits (or the orcs would eat them).

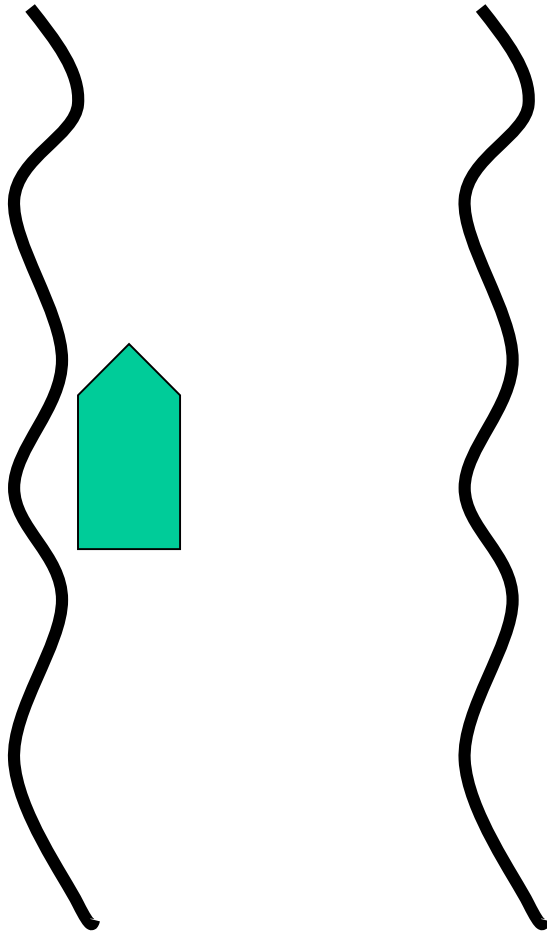
# Hobbits and Orcs

Alcuin (d. 804)

aka “Missionaries and Cannibals”, “Jealous Husbands”, “Brothers and Sisters”



H  
H  
H  
O  
O  
O



**The problem is to transport all six creatures across the river without the hobbits ever being outnumbered.**

# Well-Defined Problems

- Completely Specified
  - Initial Conditions
  - Goals
  - Intermediate Operations

$$3x + 3 = 12$$

$$x = ?$$

*Only One Possible Representation of Problem*

*Only One Correct Solution*

# III-Defined Problems

- Incompletely Specified
  - Initial Conditions
  - Goals
  - Intermediate Operations

$$3x + 3y = 12$$

$$x = ?$$

*Many Possible Representations of Problem*

*Many Possible Correct Solutions*

# Conditions of Uncertainty

- Ill-Defined Problem
- Algorithm Unknown
- Insufficient Information
- Insufficient Opportunity
  - Time
  - Motivation



# Judgment Heuristics

Kahneman & Tversky (1973)

Tversky & Kahneman (1974)



- Shortcuts, “Rules of Thumb”
  - Bypass Logical Rules
- Permit Judgments Under Uncertainty
- Also Permit Judgments Under Certainty
  - Use Increases Probability of Error
    - Infer Heuristics From Judgment Errors

# Common Judgment Heuristics

Kahneman & Tversky (1973); Tversky & Kahneman (1974)

- Representativeness
  - Categorization
  - Other Judgments of Similarity
  - Probability, Causality
- Availability
  - Frequency, Probability
- Simulation
  - Probability, Causality
- Anchoring and Adjustment
  - Estimation





# Applications of the Representativeness Heuristic

- Categorization (Prototype-Matching)
- Similarity Judgments
- Probability of Future Event
- Causality

# The Birth Problem

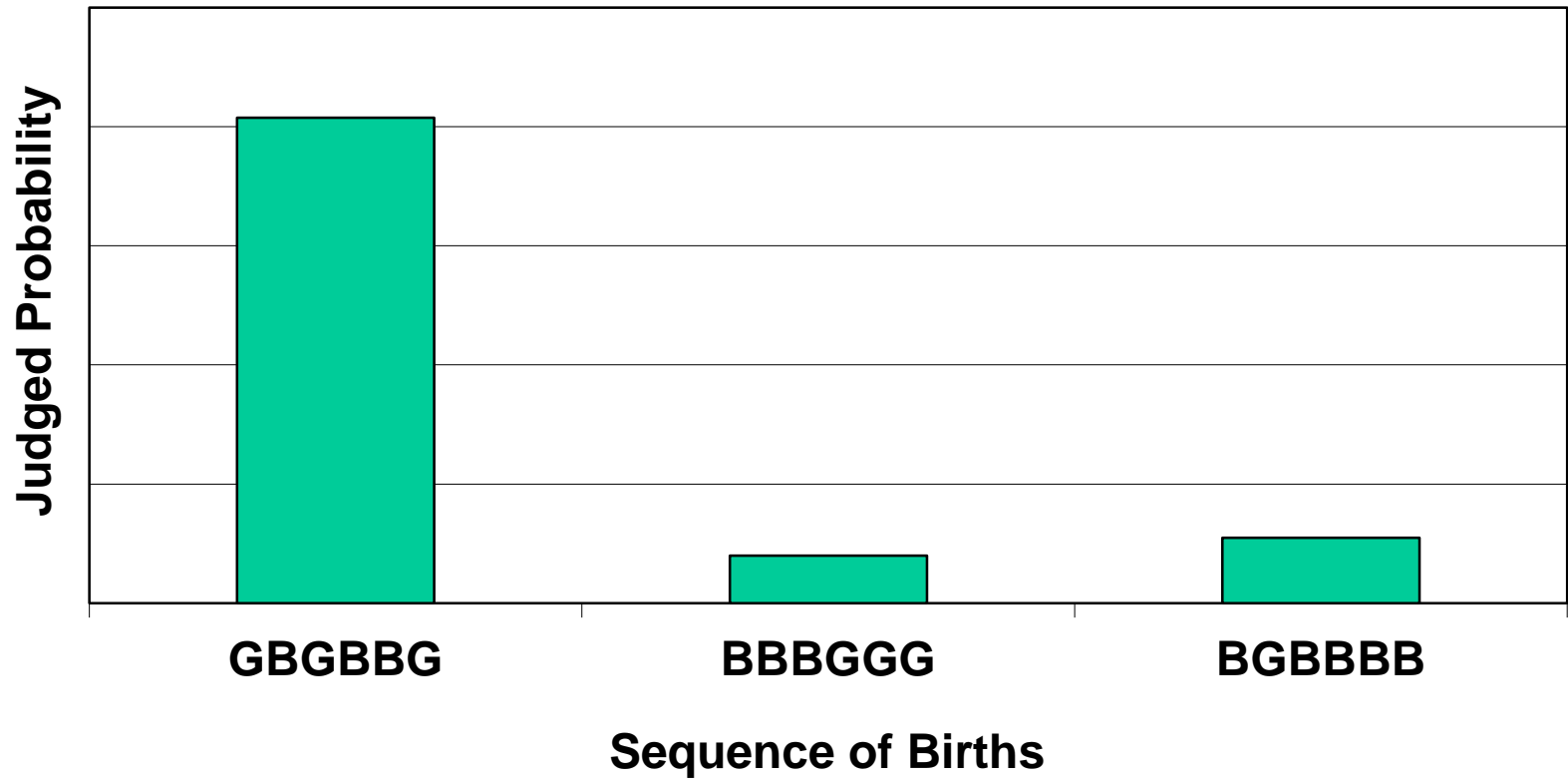
In families of six children, which is the most likely sequence of boys and girls?



- GBGBBG
- BBBGGG
- BGBBBB

# Judgment of Likelihood

After Kahneman & Tversky (1972)



# The Birth Problem

- The probability of any particular newborn being a boy is  $1/2$
- Probabilities are independent of each other
- Therefore:

$$p(\text{GBGBBG}) = (1/2)^6 = .0156$$

$$p(\text{BBBGGG}) = (1/2)^6 = .0156$$

$$p(\text{BGBBBB}) = (1/2)^6 = .0156$$

# The Gambler's Fallacy

On a roulette wheel, half the numbers are “red” and half are “black”.

Which of the following runs is more likely to end with a red?

**R****B****R****R****B****R****B**\_\_

**B****B****B****B****B**\_\_

**B****B****B****B****B****B****B****B****B****B****B****B**\_\_



# The Representativeness Heuristic

- Judgments are Based on the Extent to Which an Event...
  - Resembles Its Parent Population
  - Reflects Salient Features of the Generating Process
- Representativeness = Similarity
- Problems
  - Failure to Appreciate Base Rates
  - Failure to Calculate Prior Probabilities

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# Representativeness in Judgments of Causality

## Causes Should Resemble Effects

- Sex Education, Birth Control
  - Causes Sexual Behavior to Occur
- Violence in Movies, TV, Videogames
  - Causes Violent Behavior to Occur
- Arts and Music Education in Schools
  - Causes Academic Achievement





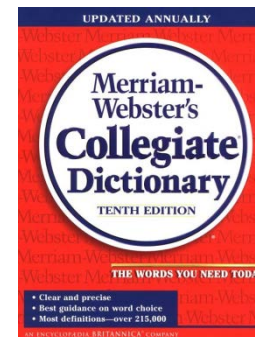
# Applications of the Availability Heuristic

- Judgments of Frequency
- Judgments of Probability

# The Word Problem

After Kahneman & Tversky (1973)

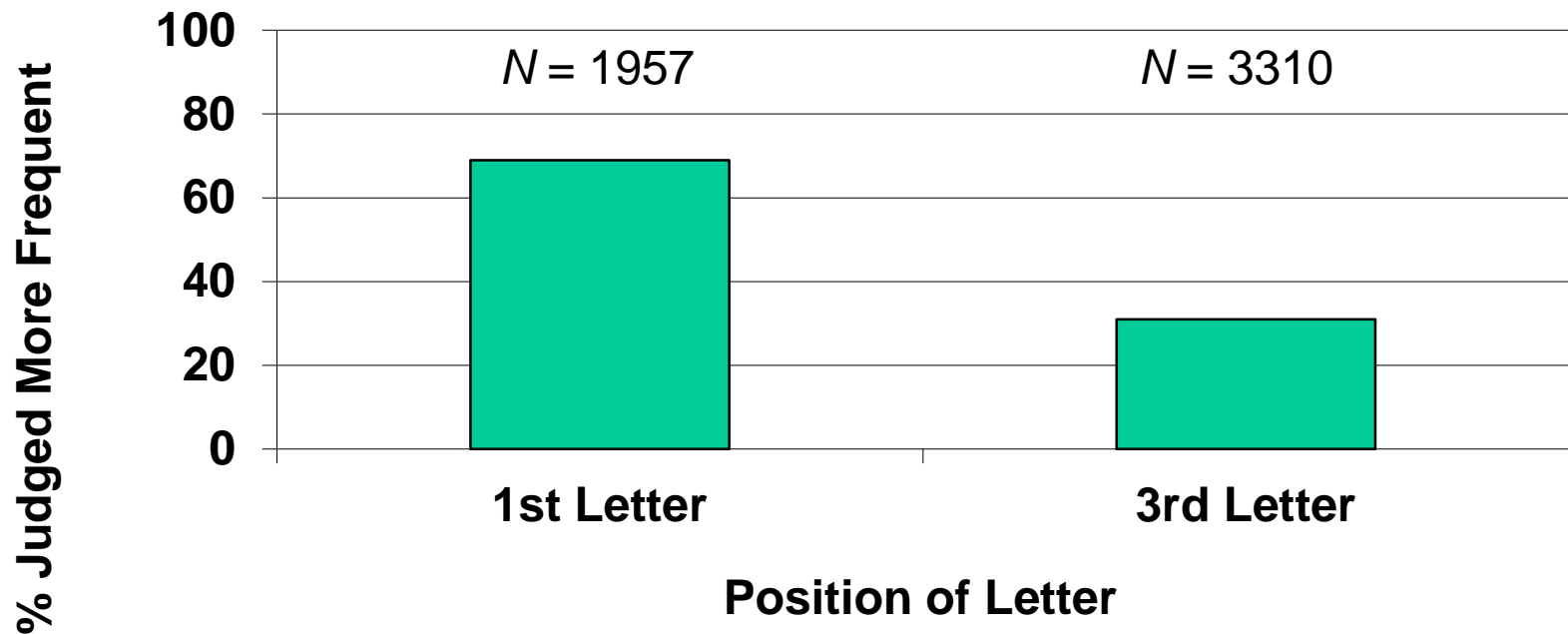
- Estimate the Number of Words in English...
  - Beginning with the Letter “K”
  - With “K” as Their Third Letter
- Repeat Estimates for Other Letters
  - L, N, R, V



# Letter-Frequency Judgment

Tversky & Kahneman (1973)

**Letters K, L, N, R, & V**



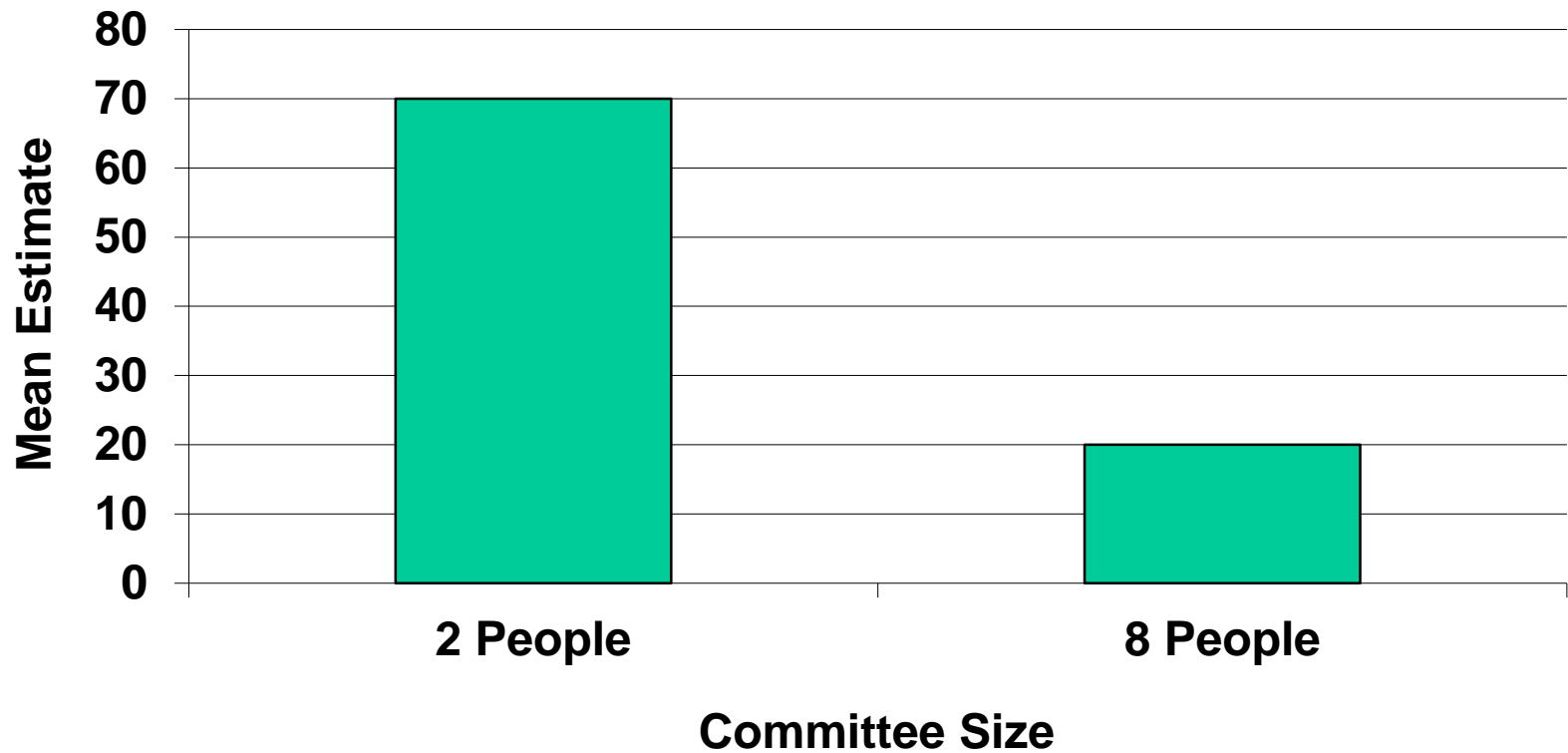
# The Committee Problem

- Given a Group of 10 People...
  - How Many Different Committees of 2 People Can You Create?
  - How Many Different Committees of 8 People?



# Number of Committees Created From 10 People

Tversky & Kahneman (1973)



# The Committee Problem

- The number of committees of  $k$  people that can be formed from a group of  $N$  people is given by the binomial coefficient

$$\binom{N}{k}$$

For committees of 2:  $\binom{10}{2} = 45$

For committees of 8:  $\binom{10}{8} = 45$

- From a group of 10 people, the creation of every committee of 2 automatically creates another committee of 8  $(10-2)$ !

# The Availability Heuristic

- Judgments are based on the ease with which instances can be brought to mind
- Frequency Affects Availability
- Problem
  - Ignores factors other than frequency that can affect availability

# Applications of the Simulation Heuristic

- Estimates of probability
- Judgments of causality



# The Undoing Problem

- Two Different Travelers
  - Heading for Airport
    - Different Flights, Leave at the Same Time
- Decide to Share Cab
- Caught in Traffic Jam
  - Expect to Miss Plane
- Get to Airport 30 Minutes Late
  - A's Plane Left on Time
  - B's Plane Was Delayed, But Left 5 Minutes Ago

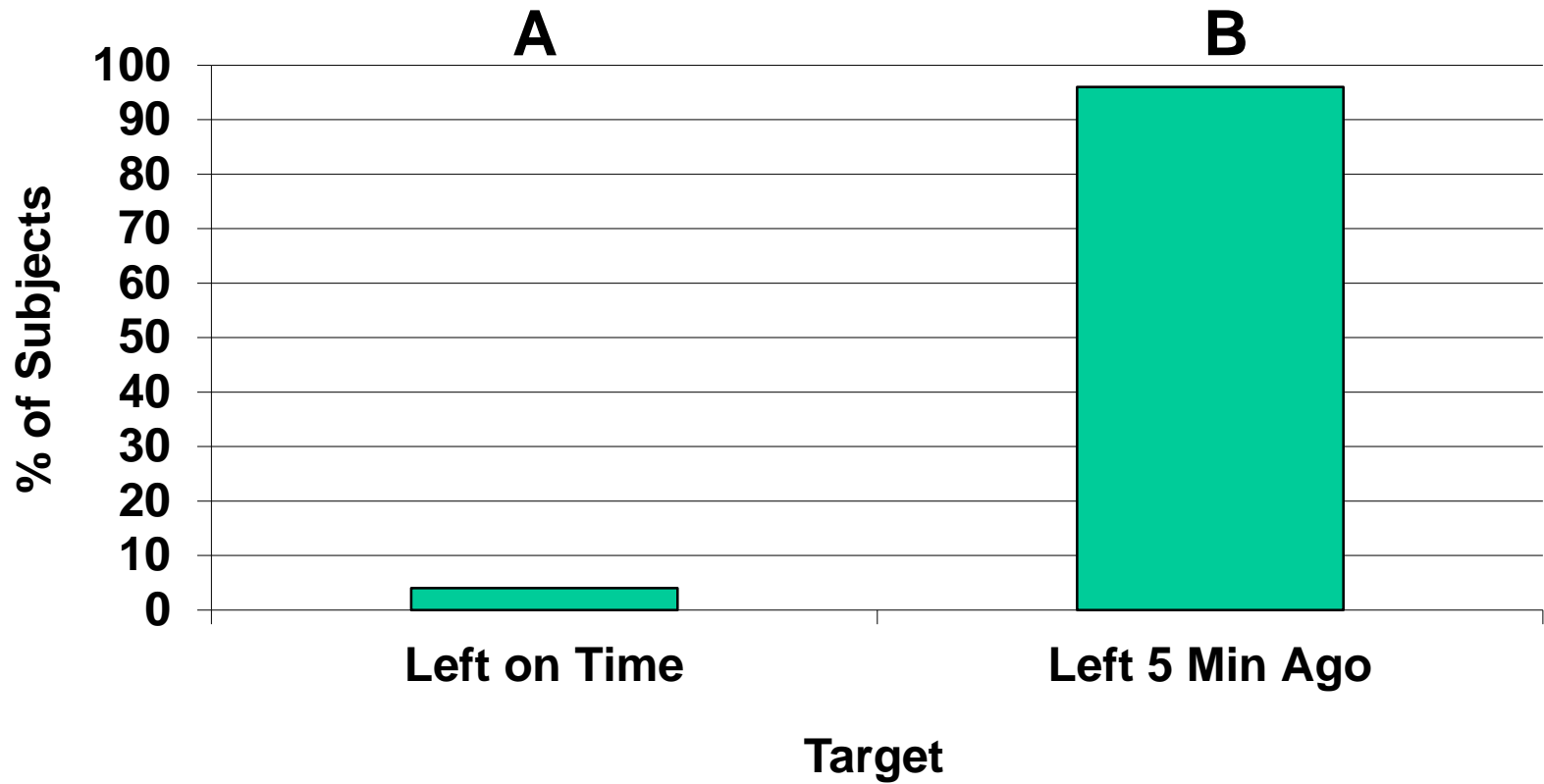


cityunlisted.com

*Who is more upset?*

# Judgments of “Upset”

Kahneman & Tversky (1979)



# The Simulation Heuristic

- Judgment is Based on the Ease with which a Plausible Scenario can be Constructed
- Similar to Availability
  - But Ease of Imagination
  - Not Ease of Retrieval from Memory
- Problem
  - No Guarantee that Imagined Scenario Might Have Occurred

# Simulation and the “Counterfactual Emotions”

- Compare Actual Outcome with “What Might Have Been”

Frustration

Regret

Grief

Indignation

# Applications of the Anchoring and Adjustment Heuristic

- Estimates

# The Extrapolation Problem

- Quickly Estimate the Following Product:

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

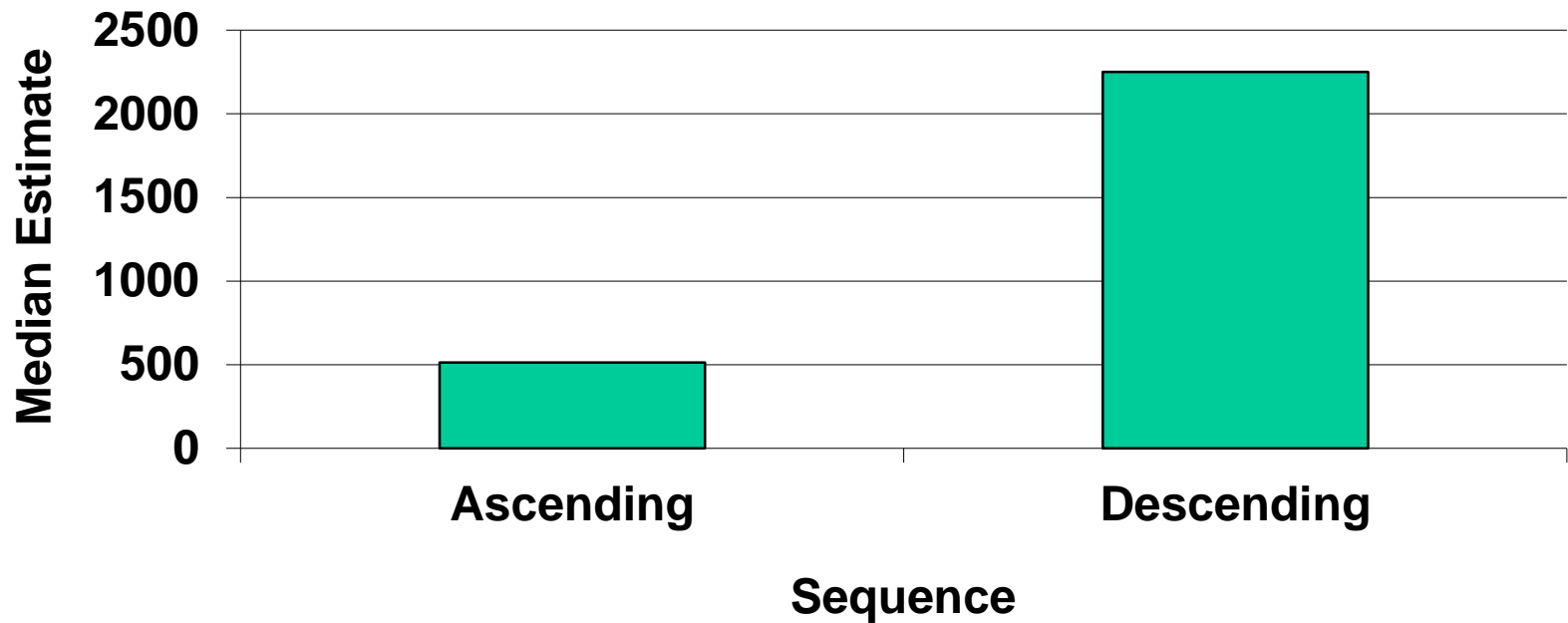
or

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$$

# Extrapolation Estimates

Tversky & Kahneman (1974)

**40,320**



# The United Nations Problem

- Please Estimate the Percentage of Countries in the United Nations that are From the Continent of Africa
  - Some People Say 10%
    - *What Do You Say?*
  - Some People Say 65%
    - *What Do You Say?*

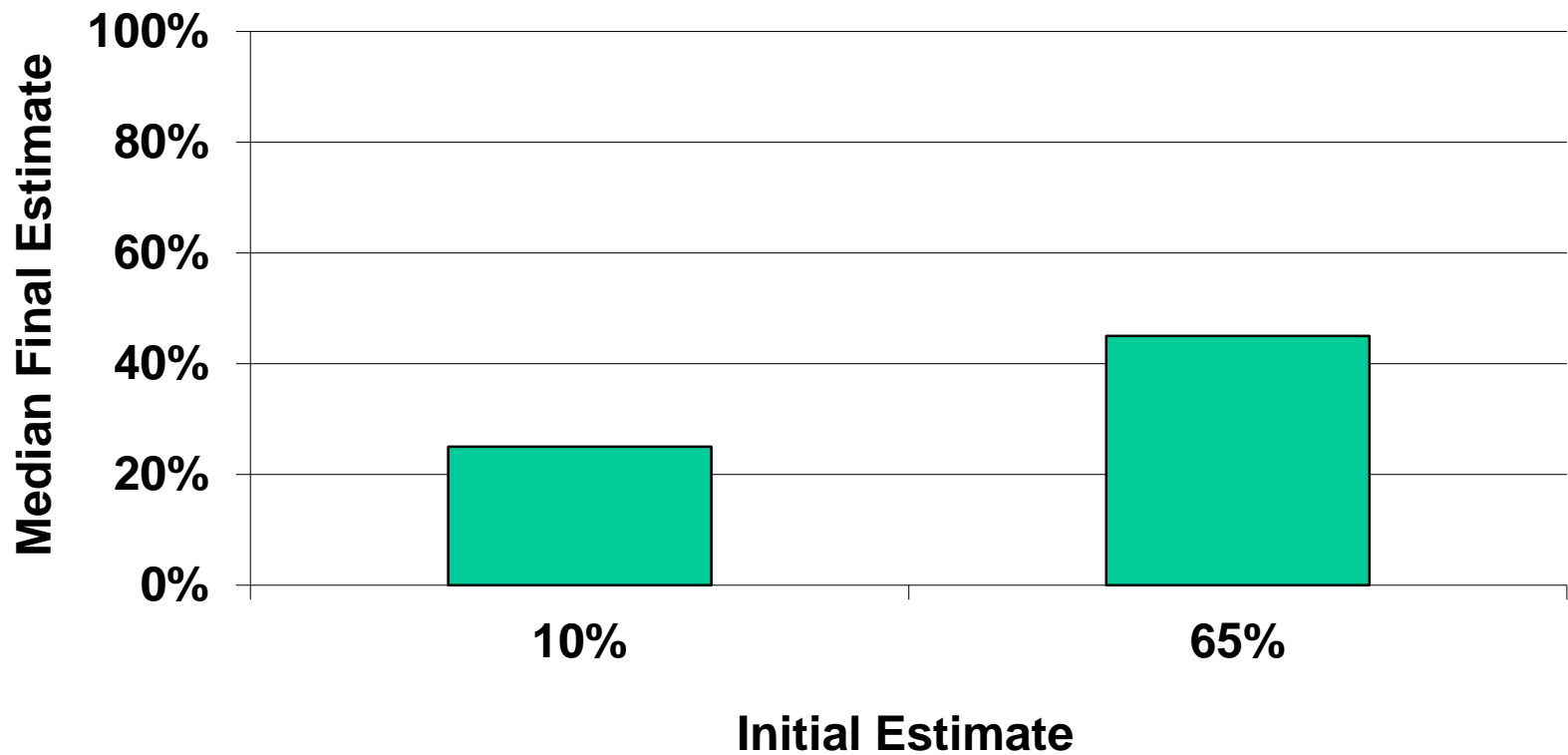


freeworldmaps.net



# African Countries in United Nations

Tversky & Kahneman (1974)



# The Anchoring and Adjustment Heuristic

- Final estimates are overwhelmingly influenced by initial estimates
  - Estimates Begin with Initial Value
  - Initial Value Serves as Anchor
- Problems
  - Formulation, Partial Computation Misleading
  - Insufficient Adjustment

The Power of First Impressions

# Judgment Heuristics in Problem-Solving

- Problem-Solving Begins with Categorization
  - New Problem Similar to Familiar One
- Representativeness
  - *Einstellung* (“Attitude”)
    - Inappropriate Problem-Solving Set
- Availability
  - Functional Fixedness
    - “Inertia” in Problem-Solving

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