Nicholas Bourbaki, Collective Mathematician
An Interview with Claude Chevalley

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Translated by Jeremy Gray*

Editor's Note: *Claude Chevalley died in Paris on June 28, 1984. He was an extraordinary mathematician and an equally extraordinary man, with a different view of mathematics and life in general. The mathematical community will miss both Chevalley and his views.*

The following is a translation of an article and interview by Denis Guedj. It originally appeared in the November 1981 issue of Dédales, *a short-lived journal edited by Denis Guedj. We thank* Dédales *for permission to publish this translation.*

The mystery which initially surrounded Bourbaki was cleared up long ago. Nicolas Bourbaki was born in 1935 from the incestuous love of seven young French mathematicians: Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, Szolem Mandelbrojt, Rene de Possel, André Weil. These seven alone are considered the founding fathers of the group, and the bonds of friendship which united them were important for the success of the project. In the 1930's, on the other side of the Rhine, rigour was rigour . . . in mathematics; the German school was highly regarded, particularly by those mathematicians who would no longer accept the way in which their art was practiced in France. They decided to end the blur, the approximation, the poor (or simply false) proofs which prevailed in a good number of works.

The treatise by Goursat "Calcul différentiel," with all its deficiencies, set the powder alight. They decided to re-write it; that is to say, to define the objects used, to complete the proofs, to establish them when they were sadly lacking—in brief, to write a "true" mathematical text. This took much longer than expected. During this time the Bourbaki group was born. The first congress took place in the Auvergne in July 1935 and only those present were crowned members of Bourbaki. Little by little the project became more precise; it grew considerably, but never lost its humour or its prankishness.

The Horizon of Perfect Rigour

Animated by a profound faith in the unity of mathematics, and wishing to be "universal mathematicians," they undertook to derive the whole of the mathematical universe from a single starting point. It was necessary to have a rock on which to build the edifice: they found it in the theory of sets. They added to it the elements of logic necessary to a rigorous practice of mathematical reasoning, and "thus, prepared according to the axiomatic methods, and always maintaining, as a sort of horizon, the possibility of a total formalization, our Treatise aims at a perfect rigour." (Quotation from Bourbaki, Introduction, *Théorie des ensembles.*)

They were not the first to have entertained such a project, but they remain the only ones to have advanced so far towards its realization. To carry it out they chose two powerful weapons, axiomatization and the general notion of a mathematical structure. The first they drew from Euclid, Hilbert, etc., but the second they had to invent, and it counts as one of the most beautiful jewels of 20th-century mathematics.

A Collective Work Amongst Equals

The following axiom was always present during the editorial work: "Among all the possible ways, there is for each mathematical question a best way of treating it, an optimal way." The search for this optimum made up (and still makes up) the greatest part of the menus at the Bourbaki congresses—the more so because each draft has to be accepted unanimously by those present. But a text accepted at one congress could be called into question the following year, new notions having been defined in the meantime. And these Sisyphean mathematicians had to begin their work again, cursing no doubt the gods who condemned them to their incessant labour.

An age limit was fixed. Above fifty, one had to leave the group. One by one the founders left their places and Bourbaki began to renew himself. (But no woman had the honour of being crowned Bourbakie.)

"Guinea pig" was the name given to non-bourbakist mathematicians invited to participate in the con-

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gresses. It was from this group that the new members were chosen. Those elected were those who, by their personal and mathematical qualities, would preserve the unanimity. Since then, numerous guinea pigs have entered the group. The bourbakist nomenclature is thus: first the "Founders," then the "Middles," then the Bourbakis of the third age called the "Young Ones" [in English, translator's note], and finally the "Bourbakis of today."

The list of members has always been a secret (an open secret). Without having any criterion of belonging in our possession, here, somewhat approximately, is the present composition of Bourbaki: Atiyah, Boutet de Monvel, Cartier, Demazure, Douady, Malgrange, Verdier . . . One knows when the stories begin . . . but who knows on how many generations Bourbaki will feed.

From Revolt to Honour

The Theory of Sets, the idea of structure, and many other things that one owes to Bourbaki constitute the raw material and the armory of the major part of today's mathematicians. But the bourbakist point of view, instead of contenting itself with being one among others, has become in many places the filter— the obligatory language without which there will be no salvation. Begun by a revolt against the dominant mathematics of the 30's, Bourbaki has turned steadily towards other paths that probably were not envisaged or welcomed by the founders. The bourbakists themselves have become well-known mathematicians, accepting medals and honours which they had condemned 30 years before. Inevitably, they have guided Bourbaki onto the paths that lead to power.

An Interview with Claude Chevalley, One of the Founders of Bourbaki

Dédales: You are one of the seven founders of Bourbaki and without doubt one of the most conspicuous. Almost fifty years after the birth of the group, where do you stand now?
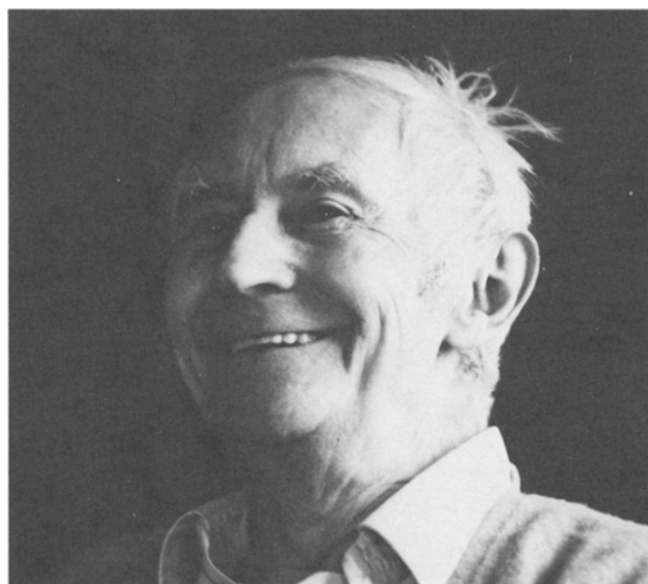**Chevalley:** How was it started? For me, at the beginning, it was by the despair of not reaching an understanding of a number of the works of mathematicians of the time. On the other hand, the necessity for me (and other mathematicians of my generation) to expound these works in a manner that I knew to be contradictory and false made the situation unbearable. My meeting with Weil was decisive. Until then I could believe that it was impossible to reason correctly in mathematics; but in him I had an example that showed it was good and indeed quite possible. If I advanced so much at that time, it was because Weil never had the tact to hide from me that I was talking nonsense. (Nor did it take him long to let me know that what I had done was correct.) It was at this point that Freyman intervened. He had a certain business-like gift. He was a good negotiator, a good intermediary, and since then he has become a friend. He gave the impression, both to Weil and me, that we would get the thing started. Which showed, evidently, that we were almost ready to set off.

Dédales: What thing was that?
Chevalley: The thing? The project, at that time, was extremely naïve: the basis for teaching the differential calculus was Goursat's *Traité*, very insufficient on a number of points. The idea was to write another to replace it. This, we thought, would be a matter of one or two years. Five years later we had still published nothing. The project was born in 1933. I was 24, the others between 24 and 30. No one was then well-known.

Claude Chevalley

Dédales: When was Bourbaki born?
Chevalley: Even before our project. Weil had spent two years in India and for the thesis of one of his pupils he needed a result he couldn't find anywhere in the literature. He was convinced of its validity, but he was too lazy to write out the proof. His pupil, however, was content to put a note at the bottom of the page which referred to "Nicolas Bourbaki, of the Royal Academy of Poldavia." When we needed to deck ourselves out with a collective name, Weil proposed to revive this tall story. Throughout his youth Bourbaki tried to play at being a secret society. It was quite ridiculous because, of course, we could not remain clandestine. Everyone knew it, but we refused even to reply to questions about the list of members, about the origin of the word Bourbaki, or about our projects.

There were two congresses at my parents' house at Chançay. The Bourbakis arrived at the station at Amboise. Those who were already there let out a frightful howl: Bourbaki! Bourbaki! You would have taken us for a band of madmen. There, that was the Bourbaki style! (An anecdote: on one occasion we had failed to reach agreement on a choice of definition—we asked my daughter Catherine, who was very small, to choose.)

Strong bonds of friendship existed between us, and when the problem of recruiting new members was raised we were all in agreement that this should be as much for their social manner as their mathematical ability. This allowed our work to submit to a rule of unanimity: anyone had the right to impose a veto. As a general rule, unanimity over a text only appeared at the end of seven or eight successive drafts. When a draft was rejected, there was a procedure foreseen for its improvement. "The Tribe," a report of the congress, related the discussions and decisions on the subject. The general lines in which the new draft should go was indicated in such a way that the new author would know what he had to do. It was always someone else who was charged with the next draft. There never was an example of a first draft being accepted.

The decisions did not take place in a block. In the Bourbaki congresses one read the drafts. At each line there were suggestions, proposals for changes were written on a black-board. In this way a new version was not born out of a simple rejection of a text, but rather it emerged from a series of sufficiently important improvements that were proposed collectively.

Bourbaki had a great advantage: one always accepted the possibility of a sharp change of opinion. This was very clear in the final agreement concerning the acceptance of mathematical logic. In the beginning, people were against it; they thought that logic would not be interesting. I was the one who imposed it and who wrote the first versions of the book on logic. Bourbaki was capable of changing his point of view. The rejection of logic, which certainly was a part of the way of thinking of many members, was eroded in the end. No one in Bourbaki had the impression of talking to a wall. In this sense it was a very remarkable phenomenon of collaboration.

Dédales: Did you suspect at the start that you could be embarking on an ever-expanding, unrealisable project?
Chevalley: It took us about four years to bring out the first fascicle, the one on results in the theory of sets. The writing of the complete *text* on the theory of sets had been put back to later. The first fascicle had been published so that readers would understand the ideas of the theory that would be employed constantly by Bourbaki. At the start, our ambition was very modest; only during the second congress did we become con-

scious of the size of the project. It was our purpose to produce the general theory first before passing to applications, according to the principle we had adopted of going "from the general (*generalissime*) to the particular."

Dédales: When you realized the enormity of the project did you feel proud?
Chevalley: Certainly. I absolutely had the impression of bringing light into the world—the mathematical world, you understand. It went hand in hand with the absolute certainty of our superiority over other mathematicians—a certainty that we held something of a higher level than the rest of mathematics of the day. For example, there is a word which was—which still is—in current usage, to *bourbakise* (*bourbachiser*). This means to take a text that one considers screwed up and to arrange it and improve it. It's more than just to improve; it's to treat it according to the norms which Bourbaki wanted to introduce in mathematics, essentially the theory of sets and the notion of structure. It is the notion of structure which is truly bourbakique. But with this feeling of accomplishing a gigantic task we came to the certainty that it would be impossible to achieve.

Dédales: Were you aware of constructing a sort of bible?
Chevalley: It seemed very clear that no one was obliged to read Bourbaki. We believed sincerely that if success came to us it would only be by virtue of the proper value of our text and would not become an "obligation," as it has become now. But a bible in mathematics is not like a bible in other subjects. It's a very well arranged cemetery with a beautiful array of tombstones, and that does not have the oppressive overtones of a bible in the religious sense. There was something which repelled us all: everything we wrote would be useless for teaching. I have no memory that we ever discussed this amongst ourselves. Mathematics at that time was so weak that the mathematical establishment and its eventual power seemed derisory to us. Therefore this wasn't a topic for discussion, and we had absolutely no idea that one day power could become bourbakised.

Dédales: Before getting there, what were the struggles for power inside Bourbaki?
**Chevalley:** There were lively disagreements, that's for sure. For example, on integration, which was the subject of eight volumes, there was a terrific struggle for years between Weil and De Possel. I inclined somewhat to the side of De Possel; it was concluded in Weil's sense but he doesn't recognize it now because he finds, rightly, that his ideas have been somewhat emasculated.

Dédales: For yourself, who is somewhat anarchistic, how does it feel in retrospect to have participated in an undertaking which led to power?

Chevalley: A large amount of resentment towards certain other members who originated this slide. You will say to me that this slide was inevitable. I'm not so sure. For example, Samuel never turned that way. (In fact, he was the only one who exercised his right of veto on a draft.) The existence of Samuel is a proof that this slide towards power wasn't inevitable.

Dédales: Do you think that one can give birth to such an undertaking without being transformed unavoidably into a tool of power, a tool of the dominant ideology? Isn't there a logic inherent in projects of this type that transforms people who participate in them into "masters"? You, for example, didn't you try to oppose this deviation?

Chevalley: If I had been sure it would happen, if I had had the perspicacity, if I had not been so weak as not to ask myself that question, I think that that could have been. . . .
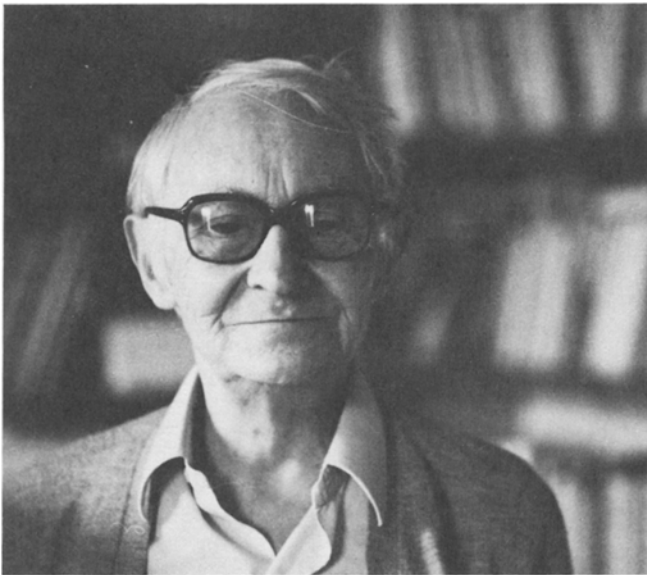
Chevalley

Dédales: But in fairness one never knows in advance. . . .

Chevalley: I never asked myself about the future. I believed it would continue in the same way as it had begun. But if I had known I think I would not have been able to fight against it. I have a sense of remorse at not having tried to point it in a direction that wouldn't have led to power. But I didn't try.

Dédales: Was one political in Bourbaki?

Chevalley: When I was preparing for the Ecole I read the anarchists. At the time of Bourbaki I was in a group that called itself "Le Ordre Nouveau Libertaire" which was anarchistic.

Dédales: Politics seem to have been excluded from Bourbaki. How did you live this dichotomy between your political involvement outside, and your almost complete investment in Bourbaki, above all at a time

when in Germany the Nazis were beginning to enjoy themselves to their hearts content?

Chevalley: I don't know what to say; it's a mistake. What I wrote in the political arena never satisfied me completely. It was only in Bourbaki that I was truly satisfied with what I wrote. It was probably because of the satisfaction that I could have, in Bourbaki, an activity without any relationship to what was going on politically in the world. Almost everyone mocked me for my participation in the "Ordre Nouveau," It seemed to them to be an activity unworthy of a serious spirit. That rather hurt me. Andre Weil called "Ordre Nouveau" the mind on the water. I'm trying to think what I would have replied if anyone had asked me that question at that time. What I would have said is that my political aspirations were aimed at letting everyone lead the best life possible, and that as it was possible for me to lead it, I did not dream of objecting. Now, I could still reply like that.

Dédales: How did the Bourbaki group evolve afterwards?

Chevalley: At the start we paid the costs of the congresses out of our own pockets. Then there were very substantial authors' royalties. We began to use them for Bourbaki's collective expenses. That certainly took a lot away from the feeling of being part of a collective activity. That was one of the causes of the degeneration. There were others. During the first congresses, up to the war, it was tacitly understood that one did not talk about matters having to do with a university career; it was simply not done. If anyone began, you stopped him and talked of something else. Unfortunately, the problem became absorbing after the war; probably this was due to the fact that when we began to call in young people, we naturally felt concerned with their careers. We were caught in the toils. Little by little we talked of everyone's career; it was complete decadence.

Before the war the mathematical world had decided to reward a French mathematician every year; Bourbaki was involved passionately and had declared war on the medals. The medal was never created. It was agreed amongst us—I perfectly recall conversations on the subject—that no member would be presented to the Academy. Now they almost all are. I myself was nominated as corresponding member without anyone asking my opinion. I have always refused to join anything like that.

Dédales: You have steadily drawn away from Bourbaki? How did this happen?

Chevalley: My drawing apart from Bourbaki was progressive. The last straw was the position taken by Dieudonné in 1968. At that time he took a position that I judged unacceptable; he favoured a cleaning-out of the university, making it a true university, etc. I was naïve, but I never imagined a member of Bourbaki

could hold such a position.

Dédales: And in the domain of mathematics?
Chevalley: I still adhere to the essentials of the Bourbakist position, and as someone has said: the balance is globally positive. One mustn't forget that it was Bourbaki who introduced axiomatization into France. I would also claim something else: the principle that every fact in mathematics must have an explanation. This has nothing to do with causality. For example, anything that was purely the result of a calculation was not considered by us to be a good proof.

Dédales: A good proof? Is that something which connects to meaning? I ask the question because often one opposes formalism to meaning.
Chevalley: If meaning is understood as reflection on an existing reality, then it is right to oppose them. But the meaning of which I speak is ONE meaning, acquired by an individual, and which could be totally different for another individual.

Dédales: Then what is this subjective sense of ONE formalism?
Chevalley: It's very difficult to say. Weil had a dictum: "When there is a difficulty, look for the group" (mathematical!). That swept me away. I told this to an American, a good probability theorist, who told me that meant nothing to him. What I certainly don't agree with any longer is the method of exposition. It's use-less for a beginner, for example.

At the level of mathematical logic, there's a point on which I am totally separated from them. Curiously, it concerns an idea on the subject of formalization that mostly owes its presence to my initiative. It's what in Bourbaki one called the horizon; you describe the formal rules, but there's no way you can apply them systematically because it would take up too much space. However, these rules can at least ideally describe "a horizon," a perfect text from the standpoint of rigour. Now, in my opinion, that's not possible. It was in reading Castoriadis that I understood this impossibility. For example, the idea of a symbol which is "the same," although written in different places and at different times, is not at all an idea that stands by itself. But it must stand by itself if one has this conception, even purely theoretically, of mathematics. Not only can this idea not possibly be realized, but its content is absurd. A symbol cannot possibly be "the same" if it does not have an aura of signification. There, there is an appeal to something human that contradicts the idea of a perfect "horizon."

Translator's Note: The "Ordre Nouveau" which flourished between the wars was a political movement in the tradition of French personalism, initiated by the ideas of Arnaud Dadieu and Denis de Rougemont. It is not connected at all with the present-day Ordre Nouveau, which is an extremely right-wing group.
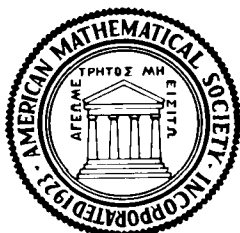
Chapter 9 of Ramanujan's Second Notebook:
Infinite Series Identities, Transformations, and Evaluations
Bruce C. Berndt and Padmini T. Joshi

Professor Bruce Berndt is successfully pursuing the important task of presenting to the mathematical public a complete, edited version of Ramanujan's famous notebooks. In this instance he and P. T. Joshi present the material in Chapter 9 of the Second Notebook together with the proofs (for the most part omitted by Ramanujan). The material has special interest today. There are many formulas for the Riemannian Zeta function with integer argument. Also the polylogarithm occurs often. Despite the wide-spread interest in these topics, it is clear that Ramanujan was able to discover many things which would probably have gone unnoticed. Formulas like

$$f\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi^2}{20},$$

where

$$f(x) = \sum_{k=1}^{\infty} \frac{h_k x^{2k-1}}{2k-1}$$

and

$$h_n = \sum_{k=1}^{n} \frac{1}{2k-1},$$

are truly wonderful.

— *George E. Andrews*
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