The Mechanics of Arrow Flight upon Release

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Abstract

The dynamic behavior of arrows upon release in Olympic archery is explored. Mechanical properties that may affect the tune of the arrow to the archer and the bow, the forgiveness of the arrow to errors, and the clearance of the arrow from the bow are examined and discussed. In addition to the classic properties of geometry and static spine, the mass distribution and natural frequencies of the arrow are considered. Initial experimental results are presented. Recommendations for future arrow design are forwarded.

I. INTRODUCTION

The equipment used in Olympic archery has improved dramatically over the past three decades. Most of these improvements were a result of new materials, such as carbon foams and fibers, which enabled arrows to be shot at greater velocities with less effort than previously possible, and improved manufacturing techniques, which have reduced the variability in performance between arrows in any given set. Modern equipment, as shown in Figure 1, and the rule that the string must be drawn and released by the fingers have made the skill of the archer the greatest contributor to misplaced shots.

Figure 1. An Olympic style recurve bow.

In competition, arrows are shot in sets of up to six arrows. The design of arrows in the past and present have thus focused on reducing the variability in the geometry, mass, and flexural properties between similarly constructed arrows. The reasoning was that if such variation were minimized, so would be the shot errors created by such variation. A typical arrow construction is shown in Figure 2. Many archery competitors spend a great deal of effort in selecting and matching the aforementioned properties in their sets of arrows.

An arrow shaft is typically tubular in geometry, and constructed of aluminum, carbon fiber and epoxy, or a combination of these materials. Shafts designed for outdoor competition are typically as thin as practical, to be less affected by crosswind. The geometry, mass, and flexural properties of an arrow shaft are easily measurable quantities. Thus, manufacturing techniques have been developed to reduce the variation in these qualities such that shafts of the same model perform essentially identical to one another.

The ability of the equipment to resist errors induced by the archer, or by variation in the equipment itself, is called its "forgiveness". Equipment that is forgiving will produce less error in arrow flight than equipment that is un-forgiving, for the exact same mistake made by the archer.

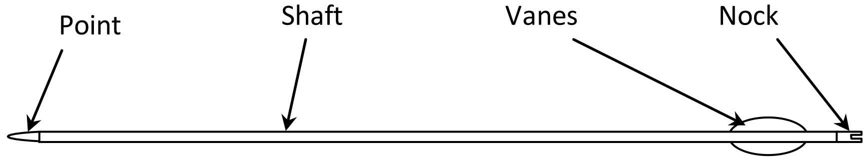

Figure 2. Parts of an arrow.

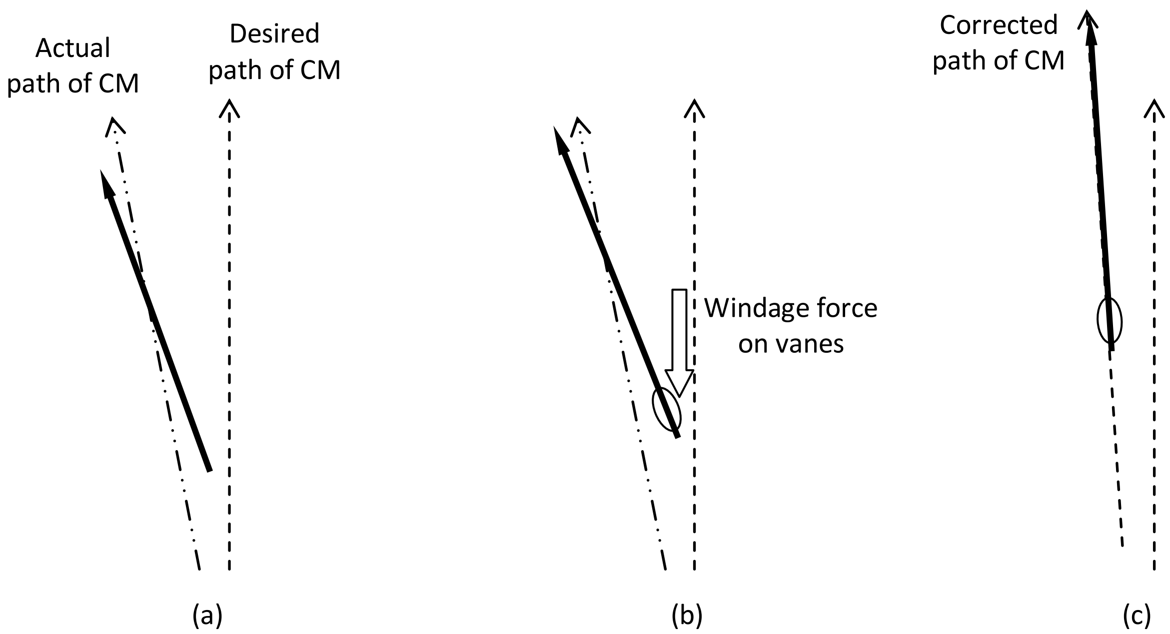

Figure 3. Effect of vanes on arrow flight.

The classical method of improving the forgiveness of the equipment is by "tuning" the arrow set to the bow and the archer. There are many tuning methods that are used, some of which are only qualitative in nature. Most tuning methods involve a process where the flexural stiffness of the shaft is selected based upon its length and draw weight of the bow from which it is shot. Further tuning involves selection of a mass for the arrow point, adjustment of the sideways force on the arrow produced at the arrow rest, and fine adjustment of the draw weight of the bow. Proper tuning of the equipment should minimize the probability that the arrow will strike the bow as the arrow is launched.

It is possible, however, that classical methods for tuning the equipment for improving its forgiveness may not also minimize the probability of secondary contact between the arrow and bow upon launch. A properly designed and properly selected arrow shaft should yield both qualities. An improperly designed or improperly selected shaft may be difficult or impossible to tune. The following discussion describes a classical tuning technique, followed by a proposed design or selection technique that can be used to ensure that the arrow proper clears the bow upon launch. The technique is confirmed by experimental measurements on two commercial arrow shafts that are known to perform well.

II. A QUALITATIVE REVIEW OF CLASSIC TUNING

The most widely accepted method of equipment tuning is a process known as "bare-shaft" tuning. The purpose of bare-shaft tuning is to minimize the effort required for the vanes to correct the flight of the arrow. This method thus minimizes the error dependence on the vanes, and possible variations in their construction and placement.The effect of the vanes in correcting initial arrow flight is shown in Figure 3. In (a), without vanes, the path of the center-of-mass of the arrow and the axis of the arrow may not be aligned. When vanes are added to the arrow in (b), any misalignment between the path of the center-of-mass and the axis of the arrow will result in a windage force on the vanes. The windage force is highest at arrow launch, when the misalignment is the greatest. When the misalignment switches to the other side of the arrow, the windage force also switches to that side, causing the arrow to wobble. As shown in (c), the path of the arrow will be corrected in the direction of the original tail misalignment, which is the direction of the large initial correcting force, because the wobble is quickly damped out.

Since the initial correction forces are greater when the initial misalignment is greater, the vanes tend to reduce the error to the desired arrow path as produced by release errors. The process of bare-shaft tuning involves shooting a group of arrows that have vanes, and then shooting a set of similarly constructed arrows that do not have vanes. Assuming that there are no gross errors made by the archer, the vaned shafts and the bare shafts will typically land in two separate groups, as shown in Figure 4.

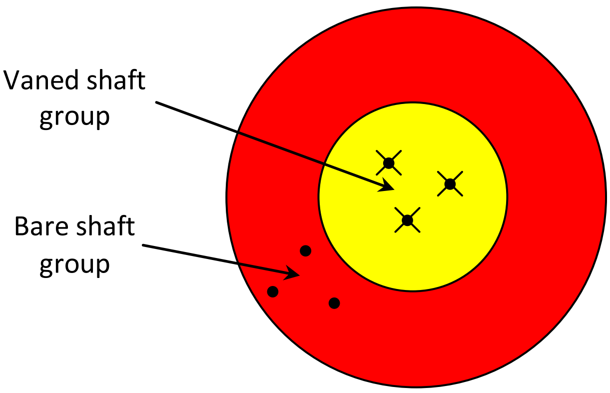

Figure 4. Bare shaft tuning on a target face.

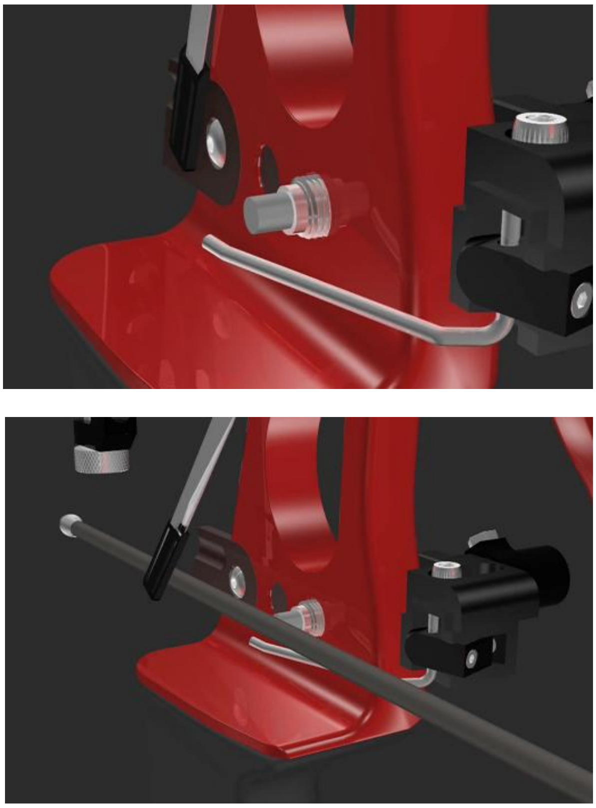

Figure 5. The plunger button.

In the case of Figure 4, the bare shafts have landed to the left of the vaned shafts. This case indicates that the original misalignment of the arrow upon launch was the tail to the right of the original path of the center-of-mass, because the vaned shafts were corrected in that direction. Had the bare shafts landed to the right of the vaned shafts, the original misalignment of the arrow upon launch was the tail to the left of the original path of the center-of-mass.

The bare shaft group and the vaned shaft group can be forced to merge by adjusting the side pressure on the arrow upon launch with the plunger button at the arrow rest. The plunger button, as shown in Figure 5 before and after an arrow is inserted, is spring loaded with a preload. Both the preload and the spring constant are adjustable.

When an arrow is released from the fingers, the bowstring and tail of the arrow are deflected slightly sideways (to the left for an archer drawing with the right hand) as the string must still travel around the fingers after they are relaxed. As the string pushes the arrow forward,

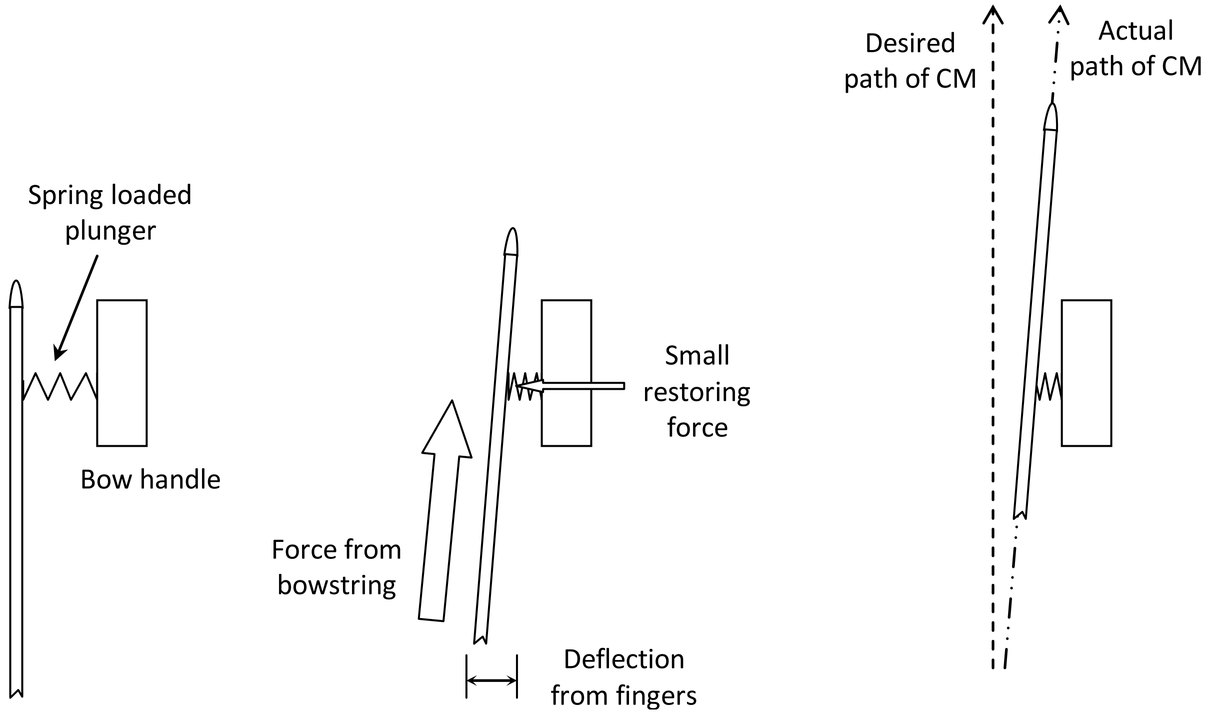

Figure 6. Overly compliant plunger button.

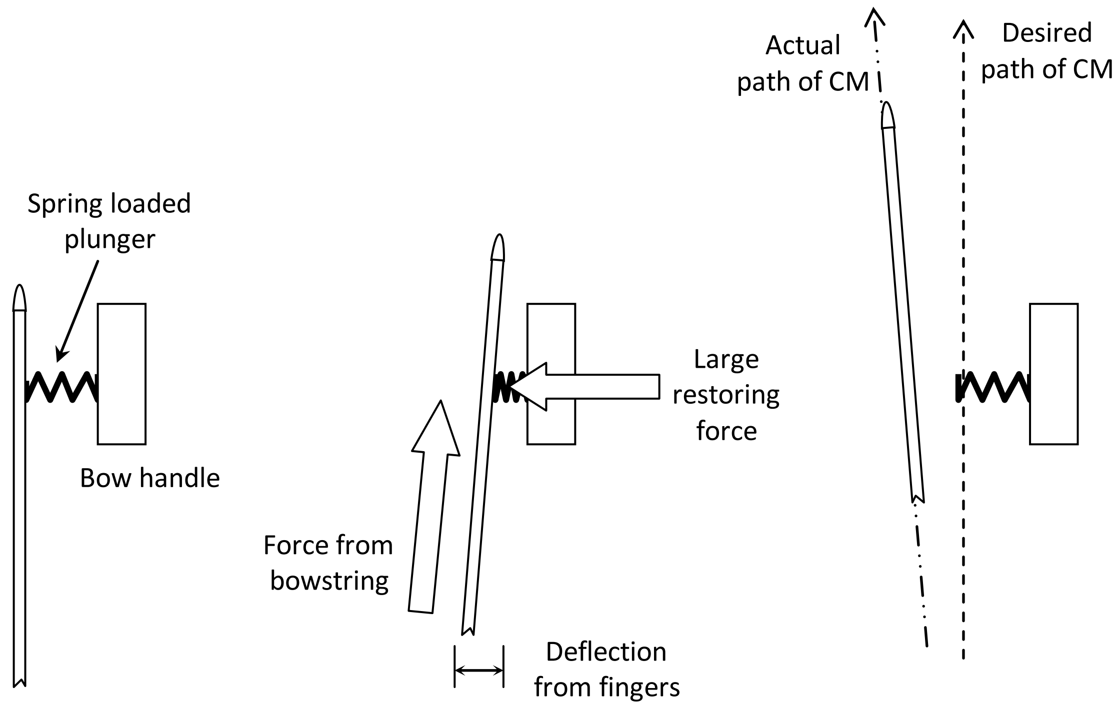

Figure7. Overly stiff plunger button.

it also imparts a force directed sideways at the bow handle. As a result, the plunger button is depressed. Figure 6 shows the result if the preload or spring constant is overly compliant. The tip of the arrow moves toward the bow handle, causing the tail of the arrow to be oriented to the left of the intended flight path. The error in the flight path would then be partially corrected by the vanes. The amount of correction can be minimized by adjusting the stiffness or pre-load of the plunger spring.

If the plunger spring is overly stiff, as shown in Figure 7, the tip of the arrow rebounds off the plunger, causing the tail of the arrow to be oriented to the right of the intended flight path. If however the stiffness of the plunger button is correctly chosen to compensate for the deflection error caused by the fingers upon release, the arrow would be aligned to the path of the center-of-mass, the bare shafts would group with the vaned shafts and the equipment would be tuned.

The stiffness of the plunger alone cannot be used to tune the arrows to the bow, because the arrows themselves are elastic. In the arrow industry, the static stiffness is measured in terms of a flexural deflection called “spine”, which is defined when a standard force is applied to the center of a standard length shaft, as defined in Figure 8. For a simply supported Euler-Bernoulli beam that is loaded in the center with a concentrated force, the maximum deflection of the beam (at the center) is Gere and Timoshenko (1997)

$$w_{max} = \frac{PL^3}{48EI} \qquad (1)$$

where w = transverse deflection
P = static load applied at the center
L = distance between the supports
E = elastic modulus
I = area moment-of-inertia about the neutral axis for the cross section of the beam

For a tube, which is the most common shape for an arrow shaft, the area moment-of-inertia is

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$

where D_o = outer diameter of the tube
D_i = inner diameter of the tube

It is important to note that the static flexural stiffness of the beam, which is the arrow shaft in this case, is independent of the mass off the shaft, and also independent of the mass of any components added to the shaft (that do not also contribute to the elastic modulus or area moment of inertia).

It is evident from Equation (1) that, with the length of the shaft held constant, that the flexural stiffness may be increased by increasing the elastic modulus, as by changing the material from aluminum to carbon composite. The flexural stiffness can also be dramatically increased by increasing the outer diameter of the shaft, which can be done without increasing the mass of the shaft if the inner diameter is also increased appropriately.

The static spine of an arrow shaft is most often used as a reference, or starting point, for the selection of an arrow shaft that can be tuned to the bow. The spine of an arrow shaft is a reference indication of the relative stiffness or flexibility of the shaft, in itself cannot be used as a means of selecting a shaft that will tune to the bow with a given draw weight. The flexibility of the shaft acts in conjunction with other factors, including the stiffness of the plunger button and the mass placed at the tip and tail of the shaft.

The mechanics of arrow deformation at the instant of release is illustrated in Figure 9. The tail of the arrow is deflected sideways by the fingers. The amount and the repeatability of the deflection is determined by the skill of the archer. The sideways deflection and the load from the bowstring in a direction along the shaft causes the shaft to buckle slightly. The amount of buckling depends on the flexural stiffness of the shaft, the amount of any mass pile at the tip or tail, and (to a lesser extent) the mass of the shaft itself. In a typical application, the mass of the point placed at the tip ranges from 40-80% of the mass of the shaft alone. A larger mass at the tip of the arrow

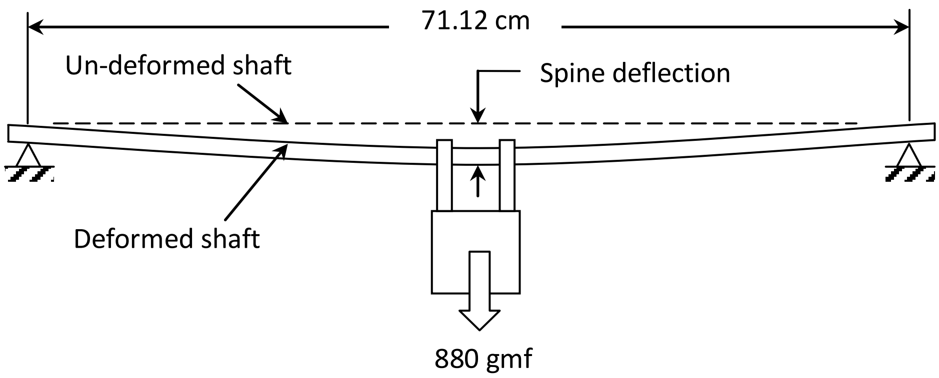

Figure 8. Definition of static spine.

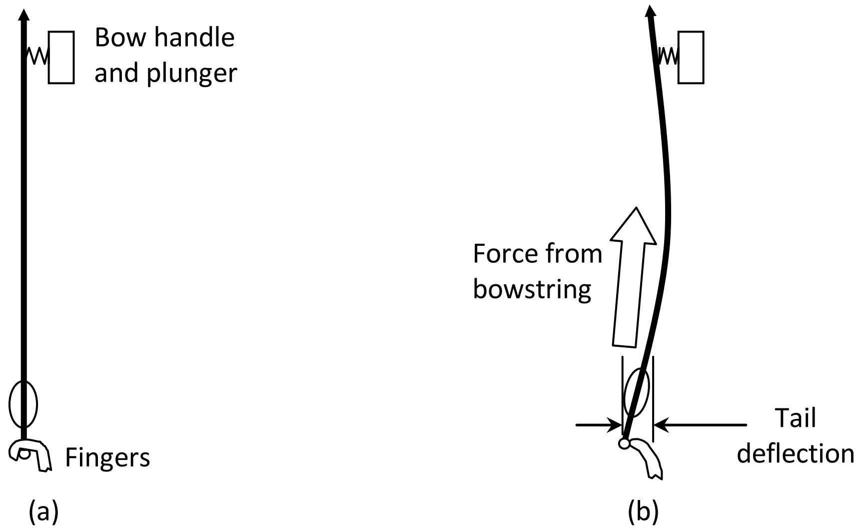

Figure 9. Buckling deformation of an arrow upon release.

will causes the tip to resist the initial change in motion from the force applied by the shaft, causing the shaft to deform more, with the same effect as decreasing the stiffness of the shaft. A smaller tip mass would have the opposite effect. Conversely, a larger mass at the tail of the arrow creates a tendency for the tail to resist the initial change in motion from the force applied to the bowstring, causing the shaft to deform less, with the same effect as increasing the stiffness of the shaft. Reducing the tail mass would have the opposite effect.

The deformation of the arrow upon release creates the same effect as depressing the plunger button in altering the flight path of the arrow by changing the path of its center-of-mass. The compliance of the arrow can thus be added to the compliance of the plunger button in much the same way as would be springs in series. If their combination is overly stiff, the tail of the arrow initially will leave the bow oriented to the right of trajectory of the arrow center-of-mass (for a right-hand release) and the bare shafts will group to the left of the vaned shafts. The two groups can be forced to merge by reducing the flexural stiffness of the arrow, or the plunger button, or both. If the combination of the shaft and the plunger button is overly compliant, the tail of the arrow will leave the bow oriented to the left of the trajectory of the arrow center-of-mass (for a right-hand release) and the bare shafts will group to the right of the vaned shafts. In this latter case, the two groups can be forced to merge by increasing the flexural stiffness of the arrow, or the plunger button, or both.

III. THE ARCHER'S PARADOX AND CLEARANCE FREQUENCY

Following the initial transverse deflection cause by the necessity for the bowstring to travel around the fingertips upon release, and the buckling caused by the force of the bowstring, the arrow shaft rebounds by deforming in the opposite transverse direction. This rebound is illustrated in Figure 10.

In the absence of any further transverse forcing function, the transverse response of the arrow becomes a summation of its transverse Eigen-modes at their natural frequencies. The first three Eigen-modes for arrow flexure are illustrated in Figure 11. Generally, the first Eigen-mode is the most significant because it carries the largest amplitude. If the frequency of the first Eigen-mode is correctly selected, the arrow will be launched from the bow without striking any part of it (including the plunger button) despite its transverse deformation. This phenomenon is known in the archery trade as the "archer's paradox", where the arrow "bends around the bow".

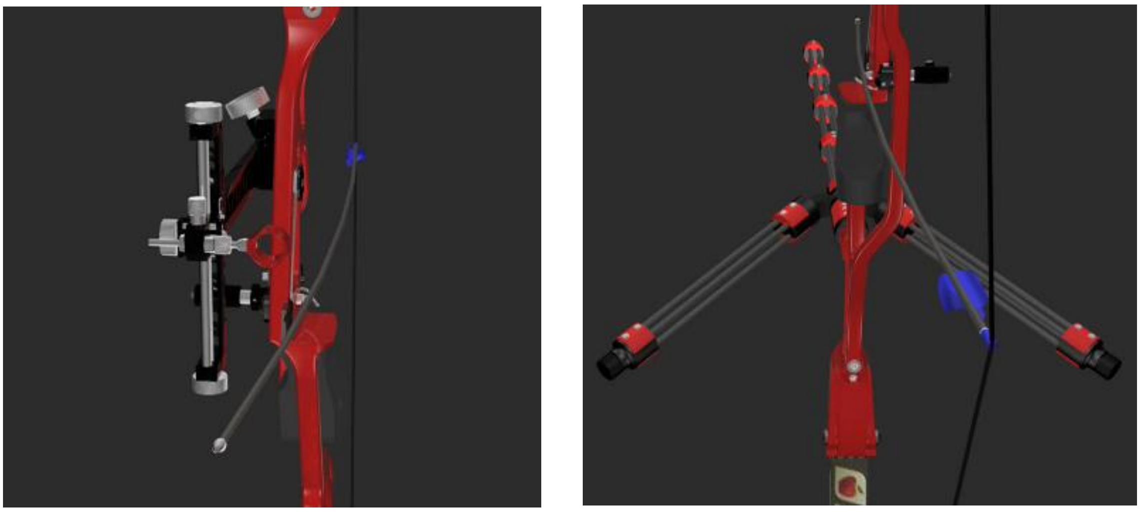

Figure 10. Arrow shaft rebound in the Archer's Paradox

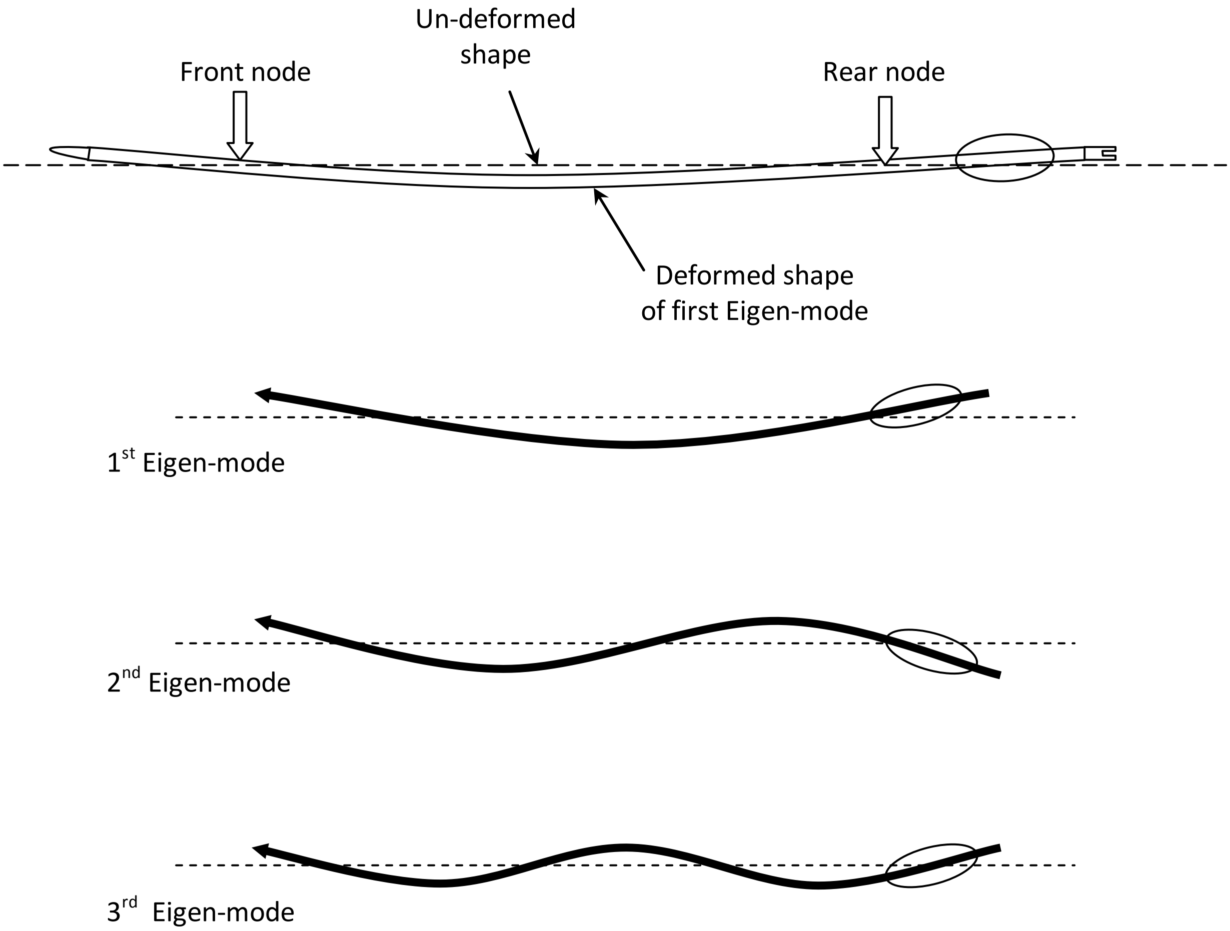

Figure 11. First three vibration modes of an arrow.

The ideal transverse arrow flexure for bow clearance, considering only the first mode, is shown in Figure 12. For clarity of illustration, the bow handle in this figure is shown moving with respect to the arrow so that the arrow vibration can be more clearly visualized. The arrow begins its vibration cycle upon release. If one complete cycle is concluded by the time the rear node of the arrow passes the plunger button, as shown in Figure 12(e), there should be no contact between the two objects between the nodes of the arrow. At this time, the part of the arrow rearward of the rear node is also moving transversely away from the plunger button. The vibration frequency for this occurrence can be found by calculating the time needed for the rear node of the arrow to reach the location of the plunger button from its starting position when the bow is at full draw. This time would be the time required for one full cycle.

Another situation of interest occurs if the tail end of the arrow passes the plunger button at 1.25 cycles, as shown in Figure 12(f). This condition presents the greatest amount of clearance between the tail and the plunger button, and offers the greatest clearance for vanes that are mounted to the tail. The vibration frequency for this occurrence can be found by calculating the time needed for the rear end of the arrow to reach the location of the plunger button from its starting position when the bow is at full draw. This time would be the time required for 1.25 cycles.

Examination of the theoretical solution for vibration of the shaft offers considerable insight into the factors that affect the frequency of vibration of an arrow. The Euler-Lagrange equation that describes the dynamic behavior of an Euler-Bernoulli beam is according to Craig (1981)

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = \mu\frac{\partial^2 w}{\partial t^2} + q \qquad (2)$$

where w = deflection
x = location along the beam
t = time
q = load (a function of time and location along the beam)
μ = mass density per unit length of the beam

In this case, the arrow is attached to the bowstring at the nock, but the sideways deflection is slight, and thus offers very little constraining load in the transverse direction once the arrow has been released. Not yet considering the concentrated masses at the tip and tail of the arrow, the arrow vibrates with free-free boundary conditions in the transverse direction. The boundary conditions for Equation (2) are thus

$$\frac{d^2 w}{dx^2} = 0 \quad \text{and} \quad \frac{d^3 w}{dx^3} = 0 \qquad (3)$$

When Equation (2) is integrated over the length of the shaft, the natural frequencies of the shaft for the j^{th} mode are

$$\omega_j = \frac{\lambda_j}{L}\sqrt{\frac{EI}{\mu}} \qquad (4)$$

where ω_j = the j^{th} natural frequency
λ_j = the j^{th} Eigen-value

Generally, only the first natural frequency and mode shape are of significance when considering the proper clearance of the arrow from the bow. Later testing with an accelerometer and spectrum analyzer would show that the second and third modes have less than 10% and 1%, respectively, the energy of the first mode.

The form of Equation (4) shows some important relationships between the construction of an arrow and it natural frequencies. The natural frequencies are inversely proportional to length, proportional to the square of the outer radius, proportional to the square root of the elastic modulus, and inversely proportional to the square root of mass per unit length. All the relationships need to be considered when designing or modifying an arrow for the proper clearance frequency. In addition, when masses are placed at the tip and tail of the shaft, the natural frequencies will certainly be decreased as the added mass increases.

The variables that affect static flexure and dynamic vibration of an arrow are closely related. For example, increasing either the elastic modulus of the shaft material or the outer radius of the shaft (even without increasing its mass) would increase both static spine and the frequency of vibration. Increasing the length of the shaft would decrease both static spine and vibration frequency. However, adjustment of other variables would create counter affects.

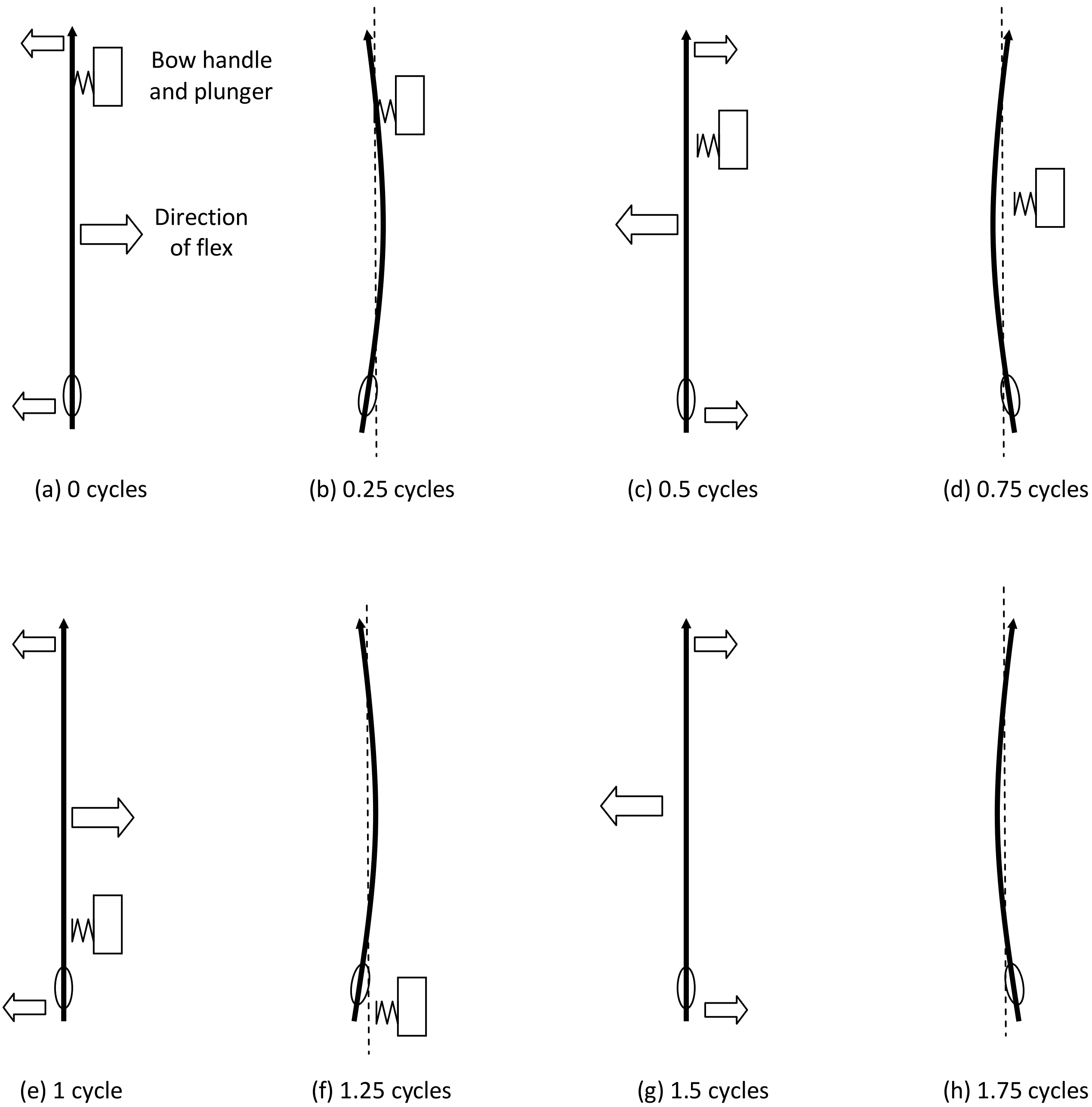

Figure 12. Illustration of the ideal clearance frequency for the first Eigen-mode.

For example, decreasing the density of the shaft material has no affect on static spine, but would increase the vibration frequency. Increasing the mass at the tail of the shaft would have no effect on the static spine, would cause the arrow to appear stiffer when launched, but would decrease the frequency of vibration. Clearly there are sufficient variables that can be adjusted that would cause an arrow to be tuned by classic bare shaft testing, but can make the clearance of the shaft and vanes either optimal or sub-optimal.

IV. EXAMPLES OF CLEARANCE FREQUENCY CALCULATION

Two cases were considered as examples of calculating the ideal clearance frequencies. One was the case of an arrow made with an Easton ACC 3L-18 shaft. The second was an arrow made from a Carbon Tech McKinney II 600 shaft. Both arrows were 76.5 cm (30.12 in) long from the nock groove to the tip of the point. Both types of arrows were shot from AMO 68" recurve bow made from a 25" Hoyt Matrix handle with Hoyt M1 limbs and 16 strand BCY 8125 Dyneema bowstring. The final draw weight was 169 N (38 lb). The distance from the plunger button to the neutral position of the string, known in the industry as the "brace height" was 22.5 cm (8.87 in). The bow was drawn an additional 48.6 cm (19.13 in) to achieve the final draw weight. Both arrows were shot for several weeks by an experienced archer, and were successfully tuned using the bare shaft method. High-speed video and close inspection of the arrows for signs of contact confirmed that both types of arrows cleared the bow well. The location of the nodes for the first Eigen-mode for an arrow were found by gently holding the arrow shaft between two fingers and continually tapping the far end of the arrow against a padded surface, such as upholstered chair. The nodes were located where the resulting vibration was felt to be at its minimum.

The final velocity of the arrow at launch was calculated with an energy balance. The limbs were assumed to produce a draw force that was linear with draw distance. The bow was assumed to be a modern recurve bow with a limb and string efficiency of 90%. A balance of the kinetic energy of the arrow with the potential energy in the limbs gives

$$\frac{1}{2}m_a v_o^2 = \varepsilon \frac{1}{2} F_o h_o \qquad v_o = \sqrt{\frac{\varepsilon F_o h_o}{m_a}} \qquad (5)$$

where m_a = total mass of the arrow
v_o = final speed of the arrow
ε = efficiency of the bow (90%)
F_o = final draw weight
h_o = draw beyond the brace height

The time required to achieve the final arrow velocity is the time required for the bowstring to return to its neutral position, where the arrow is assumed to separate from the string, from being released from the drawn position. The arrow on the string is analogous to a mass on a linear spring. The frequency for such a system is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ Hz}$$

$$\text{or} \qquad f_s = \frac{1}{2\pi}\sqrt{\frac{\varepsilon F_o}{m_a h_o}} \text{ Hz} \qquad (6)$$

where k = spring constant for the arrow on the bowstring = F_o/h_o

When the bowstring completes 0.25 cycle, it has returned to its neutral position. Thus the time for it to reach this position is

$$t_o = \frac{\pi}{2}\sqrt{\frac{m_a h_o}{\varepsilon F_o}} \qquad (7)$$

Once this time and position has been reached, the arrow continues forward at the velocity specified by Equation 5. The time for the first node to pass the plunger button becomes

$$t_1 = t_o + \frac{h_b + h_n}{v_o} \qquad (8)$$

where h_b = brace height
h_n = distance from the tail to the rear node

The clearance frequency for the arrow requires that one cycle be completed in the time specified by Equation (8). The clearance frequency is thus

$$f_1 = \frac{1}{t_1} \qquad (9)$$

The time for the tail end of the arrow to pass the plunger is

$$t_2 = t_o + \frac{h_b}{v_o} \qquad (10)$$

Maximum clearance between the tail and the plunger requires that 1.25 cycles be completed in the time specified by Equation (10). The frequency for maximum tail clearance is thus

$$f_2 = \frac{1.25}{t_2}$$

The results of the calculations for the two test cases are shown in Figure 13. For the bow, draw length and draw weight specified, the calculations predict a final arrow velocity of 57.9 ms and 63.9 m/s for the ACC and McKinney II arrows,

Arrow Type:	Easton ACC, 3L-18
Spine:	620 (1.58 cm)
Shaft Weight:	222 gr
Shaft Length:	74.93 cm
Point:	100 gr
Nock:	Easton G, 7 gr
Vanes:	4.44 cm Spin Wings, 2 gr
Rear node:	12.7 cm from tail (5 in) (measured)
Total Mass:	341 gr
Brace height:	22.5 cm (8.87 in)
Draw height:	48.6 cm (19.13 in)
Draw weight:	169 N @ 48.6 cm (38 lb @ 19 in)
Predicted results:	
Arrow speed:	57.9 m/s (190 ft/s)
Clearance frequency:	66 Hz
Max vane clearance:	72 Hz

Arrow Type:	Carbon Tech MK 2
Spine:	600 (1.52 cm)
Shaft Weight:	161 gr
Shaft Length:	73.66 cm
Point:	110 gr
Nock:	Easton G, 7 gr
Vanes:	5.08 cm Spin Wings, 2 gr
Rear node:	17.8 cm from tail (7 in) (measured)
Total Mass:	270 gr
Brace height:	22.5 cm (8.87 in)
Draw height:	48.6 cm (19.13 in)
Draw weight:	169 N @ 48.6 cm (38 lb @ 19 in)
Predicted results:	
Arrow speed:	63.9 m/s (209 ft/s)
Clearance frequency:	78 Hz
Max vane clearance:	80 Hz

Figure 13. Arrow construction and predicted clearance frequencies for two cases.

respectively. The clearance frequencies were 66 Hz and 78 Hz, respectively. The maximum vane clearance frequencies were 72 Hz and 80 Hz, respectively.

V. EXPERIMENTAL MEASUREMENT OF ARROW FREQUENCIES

The natural frequencies of the arrows used in the two test cases were measured using an accelerometer and a spectrum analyzer. The accelerometer was an Endevco model 22, which was mounted on the arrow on its point using a thin layer of mounting wax and secured with Mylar tape. The arrow was excited by holding it gently between two fingers at the approximate location of the front node and tapping the tail against a padded surface. A total of 6 arrows of each type were tested. The experimental setup is shown in Figure 14.

The first three natural frequencies were easily identifiable. The repeatability and variation of the frequencies between arrows of the same type were less than 0.5 Hz for the first mode, 1 Hz for the second mode, and 2 Hz for the third mode. The results of the tests, with the average measured frequencies are shown in Figure 15. The results show a remarkably good match with the predicted frequencies required for proper clearance of the arrow and its vanes.

VI. CONCLUSIONS AND IMPLICATIONS FOR FUTURE ARROW DESIGN

The prediction and measurement of at least the first natural frequency of an arrow may offer new clarity in proper arrow design and proper tuning of arrows for optimal forgiveness and proper clearance from the bow. Although many arrow shafts on the market today appear to offer proper clearance at the same time that the arrow is tuned by the classic (bare-shaft) method, these designs may have been the result of natural evolution of arrow shaft design, as unsuccessful designs are soon eliminated from the market for their poor performance. The examination of arrow frequencies may become more important as lighter stiffer materials are developed and used in shaft construction. Future arrow design may see the development of shafts that are very thin (for cross-wing resistance), but also very light and very stiff. Tip and tail masses may then be added as needed to achieve the desired total arrow mass, point mass for stability and tunability by classical methods, tail mass for additional forgiveness and tunability by classical methods, and arrow frequency for proper clearance.

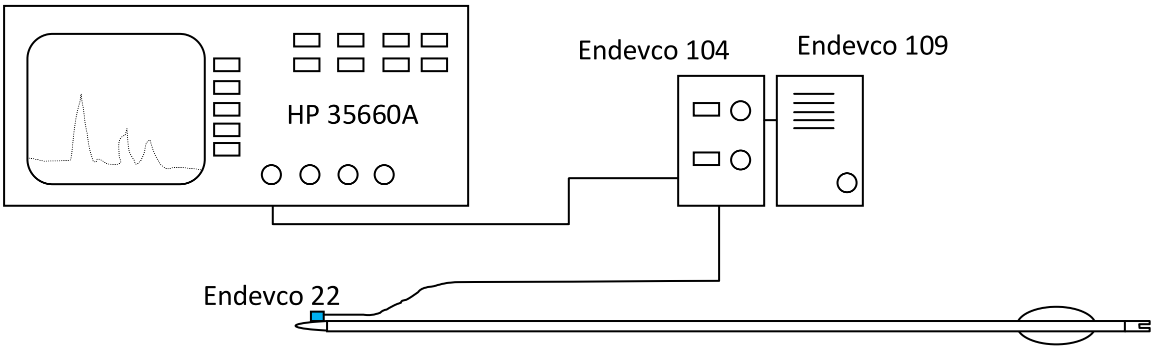

Figure 14. Experimental setup for measuring the natural frequencies of an arrow.

Arrow Type:	Easton ACC, 3L-18
Spine:	620 (1.58 cm)
Shaft Weight:	222 gr
Shaft Length:	74.93 cm
Point:	100 gr
Nock:	Easton G, 7 gr
Vanes:	4.44 cm Spin Wings, 2 gr
Rear node:	12.7 cm from tail (measured)
Total Mass:	341 gr
Measured Frequencies	
1st Mode:	68 Hz
2nd Mode:	207 Hz
3rd Mode:	424 Hz

Arrow Type:	Carbon Tech MK 2
Spine:	600 (1.52 cm)
Shaft Weight:	161 gr
Shaft Length:	73.66 cm
Point:	110 gr
Nock:	Easton G, 7 gr
Vanes:	5.08 cm Spin Wings, 2 gr
Rear node:	17.8 cm from tail (measured)
Total Mass:	270 gr
Measured Frequencies	
1st Mode:	78 Hz
2nd Mode:	244 Hz
3rd Mode:	492 Hz

Figure 15. Arrow construction and measured frequencies.

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