Worksheet 11 (7.4, B4.2 - B4.3)

1. Find a singular value decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

2. Find the general solution to the differential equation

$$y^{(4)} + 2y'' + y = 0$$

3. Explain why a linear homogeneous differential equation (of any order) with constant coefficients can never have $y = \ln t$ as a solution.

4. (a) Say we have a second order linear homogeneous differential equation with constant coefficients (the coefficient of y'' must be nonzero so that this is actually second order). Explain why it is not possible for e^t, e^{2t}, and e^{3t} to all be solutions of such an equation.

   (b) Does this remain true if we remove the condition that the equation has constant coefficients?

5. (a) Say A is an orthogonal matrix. What will a singular value decomposition of A look like?

   (b) Say A is a nonsquare matrix whose columns are orthonormal. What will a singular value decomposition of A look like?

   (c) Say A is such that $A^T A = 0$. What will a singular value decomposition of A look like?

6. Say we have a linear homogeneous differential equation (of any order) with constant coefficients.

   (a) If $y = \sin t$ is a solution for this equation, must $y = \cos t$ also be a solution? If so, prove why, and if not give an example where this is not the case.

   (b) Say same question with $y = e^t \sin t$ being a solution, is the same forced of $y = e^t \cos t$?

7. Is it possible to have two singular value decompositions of some nonzero matrix A such that in the two decompositions the $\sum$ and V matrices stay the same, but the U matrix is different (in order to be considered a singular value decomposition U and V must be orthogonal matrices and $\sum$ must be a "diagonal" matrix).