Lab 1: Introductory Experiments and Linear Circuits I

Christopher Agostino
Lab Partner: MacCallum Robertson
February 19, 2015

Introduction

In this lab, we intend to learn how to properly use the equipment that we will use in this course and in any setting where understanding and debugging circuits are essential skills. In addition to this, we seek to learn about linear circuits which comprise some of the simplest circuits we can create. A mastery of linear circuits, their components, such as resistors, capacitors, etc. and various measuring tools is necessary for understanding and debugging more complex circuits. Specifically we will learn the basics of breadboards, Digital Multimeters, an Oscilloscopes, function generators, and power supplies. We will be introduced to the concepts of complex impedance, the generalized Ohm’s Law, voltage dividers, Thevenin equivalent resistance and voltage. We will try to distinguish the differences between AC and DC signals and their associated properties and figure out the best ways to measure their properties such as the peak voltage and the root mean square voltage. We will also begin to combine resistors and capacitors to make useful devices such as filters.

1 Lab Exercises: Part I

1.1

Breadboards often have two rails along the side of them separated from the rest of the board. These rails are typically where voltages are connected and floated along the rails, horizontally in our picture, whereas on the rest of the breadboard any connection is made perpendicularly along each separate row.

![Breadboard connections](image.png)

Figure 1: Breadboard connections. Source: Lab manual

1.2

(a) In this scenario, one would connect the +12V terminal to the red terminal on the Digital Multimeter as well as connecting the −12V terminal to the GND one on the power supply before connecting that to the ground(black) terminal on the Digital Multimeter. The DMM then reads +24V
(b) In order to have the DMM read $-24V$, one must connect the $+12V$ terminal to the $GND$ terminal on the power supply then to the black terminal on the DMM and then connect the $-12V$ terminal directly to the red terminal on the DMM.

Figure 2: +24 V. Source: MacCallum Robertson

(c) This situation is asking for $+12V$. To do this we connect the $+12V$ terminal to the red terminal on the DMM and connect the $0V$ terminal to the $GND$ terminal and then to the black terminal on the DMM.

Figure 3: -24 V. Source: MacCallum Robertson

(d) Similarly to the above situation, we connect the $-12V$ terminal to the red DMM terminal and the $0V$ to the $GND$ and then to the black DMM terminal.

Figure 4: +12 V. Source: MacCallum Robertson

(e) The $+17V$ one is the fun case. In order to get this voltage, one connects the $+12V$ terminal to the red DMM terminal. Then one connects the $0V$ terminal to the $+5V$ terminal, which sets the $5V$
as the common point in the float supply so the +12V is now 12V relative to 5V, making it 17V. We are not done there yet, we must also connect the GND terminal to the black terminal on the DMM to complete the circuit.

![Figure 6: +17 V. Source: MacCallum Robertson](image)

1.3
If one were to try to measure the potential between the +12V output and the 5V supply ground, one would measure 0V because the 12V supply is a floating power supply so it is not electrically connected to ground whereas the 5V supply is relative to ground. The 12V is not relative to the ground supply so it would not make any sense to try to measure the difference between it and the 5V supply.

1.4 1.1.4
In order to derive the voltage divider equation for the voltage divider present in figure 7, we start with Ohm’s law, $V = IR$.

![Figure 7: Voltage Divider](image)

We note that $R_{tot} = R_1 + R_2$ and by charge conservation in this series resistor circuit, the current through each resistor has to be the same. By Kirchhoff’s Law the voltage drop through the entire circuit has to be equal to $V_{in}$. Thus we can set up this system of equations and recognize that $V_{out}$ must be equal to the voltage drop across only the second resistor.

$$V_{in} = I(R_1 + R_2)$$
$$V_{out} = I(R_2)$$

We solve each for $I$ then set them equal to each other and find that

$$I = \frac{V_{in}}{R_1 + R_2} = \frac{V_{out}}{R_2}$$

which yields the final result by solving in terms of $V_{out}$ to be

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

1.5 1.1.5
Here we are looking to find the current in our circuit given that $R_2$ is 470kΩ and $R_1$ is 10kΩ. We take $V_{in}$ to be 24V and solve accordingly using our relationships derived in 1.1.4

$$I = \frac{24V}{480k\Omega} = 5.0 \times 10^{-5} A$$
1.6 1.1.6

Here we are solving for $V_{\text{out}}$. We use our equation derived in 1.1.4 given our resistor and voltage values.

\[ V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_2 + R_1} = 24V \times \frac{470k\Omega}{480k\Omega} = 23.5V \]

1.7

a One would connect the DMM in series with the resistor to measure the current through it as shown in Figure 8 below.

![Figure 8: Ammeter in series](image)

b One would connect the DMM in parallel with the 470$k\Omega$ resistor in order to measure the voltage drop across it as shown in Figure 9 below.

![Figure 9: Voltmeter in parallel](image)

1.8

I measured the actual values of the 10$k\Omega$ and 470$k\Omega$ resistors using the DMM and this information, as well its tolerance and deviation from its supposed value, is in Table 1 below, where our Percent Error is defined as

\[ \frac{\text{Theoretical} - \text{Actual}}{\text{Theoretical}} \times 100\% = \text{Percent Error} \]
<table>
<thead>
<tr>
<th>Resistance</th>
<th>Actual Resistance</th>
<th>Tolerance</th>
<th>Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10kΩ</td>
<td>9.88kΩ</td>
<td>5%</td>
<td>1.2%</td>
</tr>
<tr>
<td>470kΩ</td>
<td>469.72kΩ</td>
<td>5%</td>
<td>.49%</td>
</tr>
</tbody>
</table>

1.9

The actual voltage we measured using the Digital Multimeter was 24.35V and the measured voltage drops across the 470kΩ and 10kΩ resistors were 23.819V and .5V respectively. Thus, using our actual voltages and actual resistor values, we can calculate the current in the circuit.

\[ I = \frac{24.35V}{9.88k\Omega + 469.72k\Omega} = 5.08 \times 10^{-5} A \]

which is off from our predicted value of 5.0 \times 10^{-5} by 1.6%. We can also use our actual voltage and resistor values to calculate what the actual value for \( V_{out} \) should be.

\[ V_{out} = 24.35 \times \frac{9.88k\Omega}{9.88k\Omega + 469.72k\Omega} = .5016V \]

which is fairly close to the value of 23.819V which we measured using the DMM. This predicted value of .501V is off from the predicted value of .5V we calculated in part 1.6 by .3%. This could be for a variety of reasons though I think it is most likely due to the difference in the \( V_{in} \) between our ideal input and our actual input as well as likely being

1.10

We measured 23.819V for the voltage drop across the 470kΩ resistor. This is .13% off from our calculated value in part 1.9. We find the range of values by using the given formula and find that

\[ .012\% \times 23.819V + .04\% \times .0001V = .00029V \]

which means our values range from 23.819-.00029 to 23.819+.00029 making our range 23.8188−23.8194 The voltage is fairly close to the one we calculated in part 9 but it unfortunately is not within the measurement uncertainty. Our measurement was .13% off from the ideal value.

1.11

We measured our current to be .0516mA through both of the resistors. Now, using the given formula like in part 10

\[ .0516mA \times .00012 + .0004 \times 100mA = .04mA \]

Therefore, the value we calculated in part 9 is within the range of uncertainty.

1.12

(a) To calculate the power dissipated by each of the resistors in the voltage divider, we use our ideal values of 5.0 \times 10^{-5} A and 470kΩ

\[ P = I^2R = (.00005A)^2 \times 470k\Omega = 1.175mW \]

and we do the same for the 10kΩ resistor.

\[ P = (.00005A) \times 10000\Omega = 25\mu W \]

Both of these resistors are rated for power up to 1/4 W so they are rated for the power output in this circuit.

(b) Each resistor has a power rating up to a 1/4W and in order to reach that, one would decrease the resistance of the resistors because resistance is inversely proportional to the power for some fixed voltage.
(c) The 470kΩ resistor would reach its max power rating first because it has a higher resistance.

(d) In order to find the max value for one of the two resistors to exceed its power rating, we must calculate the resistor value at which a 24V signal would generate .25W.

\[ .25W = \frac{24^2V^2}{xΩ} \rightarrow x = \frac{576V^2}{.25W} = 2304Ω \]

Now this is not a resistance one is likely to encounter in any real situation so I will assume a value of 2.2kΩ as it is the closest common value. Now we can use our ratio to calculate the other resistor value we would need.

\[ \frac{x}{2.2kΩ} = \frac{470000kΩ}{10000kΩ} \rightarrow x = 103400Ω \approx 100kΩ \]

So in order to do this, you would need to make both resistors smaller by a factor of about 4.5

(e) This would not be very easily accomplished in the lab as these resistances do not match those found in common resistors. It would, however, be easy to do something quite similar with 2.2k and 100k resistors.

1.13

When trying to find the signal on the scope after the settings have been messed with, it is important to check the V/div, the vertical position, the time scale, and whether or not it is AC coupled, DC input, or set to ground. It could also be the case that the scope is in XY mode in which case you would simply turn off XY mode to get back to the normal input. In addition, it is wise to check whether or not the channel you are trying to measure a signal on is actually turned on. Otherwise, there aren’t that many more commonly used settings on the oscilloscope, or at least settings which will be applicable in this course.

1.14

We generated the triangular wave by hitting the Ramp button the function generator. We then generated a sawtooth like wave by adjusting the the output parameters on the initial ramp wave. Then we generated the pulsed squares by using the pulse option and adjusting the width of the pulse output and the frequency of the signal to have three waveforms appear on the scope screen.

1.17

(a) We connected the 5V power supply to the DMM in the 10V range and measured the actual voltage to be 5.0565V. Thus we can calculate the range using the same formula we used in 1.10 and 1.11

\[ .00012 \times 5.0565V + .0004 \times .1mV = .0061V \]

Therefore we can write the voltage measurement as being 5.0565 ± .0061V.

(b) We then connected the 5V power supply to the Oscilloscope and measured the voltage to be 5.10V. The range was 200 mV on the scope so we can calculate the uncertainty.

\[ .00012 \times 5.10V + .0004 \times .2V = 6.92 \times 10^{-4}V \]

which allows to write the voltage measurement as 5.10V ± 6.92 × 10^{-4}V

(c) We then altered the settings on the scope and remeasured the voltage. Using the range of 2V/div (.4 V/tick), we found that the Oscilloscope measured 5.04 V using channel one. When we plugged the same signal in to channel 2, we also measured 5.04 V at 2V/div. We can find the uncertainty of this measurement to be

\[ 5.04V \times .00012 + .0004 \times .4V = 7.65 \times 10^{-4}V \]

giving us a range of 5.04V ± 7.65 * 10^{-4}. The measurements by the DMM and scope with different settings are not consistent with each other within the calculated uncertainty and measure slightly different values for the voltage which is likely due to the two devices having different input impedances.
(d) For this setup, the scope should be set up such that the zero of the scope is at the bottom of the graph and the V/div should be as small as possible such that the highest point of the signal is still on the screen.

1.18

(a) We had the scope connected to the 5 volt supply and turned on the A.C. coupling. This setting ignores the DC offset voltage and only displays voltages associated with the alternating current part of the signal.

(b) When you look closer at the A.C. part of the signal, one begins to notice that there is no real clear pattern in the signals displayed on the scope. It has a kind of random wave for which oscillates around 0 V. Essentially, we are seeing the electrical noise from the power supply, which senselessly varies among low voltages around 0 V.

1.19

(a) For a voltage varying sinusoidally, we have some basic form for the voltage $V(\theta) = V_{\text{peak}} \sin(\theta)$ where $V_{\text{peak}}$ is the amplitude. We choose to calculate the root-mean-square voltage over the first quarter of the period as it is symmetric to the other parts.

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi/2} \int_{0}^{\pi/2} (V_{\text{peak}} \sin(\theta))^2 d\theta}$$

We can rewrite $\sin^2(\theta)$ using our double angle identity and then we can evaluate the integral to be

$$v_{\text{rms}} = \frac{\sqrt{2} V_{\text{peak}}}{\sqrt{\pi}} \sqrt{\theta/2 - 1/4 \sin(2\theta)}$$

evaluated at the limits of 0 and $\pi/2$

$$V_{\text{rms}} = \frac{\sqrt{2}}{\sqrt{\pi}} V_{\text{peak}} \sqrt{(\pi/4 - 0) - (0 - 0)} = V_{\text{max}} \sqrt{\pi/2} = \frac{V_{\text{max}}}{\sqrt{2}}$$

(b) Now we look at the triangular wave. Similarly, we go over a quarter of the cycle for simplicity and because of symmetry. For the first quarter we can write the voltage as a function of time being $V(t) = V_{\text{peak}} * t * 4/P$ where the $4/P$ comes in as the function reaches $V_{\text{max}}$ at time $t = P/4$. We then begin a similar analysis to find the root-mean-square voltage of the triangle wave.

$$V_{\text{rms}} = \sqrt{\frac{1}{P/4} \int_{0}^{P/4} (V_{\text{peak}}^2 t/P)^2 dt} = \sqrt{\frac{1}{P} \int_{0}^{P/4} t^2 dt}$$

The integral evaluates to $t^3/3$ evaluated at the limits 0 and $P/4$ which gets rid of the $(4/P)^3$ term and introduces a $1/3$ term which at the end evaluates to

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{3}}$$

(c) For square waves, the root-mean-square voltage is equal to its peak voltage because there is no variation over symmetrical parts of the waveform. The voltage is either equal to $-V_{\text{peak}}$ or $+V_{\text{peak}}$ and when squared it eliminates the possible for any variation such that

$$V_{\text{rms}} = V_{\text{peak}}$$

We then fed in 1Vpeak sine, triangle, and square waves at 1 kHz into the DMM and measured the $V_{\text{rms}}$ for each type of wave and found that the RMS voltages we measured were convincingly close to the values predicted by the coefficients we derived in this problem.
Table 2: $V_{\text{peak}}$ vs $V_{\text{rms}}$ for various waveforms

<table>
<thead>
<tr>
<th>Type of Wave</th>
<th>$V_{\text{peak}}$</th>
<th>$V_{\text{rms}}$ (V)</th>
<th>Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>1.002</td>
<td>.71437</td>
<td>.83%</td>
</tr>
<tr>
<td>Triangle</td>
<td>1.00192</td>
<td>.57851</td>
<td>.0089%</td>
</tr>
<tr>
<td>Square</td>
<td>1.00192</td>
<td>1.00198</td>
<td>.00599%</td>
</tr>
</tbody>
</table>

1.20

Here I have supplied a 1 V peak-to-peak sine wave and varied the frequency between 10 Hz and 10MHz and I plotted the RMS voltage as measured by the Oscilloscope and the DMM with green and blue dots respectively as a function of the frequency of the sine wave.

![Figure 10: Voltage readings for Oscilloscope and DMM as a function of frequency](image)

We took measurements at geometric multiples of 10 starting at 10 and also at 50 in order to get two measurements per interval. Measuring at constant arithmetic intervals would have been tedious as the range is so wide and by using geometric intervals we can use a log plot to see a larger range of frequencies in a reasonable fashion. Assuming that the DMM is quite accurate at 10 Hz, which corresponded to a voltage of 357.6 mV, we found that the DMM starts becoming inaccurate at frequencies above 325 kHz at which we measured the voltage to be 355.8 mV, approximately 5% off from the initial value. The manual for the Keithley digital multimeter states that it should properly work over the range from 3 Hz to 300 kHz which seems to be what we are seeing here. As it gets higher away from its rated frequencies, the voltage strays from the value we should be measuring. It then quickly becomes much more inaccurate as the plot of the data shows.

1.21

We fed 1 Volt peak-to-peak square-wave into two different channels of the oscilloscope, one of them with the DC input on and the other with the AC coupling on. We used three different frequencies: 10 Hz, 100 Hz, and 10 kHz. For all three of these frequencies, the DC voltages looked how one would expect whereas the AC voltages became more square as frequency increased. At 10 Hz, the signal appeared as if it were a capacitor discharging such as in Figure 11.
We see this because AC coupling uses a capacitor in series with the normal input impedance creating a sort of high pass filter in order to block out DC and low frequency signals. Because of this, we see the signal as it has been attenuated by the scope.

2 Lab Exercises: Part 2

2.1

We grabbed a black box circuit and decoded the resistances for each resistor. Each one has a tolerance of ±5% and is pictured in Figure 12 below.

![Black Box Circuit](image)

Figure 12: Black Box Circuit

We put in the two 9V batteries and measured the output voltage by attaching leads to the output and ground pins on the black box and using the mini grabbers to find that the open-circuit voltage was 4.6376V. We then measured the short-circuit current by connecting a wire from the output to the ground and measuring the current on that wire and found it to be 1.4689mA. From this we can use Ohm’s aw to calculate the equivalent Thevenin resistance.

\[
R = \frac{V}{I} = \frac{4.6376V}{1.4689mA} = 3157\Omega
\]

Thus, our black box circuit has an equivalent Thevenin resistance of 3157Ω and we can redraw our circuit like the one below in Figure 13.
We then placed resistors of 100, 1k, and 10k Ω in the output pins, effectively creating a voltage divider where R1 is the Thevenin equivalent resistance and R2 is the variable resistance. The data is plotted below in Figure 14.

\[ V_{out} = \frac{V_{in} R_2}{R_1 + R_2} \]

which for values of R2 much greater than R1 can be Taylor expanded using a binomial expansion to be

\[ V_{out} = V_{in} \left(1 - \frac{R_1}{R_2}\right) \]

and as R2 becomes much larger than R1, the output voltage approaches the input voltage. We then calculated what the voltages for such systems should be, given our Thevenin resistance and voltage as shown in the table below.

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Ideal Voltage (V)</th>
<th>Measured Voltage (V)</th>
<th>Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>.1424</td>
<td>.14129</td>
<td>.77%</td>
</tr>
<tr>
<td>1000</td>
<td>1.116</td>
<td>1.08615</td>
<td>2.64%</td>
</tr>
<tr>
<td>10000</td>
<td>3.525 V</td>
<td>3.52</td>
<td>.43%</td>
</tr>
</tbody>
</table>

It can be seen clearly from the data in Table 3 that the output voltage is a function of the Thevenin resistance as part of the voltage divider we created to measure \( V_{out} \), albeit with some uncertainty which is likely due to the resistors not having the exact values they should. If I were to do this experiment again, I would use the DMM to measure each of the resistances that we used to make a voltage divider in order to get a more accurate prediction of the voltage.
We then removed the batteries and shorted the circuit and measured the actual resistance to be 3155Ω which is .0634% off from the value we predicted by using Ohm’s law.

2.2

We connected a four-foot BNC cable to the oscilloscope and equipped it with minigrabbers. We changed the settings such that the scope was 50 mV/div and 5 ms/div.

(a) We touched the red minigrabber’s lead and noticed a signal on the scope. The origin of the signal is because the wall outlets put out AC voltages at 60 Hertz and the light in the room hits our bodies at that frequency and when we touch the minigrabber, our body acts as a conductor for that signal and it’s channeled into the oscilloscope.

(b) We tried pinching the minigrabber’s insulation and there was no additional signal because the plastic casing insulates the minigrabber lead from outside electrical signals.

(c) We also tried pinching the BNC cable and noted that there was no added signal on the oscilloscope as the cable is electrically shielded.

(d) We connected a four foot long wire between the black and red minigrabbers and set the scope to read 2mV/div and 4µs/div and noticed a new signal appearing on the scope.

[Figure 15: Signal noise from four foot cable]

The origin of the signal pictured above in Figure 15 is the parasitic impedance of the wire as well as the length of the wire which causes the signal to oscillate back and forth in the wire.

2.3

We sought to measure the input impedance of the oscilloscope by measuring the scope’s input voltage for different resistances and plotting them as a function of the input current. I determined $I_{in}$ by dividing the input voltage by the various resistances used. The data is plotted below in Figure 16 as well as the line of best fit for these points. The data is shown in Table 4. The line of best fit was determined by using the polyfit function in the numpy python package.

<table>
<thead>
<tr>
<th>Resistance Ω</th>
<th>$V_{in}$ (V)</th>
<th>Input Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 k</td>
<td>876</td>
<td>4.38*µ</td>
</tr>
<tr>
<td>470 k</td>
<td>740</td>
<td>1.57 µ</td>
</tr>
<tr>
<td>820 k</td>
<td>604</td>
<td>.74 µ</td>
</tr>
<tr>
<td>1 M</td>
<td>540</td>
<td>.54 µ</td>
</tr>
<tr>
<td>2.2 M</td>
<td>348</td>
<td>.158 µ</td>
</tr>
</tbody>
</table>

Table 4: Voltage in, Resistance, and Input Current for measuring scope input impedance.
I used the slope of the line of best fit to find the ratio of $V_{in}/I_{in}$ and found that the slope was $104.36 \, k\Omega$ indicating that this is the input impedance of the scope. However, this is about a factor of ten smaller than the actual value of 1 $M\Omega$ that we know to be the input impedance of the oscilloscope. Because of this, I decided to use the voltage and input current associated with the two highest resistance values to determine the input impedance of the oscilloscope because these are the only two relatively close to that of the oscilloscope and because of impedance matching will lower our error. Thus we calculate the impedance by finding the slope of the line that connects those two points.

$$\frac{.540 - .348}{.54 \times 10^{-6} - .348 \times 10^{-6}} = 578 \, k\Omega$$

which although it is quite far off from the actual value (43 % off), it is a far better estimation of the input impedance than the other one we determined using the line of best fit. From this, we can infer that our measurements would be more accurate if we would have used much higher resistances.

2.4

We attempted to determine the impedance of the scope probe using the same method that we employed in 1.2.3 but foolishly chose to use the exact same resistors which led us to an incredibly inaccurate measurement of the input impedance. The data is shown in Table 5. $V_{in}$ versus the input current is plotted below in Figure 17 and obtained some very disconcerting results.

<table>
<thead>
<tr>
<th>Resistance $\Omega$</th>
<th>$V_{in}(mV)$</th>
<th>Input Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200k</td>
<td>103</td>
<td>$515 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>470 k</td>
<td>101</td>
<td>$215 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>820 k</td>
<td>99</td>
<td>$121 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>1 M</td>
<td>97</td>
<td>$97 \cdot 10^{-9}$</td>
</tr>
<tr>
<td>2.2 M</td>
<td>87</td>
<td>$38.5 \cdot 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 5: Resistance, Input Voltage and input current for measuring the input impedance of the oscilloscope probe
Whereas in 1.2.3, our initial linear fit gave us an input impedance on the order of 100kΩ, the slope here is 23.95kΩ which is 3 orders of magnitude off of the value we should be measuring which should be around 10MΩ. I believe the main reason for this is that we did not use large enough resistances such that the results were largely skewed. This is due to a lack of impedance matching which greatly decreases the power transferred from the source to the load (the Oscilloscope in this case). From these results it is easy to say that the input impedance of the scope with the scope probe is much higher with it than without it. If we look at the two points associated with the lowest input voltages and currents we are able to extract a input impedance value of around 200kΩ which shows that as you increase the resistances, the associated input impedance becomes more accurate. If we were to do this again we would use much higher resistances to get a more accurate reading.

2.5

To measure output impedance of the signal generator, we used a method similar to the one above by measuring $V_{out}$ and the resistance for several resistors and then using Ohm’s law to find the corresponding current. I then plotted $V_{out}$ versus $I_{out}$ as shown below in Figure 18. As the slope of the graph shows, the output impedance of the signal generator is 50.6 Ω, which is 1.4% off from the 50 Ω value given by the signal generator. The data for this analysis are shown in Table 6.

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Voltage (V)</th>
<th>Output Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>9.6</td>
<td>.204</td>
</tr>
<tr>
<td>82</td>
<td>12.0</td>
<td>.146</td>
</tr>
<tr>
<td>100</td>
<td>13.0</td>
<td>.13</td>
</tr>
<tr>
<td>150</td>
<td>14.8</td>
<td>.0987</td>
</tr>
<tr>
<td>220</td>
<td>16.0</td>
<td>.073</td>
</tr>
<tr>
<td>330</td>
<td>17.2</td>
<td>.052</td>
</tr>
</tbody>
</table>

Table 6: Resistance, Voltage, and Output Current in measuring the output impedance of the signal generator
Figure 18: Output Impedance of the Signal Generator

We used the potentiometer on our breadboard to vary the resistance to find when the voltage would drop in half. This was arduous because it was hard to vary the resistance precisely enough to reach the exact value when the voltage dropped by one half. We eventually measured 10.2 V, which is close to the total voltage of 20 V when the resistance is 0. The corresponding output impedance was thus 56.5 Ω, which is significantly further (13%) from the given value of 50 Ω.

2.6

Here we are building an RC circuit such as the one shown below in Figure 19. We used the DMM to measure the actual resistance of the resistor to be 9.849 kΩ. We measured the value of the capacitor using the LCR meter to be 9.26 nF.

Figure 19: RC Circuit. Source: Lab Manual

We then fed a 1 V peak-to-peak sine wave into the circuit and measured the output voltage as a function of frequency and also fed the same signal into the oscilloscope and measured the phase difference between the two signals.
The plot in Figure 20 allows us to properly see how a high pass filter works as a function of frequency. In addition to this, the roll-off point is the point at which $V_{\text{out}} = V_{\text{in}}/\sqrt{2}$ because this is the point at which Power is halved because Power is related to the square of the voltage which has many uses for circuit design to maximize or minimize power dissipation. The rolloff point occurs at 1780 Hz.

2.7

We have two signals and we use the XY mode on the oscilloscope in order to find the phase difference between two signals as given by the arc sine of the ellipse’s $Y_{\text{int}}$ divided by its $Y_{\text{max}}$.

$$\delta = \arcsin\left(\frac{Y_{\text{int}}}{Y_{\text{max}}}\right)$$

Any ellipse present in XY-mode indicates that the signals are out of phase whereas there would be a straight line if the signals were in phase.

2.8

To calculate the phase shift between two signals, we first change the scope to be in XY-Mode and use the cursors to find $Y_{\text{max}}$ and $Y_{\text{int}}$. We fed a 20 kHz signal into the RC circuit from 1.2.6 and then into the scope as well as one directly from the signal generator to the scope. We then used the phase shift method to determine $Y_{\text{max}} = 1.04V$ and $Y_{\text{int}} = .1V$. We can thus use the formula we found in 1.2.7 to find the phase shift.

$$\delta = \arcsin\left(\frac{Y_{\text{int}}}{Y_{\text{max}}}\right) = \arcsin\left(\frac{.1V}{1.04V}\right) = 5.52\,\text{deg}$$

2.9

We used the same circuit as in 1.2.6 and used the Oscilloscope’s phase difference measurement to determine the range of frequencies which corresponded to approximate phase shifts of $0^\circ$, $22.5^\circ$, $45^\circ$, $77.5^\circ$, and $90^\circ$, as shown in the following table. Similarly, we found the phase shifts associated with each frequency for the circuit in 1.2.6 and plotted it in Figure 20.
<table>
<thead>
<tr>
<th>Phase Shift</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>500 kHz - 2 MHz</td>
</tr>
<tr>
<td>22.5°</td>
<td>3.8 kHz - 4.2 kHz</td>
</tr>
<tr>
<td>45°</td>
<td>1.6 kHz - 1.7 kHz</td>
</tr>
<tr>
<td>77.5°</td>
<td>190 Hz - 250 Hz</td>
</tr>
</tbody>
</table>

Table 7: Ranges of frequency for certain phase shifts for the RC circuit in 2.6

2.10

We have here that the output voltage of a black-box decreases by 20% with a load of 1kΩ compared to the no-load output. Thus we know that \( V_{\text{no load}} = V_{\text{Thevenin}} \) and that \( V_{\text{load}} = 0.8V_{\text{Thevenin}} \). Similarly, \( I_{\text{no load}} = 0 \) and \( I_{\text{load}} = 0.8 \ast V_{\text{Thevenin}}/R^2 \) where \( R^2 \) is the 1kΩ resistor. We also know from the lab manual that

\[
Z_{\text{out}} = \frac{-\partial V_{\text{out}}}{\partial I_{\text{out}}}
\]

Our \( \partial V_{\text{out}} \) is given by the difference between \( V_{\text{load}} \) and \( V_{\text{no load}} \) which is \( -0.2V_{\text{Thevenin}} \). The change in current is \( 0.8V_{\text{Thevenin}}/R^2 \). Therefore our \( Z_{\text{out}} \) is calculated to be

\[
Z_{\text{out}} = \frac{-0.2V_{\text{Thevenin}}}{0.8V_{\text{Thevenin}}/R^2} = \frac{R^2}{4} = 250\Omega
\]

Therefore the output impedance of the black box in question is 250Ω.

2.11

A 100W light bulb has a resistance of 9Ω when not plugged into a power source. Household power is 110V so we should be able to calculate the power output of the light bulb.

\[
P = \frac{V^2}{R} = \frac{110^2V^2}{9\Omega} = 1344W
\]

which is most definitely not the power output as specified by the light bulb. Assuming the voltage is constant, it is only fair to assume that the resistance is not constant and must depend on some external factor, likely temperature. Light bulbs are therefore not linear circuit components. The 100 W power amount is the correct one whereas the 1344 W amount is not because that would likely melt the light bulb and make it not functional. The resistance of the light bulb is dependent on temperature as heat causes random motions of electrons which end up increasing the resistance of the object.

2.12

We have the transfer function of a circuit

\[
\frac{P_{\text{out}}}{P_{\text{in}}} = T = 20\log_{10}|V_{\text{out}}/V_{\text{in}}|
\]

We use our values from 2.6 for \( V_{\text{out}}/V_{\text{in}} \) and use them in our transfer function to find the measured transfer function which has units on the Y axis of decibels, which results in a Bode plot
We then plot our phase difference against our frequency, using the data from 2.6.

We now relate the measured transfer function values to the theoretical values. We use the transfer function of the high pass filter such that

\[
\frac{V_{out}}{V_{in}} = \frac{\omega^2 R^2 C^2}{\sqrt{1 + \omega^2 R^2 C^2}}
\]

where \( RC = 1 \mu F \times 10 k\Omega = 10^{-4} \). We use our frequency values into the transfer function and find the theoretical values to get the theoretical transfer function values shown in the plot below.
2.13

We first set up the waveform pictured below by using the pulse option on the function generator and adjusting different parameters.

We connect the signal to the scope using a T connector. We then add in a BNC 50Ω terminator to the BNC T and notice the signal changes to the one pictured below.

This occurs because there is an added impedance in the circuit, decreasing the amplitude of the wave. After that, we remove the terminator and short the signal to ground by using a wire and notice that the signal flatlines to 0 V with no real peculiarities. Then we connect a 100 foot long BNC cable from the T connector to another T connector which is connected to another channel of the scope, generating the signal seen below.
The signal above is the way it is because the signal travels the distance of the 100 ft. cable and is partially reflected back which can be seen by the second, smaller peak of the yellow waveform. The signal is not transmitted entirely to the second channel because of the length of the cable which introduces a parasitic impedance which starts to actually affect the amplitude of the signal.
We then connect a 50 Ω terminator to the T of the second scope channel and observe the signal below, noting that there is no longer a reflected signal due to the presence of the terminator.

Then we shorted the second T connector and notice the signal below, noting that the signal has now gained a negative component.

We then removed the shorting wire and disconnected the BNC from the function generator and connected a 200Ω resistor in series then saw the signal below.

We continue to hold the extra resistor in place and add in the 50 Ω terminator to the T of the second channel and observe the signal below.
Finally, we keep the resistor in place and short the long BNC connector to ground via the T connector and observe the signal below.

2.14

The length of the 100 ft. BNC cable is 30.48 m. It would take light

$$30.48m / 3.0 \times 10^8 m/s = 102 ns$$

to travel the length of the cable. It would take about 200 ns for it to travel the length of the cable. The signal, which travels at about 2/3 of the speed of light, would take about 300 ns to travel the length of the cable back and forth. We see extra pulses on the first channel because the signal is reflected back through the long BNC cable. The pulses are sometimes not there at all because of the terminator which seeks to match the impedance of the BNC cable and as a result the signal does reflect the signal. The peak of the voltage tries to go across ground Whether or not the pulses flip is determined by the sign of the impedance difference. The signals also experience phase shifts because of differences in impedances between the cable and the other scope channel.

2.15

We wish to create a band-pass filter which allows signals between 500 Hz and 10 kHz. The desired frequency is defined as

$$f = \frac{1}{2\pi \tau} = \frac{1}{2\pi RC}$$

where R and C are the resistors and capacitors associated with that frequency. We have two frequencies we wish to solve for and after one failed attempt at making a band-pass filter with different resistances, we realized it was probably intelligent to have matching resistances in the low-pass and high-pass parts of the filter. We decided to use 47 kΩ resistors for both and then solved for capacitance values which would work for each frequency.

$$f = 500 Hz = \frac{1}{2\pi 47k\Omega C} \rightarrow C = \frac{1}{2\pi 47k\Omega \times 500 Hz} = 6773 pF$$

which is approximately equal to the common capacitor value of 6800 pF. Now for the 10,000 Hz resistor

$$f = 10000 Hz = \frac{1}{2\pi 47k\Omega C} \rightarrow C = \frac{1}{2\pi 47k\Omega \times 10000} = 339 pF$$

which is approximately equal to the common capacitor value of 330 pF. We set up the band-pass circuit by putting the high pass filter first and then the low-pass filter after that as detailed in the circuit diagram below.
I then varied the frequency of the 1 Volt peak to peak sine wave and measured the output voltages in each case as shown.

Table 8: Frequency and Voltage for Band Pass Filter

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>V_{out}(mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12.6</td>
</tr>
<tr>
<td>500</td>
<td>37.6</td>
</tr>
<tr>
<td>1000</td>
<td>37.8</td>
</tr>
<tr>
<td>5000</td>
<td>41.0</td>
</tr>
<tr>
<td>10000</td>
<td>37.8</td>
</tr>
<tr>
<td>50000</td>
<td>24.2</td>
</tr>
<tr>
<td>1000000</td>
<td>7.40</td>
</tr>
</tbody>
</table>

When this data is plotted on a log-linear graph such as in Figure 24, a straight line at y = 1/√2 shows us that the rolloff points for this data can be found at 337 Hz and 13335 Hz, representing a slightly larger range than the one we intended, but this is likely due to our limited data set. If we had more data points at different frequencies in that region, we would likely find roll off points closer to the ones we were trying to achieve.

Figure 24: Bode plot for band pass filter
which shows that if we assume the voltage at 5 kHz to be essentially the max, the values in the band-pass filter’s range are let through without too much attenuation whereas the voltages drop a fair amount below and above the range of frequencies.

3 Conclusion

In this lab, we received an introduction to basic linear circuit components, primarily resistors and capacitors as well as their associated impedances. We also learned a great deal about how to use important electrical equipment such as digital multimeters, oscilloscopes, and function generators. We also learned how to connect the DMM in series for current readings and in parallel for voltage readings. We learned about how input and output impedances can affect a signal’s reading and how to determine these two by quantifying the relationship between the input voltage and current for some specific resistances. We learned about the importance of Voltage dividers and Thevenin equivalent resistances and voltages and how useful the the applications of the black-box are. We learned how to create basic high pass, low pass, and band pass filters.