Superconducting transition out of a FL* metal

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Collaborators



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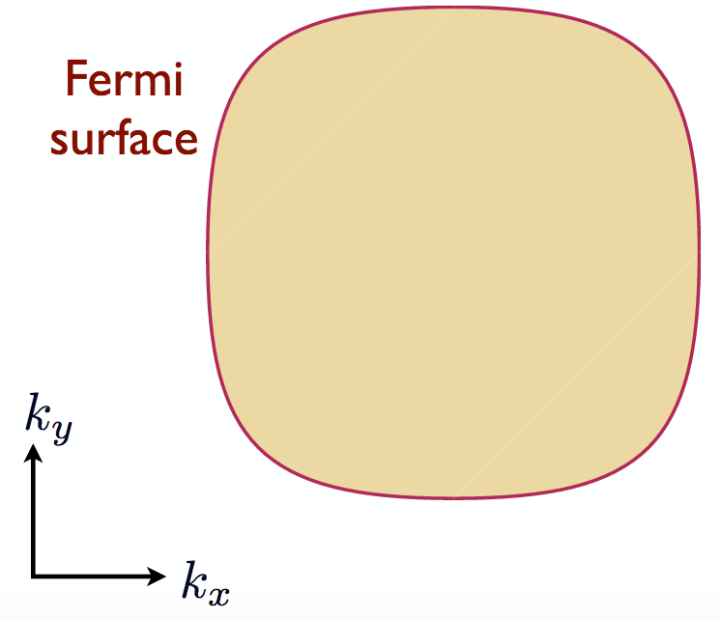
Brief Outline

- Fractionalized Fermi liquids (FL*)
- Experimental evidences for FL*
- Symmetry fractionalization: from bosons to fermions
- Superconducting transition

Conventional Fermi liquid

- Compressible state with a Fermi surface in momentum space
- Has gapless long-lived excitations across the Fermi surface
- Satisfies Luttinger's Theorem: Area enclosed by Fermi surface = total density of electrons (mod 2)



Spin liquid

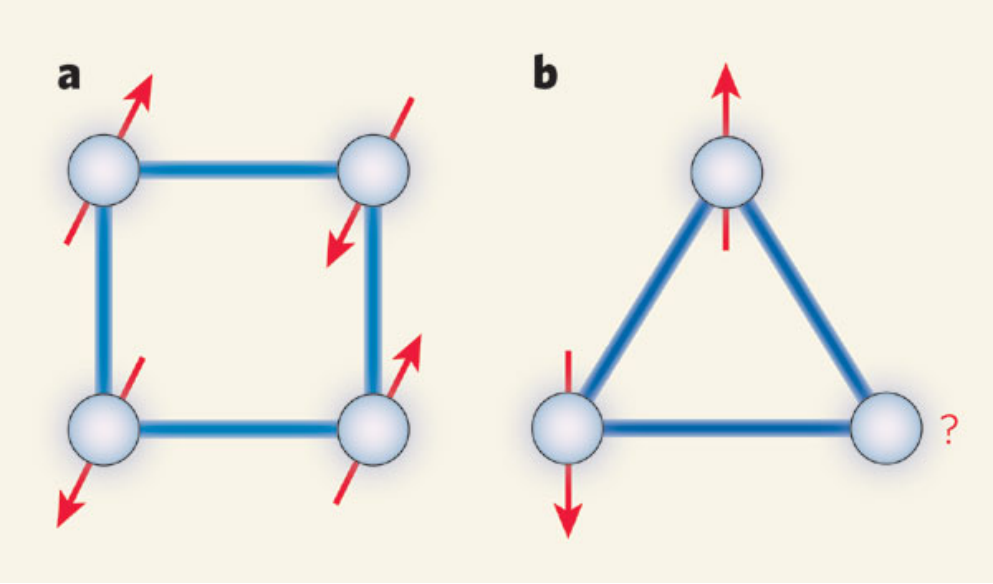


Figure: a. Neel order, b. Frustration (Nature, v456 n7224)

- Consider an insulating spin-half system with exchange interaction $H = \sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'}\, \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}'}$
- Does the ground state have long range magnetic order in presence of geometric frustation?

Spin liquid

- Neel order is destroyed by quantum fluctuations - no symmetry is broken in the ground state
- Model ground state - resonating valence bond liquid [Anderson, *Mat. Res. Bulletin*, 1973]

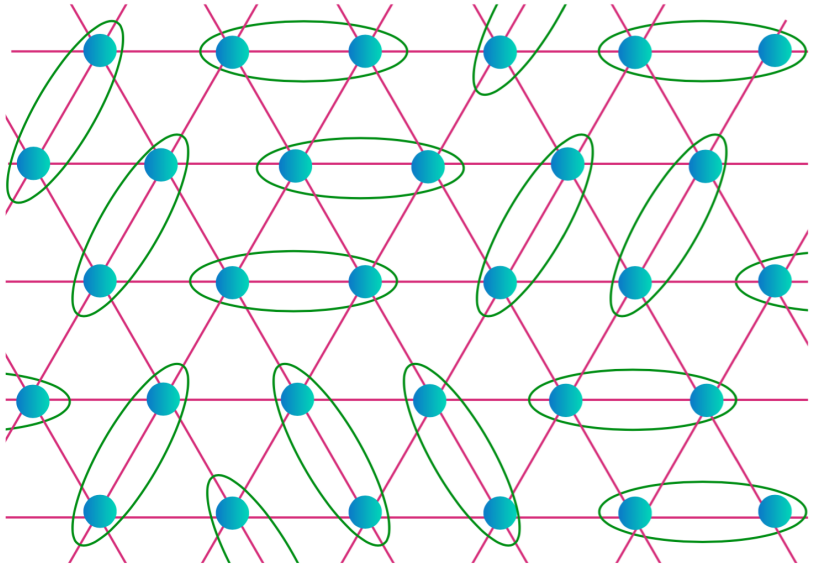


Figure: RVB state - each bond represents a singlet (S. Sachdev, Harvard)

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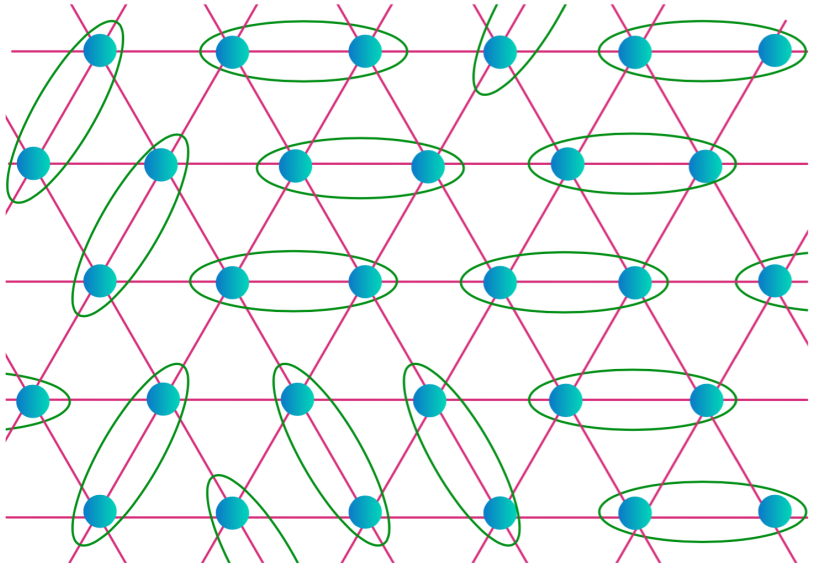

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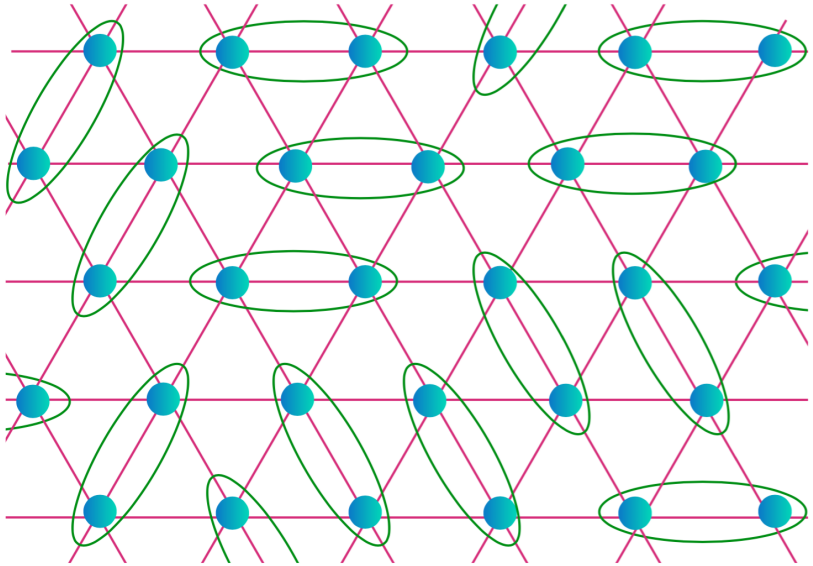

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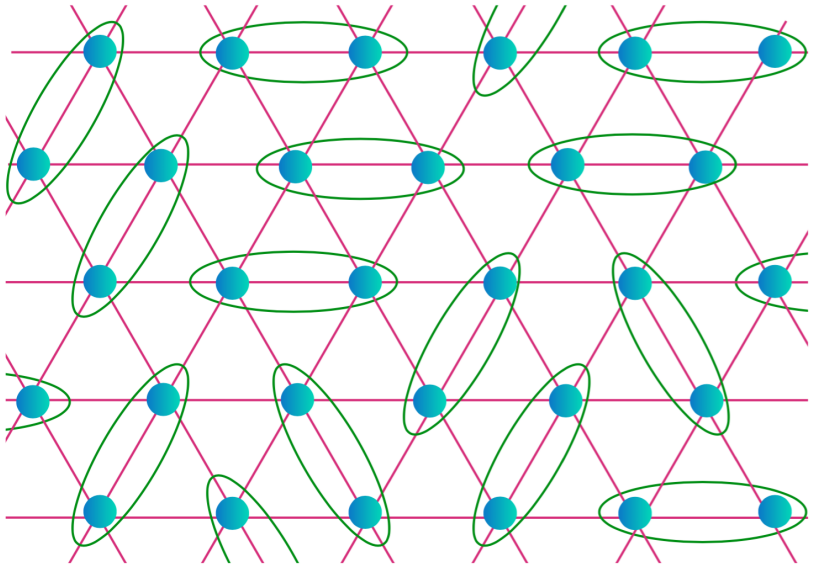

Figure: RVB state - each bond represents a singlet (S. Sachdev, Harvard)

Excitations of the $\mathbb{Z}_2$ spin liquid

- Has fractionalized spin half excitations called spinons which come in pairs
- Has emergent $\mathbb{Z}_2$ gauge field which mediates interactions between the spinons

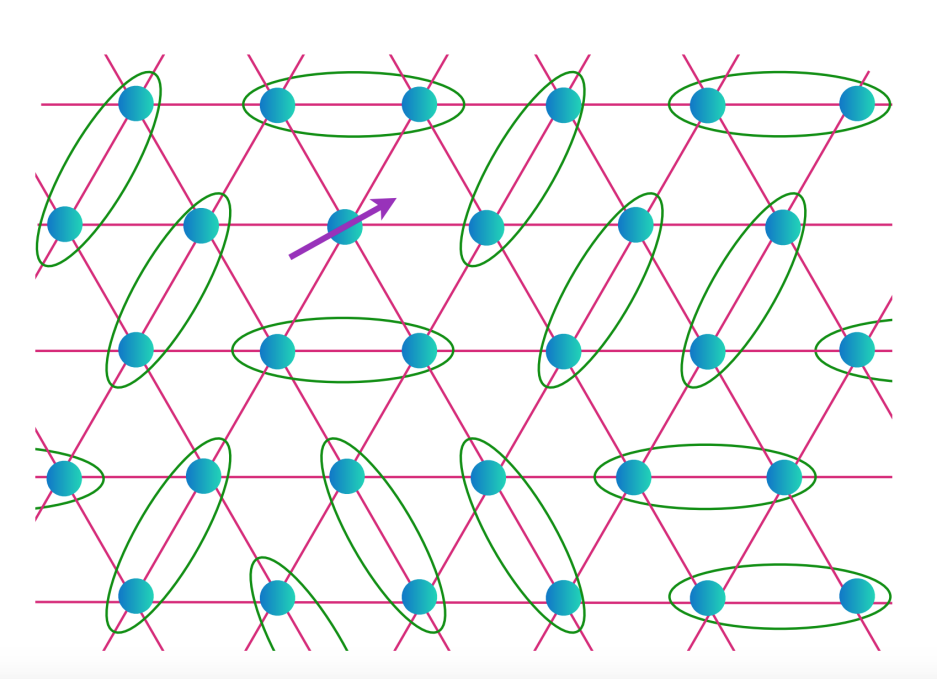

Figure: Spinon excitation of the RVB state (S. Sachdev, Harvard)

Excitations of the $\mathbb{Z}_2$ spin liquid

- Vortices in the $\mathbb{Z}_2$ gauge field are called visons
- Spinons and visons are mutual semions
- Topological order: states on a cylinder with and without a vison through the hole are topologically degenerate

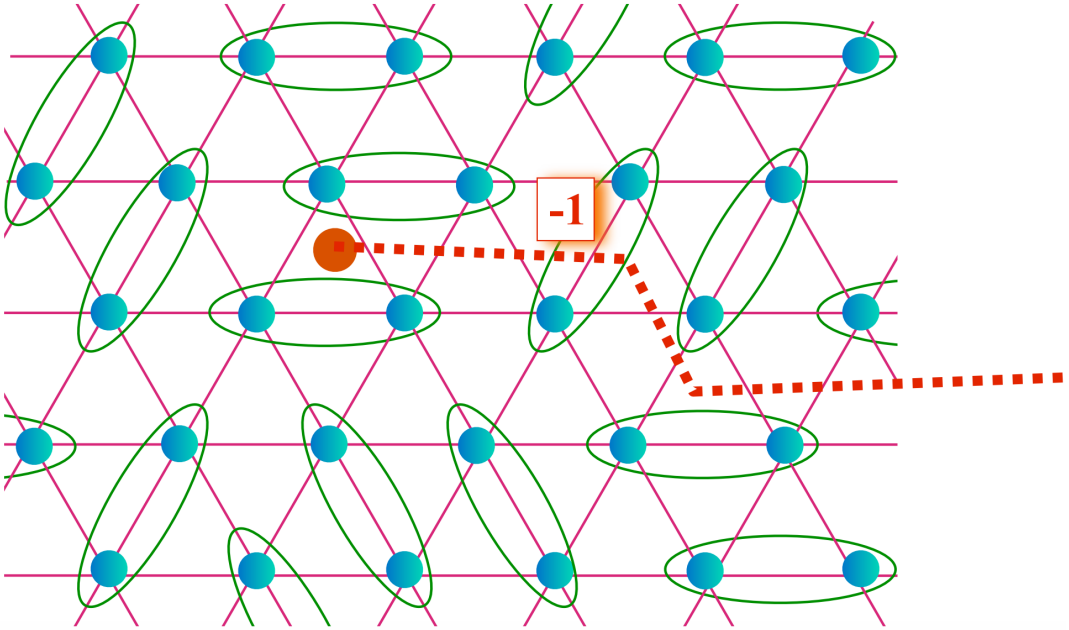

Figure: Vison excitation of the RVB state (S. Sachdev, Harvard)

Fractionalized Fermi liquid (FL*)

- Consider the Kondo-lattice problem: a band of mobile fermions interacts with a lattice of localized moments

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'}\, \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}'} + J_K \sum_{\mathbf{r}} \frac{1}{2}\, c^{\dagger}_{\mathbf{r}\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{r}\beta} \cdot \mathbf{S}_{\mathbf{r}}$$

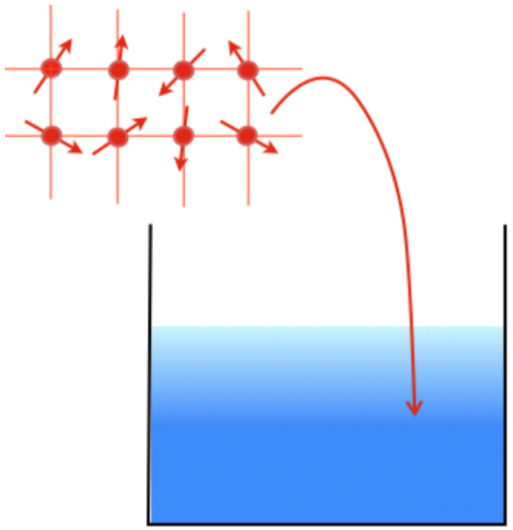

Figure: Localized moments in the Fermi sea (P. Coleman, Rutgers)

Fractionalized Fermi liquid (FL*)

- Conventionally, the non-magnetic state is a heavy-fermion metal [Doniach, *Physica B*, 1977]
- Local moments hybridize with conduction electrons - leading to formation of *heavy quasiparticles*
- Luttinger's theorem: Area enclosed by Fermi surface = (density of electrons + density of local moments)(mod 2)

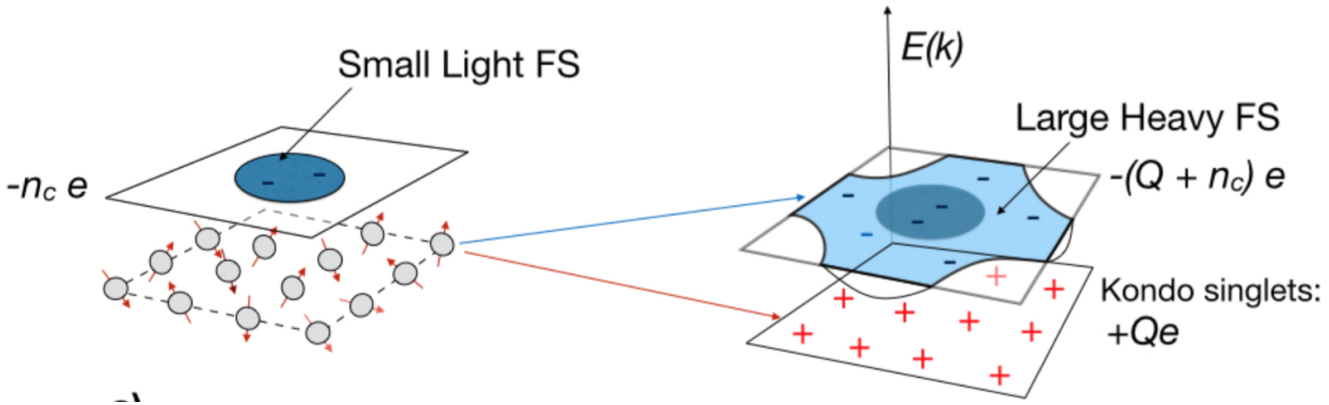

Figure: Expansion of the Fermi surface (P. Coleman, Rutgers)

Fractionalized Fermi liquid (FL*)

- What if there are strong effects of frustrations and quantum fluctuations in $2d$?
- In $J_K \to 0$ limit, we have a stable gapped $\mathbb{Z}_2$ spin liquid ground state for the local moments
- Now turn on a small Kondo-coupling: $J_K \neq 0$
- Resulting metallic state is the $\mathbb{Z}_2$ FL*: conduction electron Fermi surface + background spin liquid
- Violates Luttinger's theorem as it does not count local moments [Senthil et al, *PRL*, 2003; *PRB*, 2004]

Experimental evidence for FL*

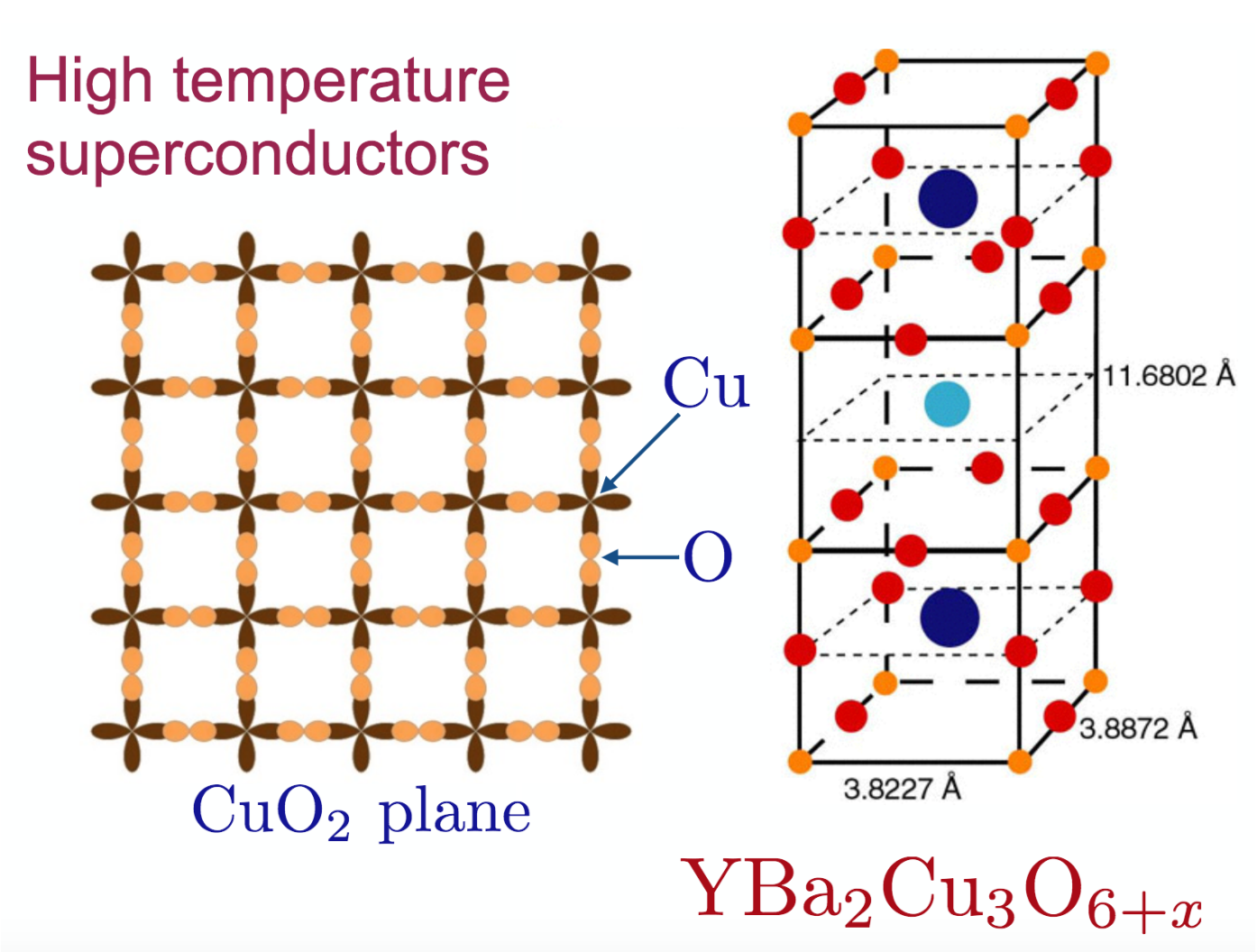

Figure: YBCO (S. Sachdev, Harvard)

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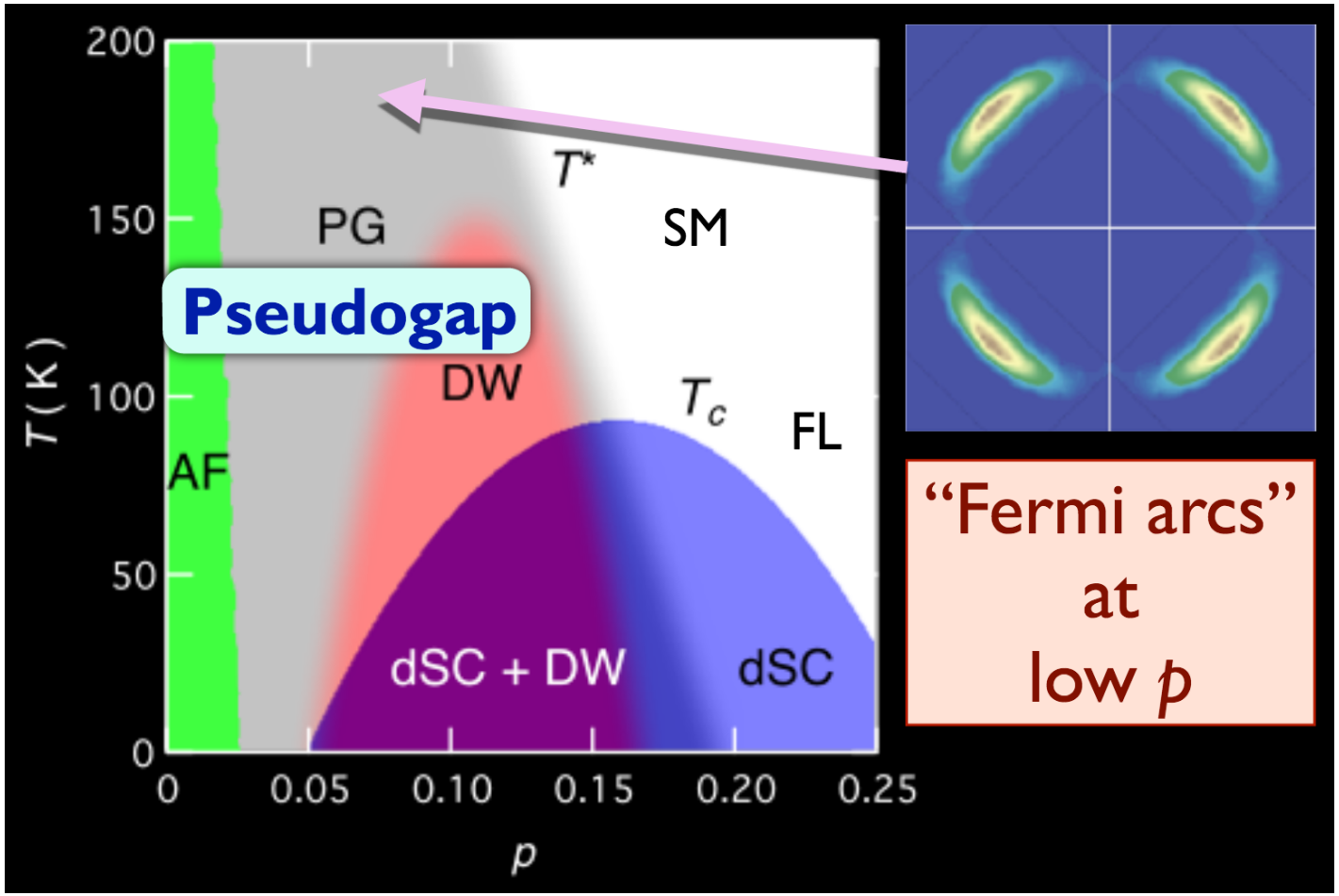


Figure: Phase diagram of the cuprates [Shen et al, *Science* 2005]

Experimental evidence for FL*

- Under hole doping p, relative to the band isulator there are $1 + p$ holes per square
- We expect a Fermi surface of size $1 + p$

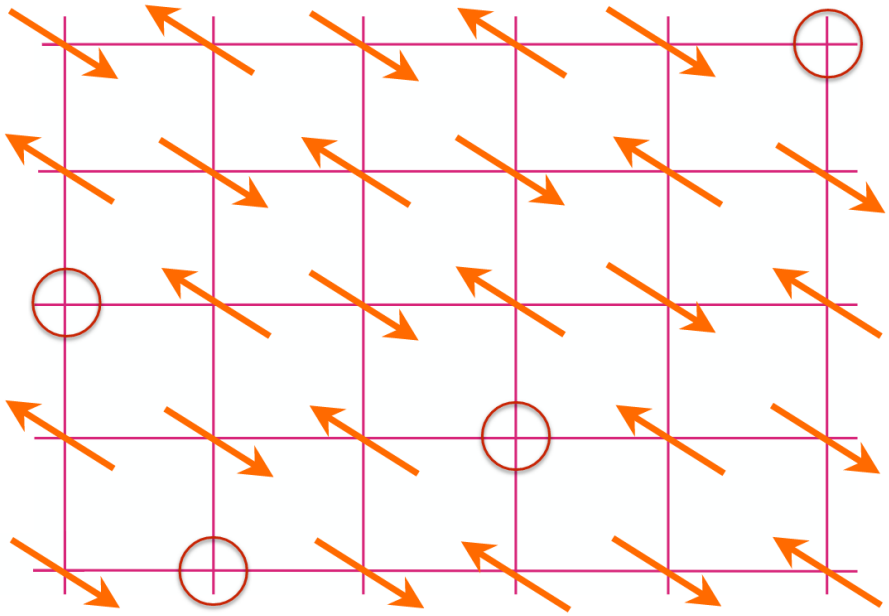
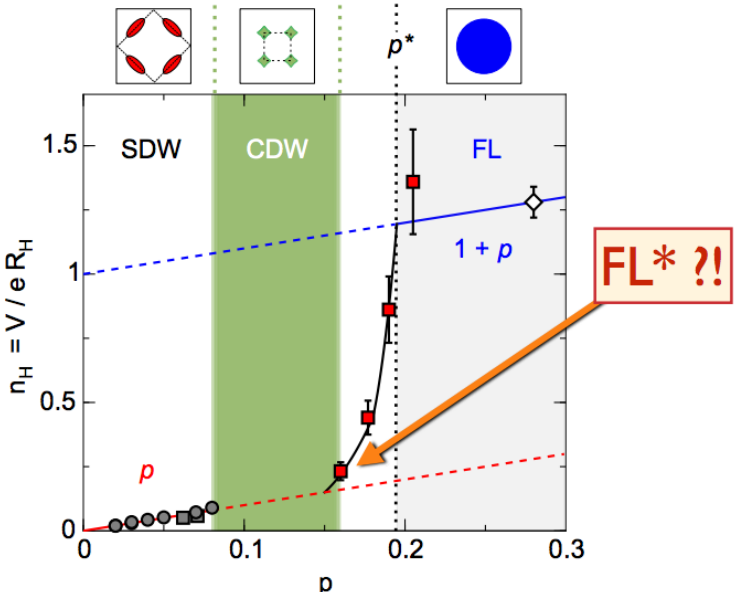


Figure: Antiferromagnets with p holes per square (S. Sachdev, Harvard)

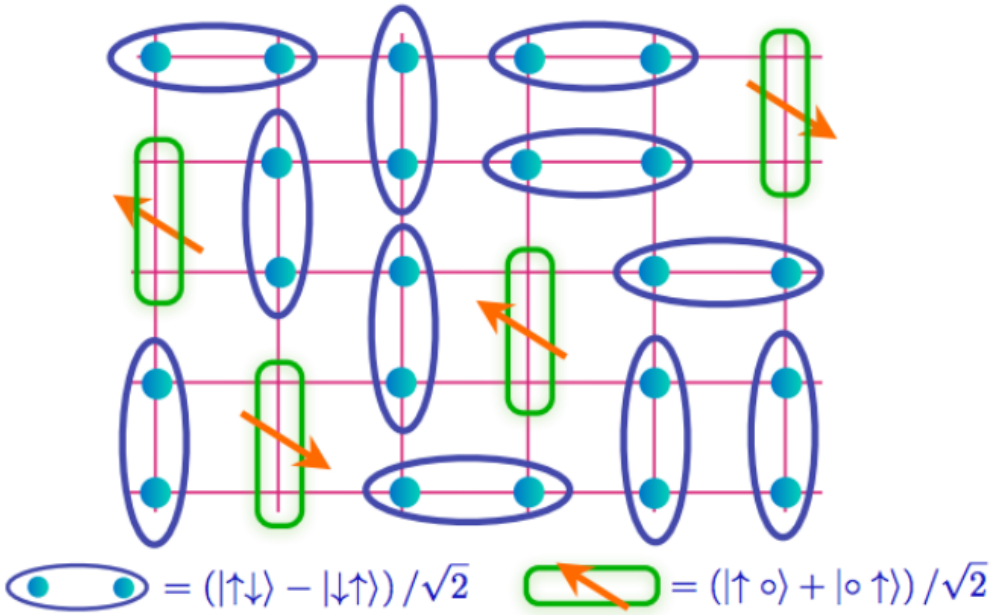
Experimental evidence for FL*

- Several experiments point to a Fermi liquid like state in the pseudogap regime, with carrier density p instead of $1+p$
  1. T-independent Hall resistance of fermions of charge $+e$ and density p [Badoux et al, *arXiv*:1511.08162]
  2. Drude-like optical conductivity with $\tau^{-1} \sim \omega^2 + T^2$ [Mirzaei et al, *PNAS*, 2013]
  3. In plane magnetoresistance obeys Kohler's rule $\delta\rho/\rho_o \propto H^2 T^{-4}$ [M. K. Chan et al, *PRL*, 2014]

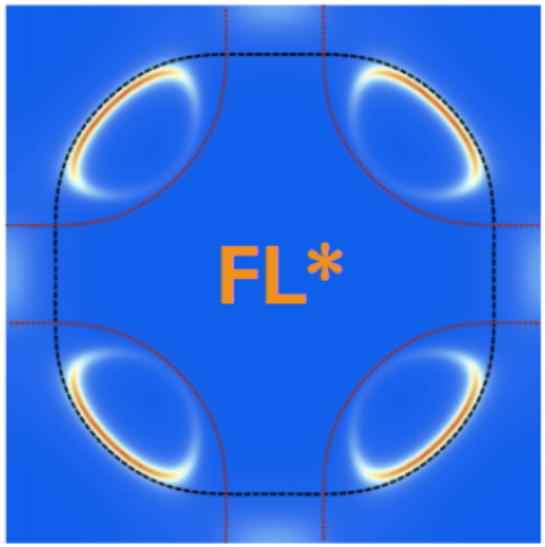


Experimental evidence for FL*

- Metallic with a Fermi surface of size p, can qualitatively reproduce results of ARPES [Qi et al, *PRB*, 2010; Punk et al *PNAS*, 2015]
- D-form factor density waves with experimentally observed wavevectors is a low temperature instability of the FL* [Chowdhury et al, *PRB*, 2014]



M. Punk, A. Allais, and S. Sachdev, PNAS 112, 9552 (2015)



Y. Qi and S. Sachdev, PRB 81, 115129 (2010)

Objective

How does the FL* metal undergo a confinement transition to a superconductor?

- An attractive insulating Mott state is the gapped $\mathbb{Z}_2$ spin liquid with bosonic spinons b, with incommensurate spin correlations and nematic order [Read et al, *PRL*, 1991]
- Need an equivalent description in terms of fermionic spinons f, which can hybridize with the hole-like c fermions
- Pairing correlations between the f spinons (naturally present in a $\mathbb{Z}_2$ spin liquid) will mediate pariring of the c fermions — leading to superconductivity and destruction of topological order.

Low energy quasiparticles in a $\mathbb{Z}_2$ gauge theory

- 4 kinds of excitations, e, m, ϵ and the trivial local excitation 1
- Have the following fusion rules:

$$e \times e = m \times m = \epsilon \times \epsilon = 1$$
$$1 \times 1 = 1, e \times 1 = e, m \times 1 = m, \epsilon \times 1 = \epsilon$$
$$e \times m = \epsilon, e \times \epsilon = m, m \times \epsilon = e$$

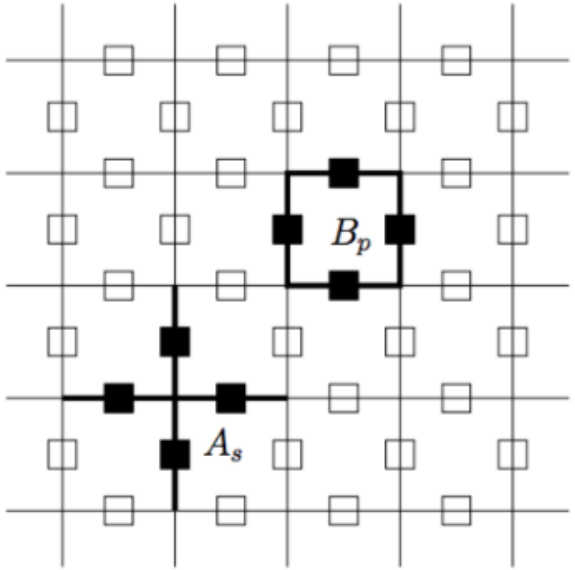
- In the context of spin liquids, e and ϵ are bosonic and fermionic spinons b and f respectively, m is the spinless vison, 1 is a local excitation with integer spin

Low energy quasiparticles in a $\mathbb{Z}_2$ gauge theory

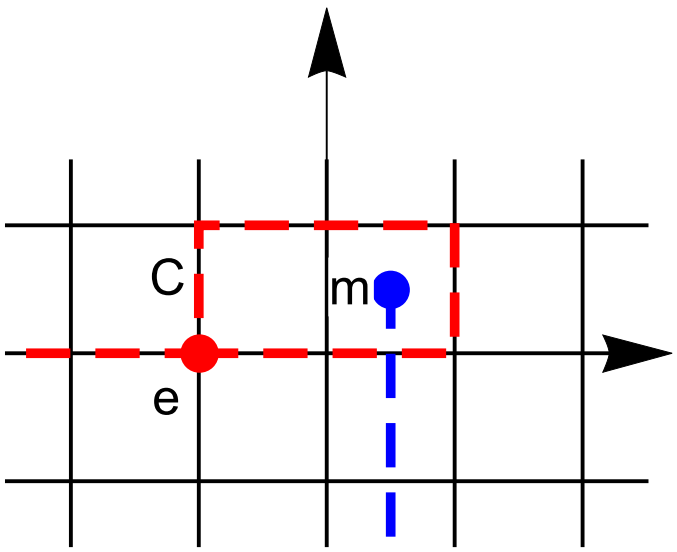
- Simplest realization: Toric code [Kitaev, *Annals of Physics*, 2003]

$$H_{TC} = -\sum_p B_p - \sum_s A_s, \text{with } B_p = \sum_{l \in p} \tau_l^z,\ A_s = \sum_{l \in \text{star}} \tau_l^x$$

- e particles are violations of $A_s = 1$, and m particles of $B_p = 1$, both can be created locally only in pairs



Low energy quasiparticles in a $\mathbb{Z}_2$ gauge theory



- Braiding an e around an m leads to a phase of -1: e and m are self bosons but mutual semions
  $|\xi\rangle \to \left(\prod_{l\in C} \tau_l^z\right)|\xi\rangle = \left(\prod_{p \text{ inside } C} B_p\right)|\xi\rangle = -|\xi\rangle$
- ϵ can be thought of as a bound state of e and m - it is a fermion

Symmetry fractionalization and symmetry quantum numbers

- $T_x T_y T_x^{-1} T_y^{-1} = 1$ on any physical state (with even no. of anyons)
- Consider a state $|\xi\rangle$ with two well-separated e particles
- Assume it is possible factorize the action of the symmetry on the two e particles locally, i.e, $T_x |\xi\rangle = T_x(1) T_x(2) |\xi\rangle$

$$\implies |\xi\rangle = \prod_{i=1,2} T_x(i) T_y(i) [T_x(i)]^{-1} [T_y(i)]^{-1} |\xi\rangle$$

- Each e particle can pick up a phase: symmetries act projectively on anyons

$$T_x(i) T_y(i) [T_x(i)]^{-1} [T_y(i)]^{-1} = e^{i\phi_i}$$

Symmetry fractionalization and symmetry quantum numbers

- Merging two e particles gives a trivial 1 particle, so $e^{i\phi_i} = \pm 1$
- This phase is the same for all e particles, and cannot change in a $\mathbb{Z}_2$ spin liquid phase without breaking translation symmetry
- Fractionalization pattern is a universal gauge-invariant property of $\mathbb{Z}_2$ spin liquids with translation symmetry
- Similar construction can be repeated for all space group and time-reversal symmetry - this gives us a classification of $\mathbb{Z}_2$ spin liquids independent of parton construction [Essin et al, *PRB*, 2013]

Symmetry fractionalization - Connecting bosons to fermions

- Assume we know the symmetry fractionalizations of the bosonic spinons b and visons v
- The fermionic spinon f can be thought of as a bound state of b and v
- Knowing the bosonic and vison symmetry fractionalization quantum numbers, can determine those of the fermions [Essin et al, *PRB*, 2013; Y. M. Lu et al, *arXiv*:1403.0575; Qi et al *PRB*, 2015]

How to fuse the boson and the vison?

- Naively, we expect that $e^{i\phi_f} = e^{i\phi_b} e^{i\phi_v}$
- Not true when the bosonic and vison strings cross during the symmetry operation - extra twist factor of -1
- $\boxed{e^{i\phi_f} = e^{i\phi_t} e^{i\phi_b} e^{i\phi_v}}$

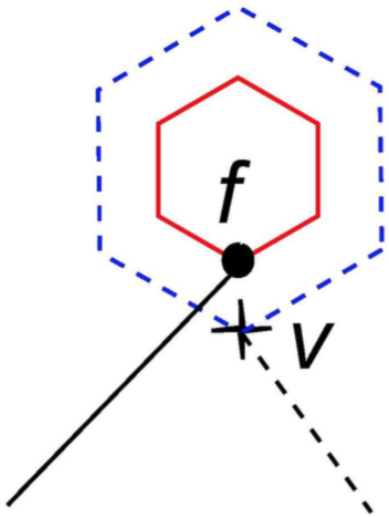


Figure: 2π-rotation - example of a fusion rule with an extra sign [Y. M. Lu et al, *arXiv*:1403.0575]

How to fuse the boson and the vison?

Commutation relation	Bosonic PSG	Fermionic PSG	Vison PSG	Twist factor	Relation
$T_x^{-1}T_y^{-1}T_xT_y$	$(-1)^{p_1}$	$\eta_{T_xT_y}$	-1	1	$(-1)^{p_1+1} = \eta_{T_xT_y}$
$P_x^{-1}T_xP_xT_x$	$(-1)^{p_2}$	$\eta_{P_xT_y}$	1	1	$(-1)^{p_2} = \eta_{P_xT_x}$
$P_y^{-1}T_x^{-1}P_yT_x$	$(-1)^{p_3}$	$\eta_{P_yT_x}$	-1	1	$(-1)^{p_3+1} = \eta_{P_yT_x}$
$P_x^{-1}T_y^{-1}P_xT_y$	$(-1)^{p_4}$	$\eta_{P_xT_y}$	-1	1	$(-1)^{p_4+1} = \eta_{P_xT_y}$
$P_y^{-1}T_yP_yT_y$	$(-1)^{p_5}$	$\eta_{P_yT_y}$	1	1	$(-1)^{p_5} = \eta_{P_yT_y}$
P_x^2	$(-1)^{p_6}$	η_{P_x}	1	-1	$(-1)^{p_6+1} = \eta_{P_x}$
P_y^2	$(-1)^{p_7}$	η_{P_y}	1	-1	$(-1)^{p_7+1} = \eta_{P_y}$
$P_x^{-1}P_y^{-1}P_xP_y$	1	$\eta_{P_xP_y}$	-1	-1	$1 = \eta_{P_xP_y}$
$\mathcal{T}^2$	-1	-1	1	1	$1 = 1$
$T_x^{-1}\mathcal{T}^{-1}T_x\mathcal{T}$	$(-1)^{p_8}$	$\eta_{\mathcal{T}T_x}$	1	1	$(-1)^{p_8} = \eta_{\mathcal{T}T_x}$
$T_y^{-1}\mathcal{T}^{-1}T_y\mathcal{T}$	$(-1)^{p_9}$	$\eta_{\mathcal{T}T_y}$	1	1	$(-1)^{p_9} = \eta_{\mathcal{T}T_y}$
$P_x^{-1}\mathcal{T}^{-1}P_x\mathcal{T}$	$(-1)^{p_6}$	$\eta_{\mathcal{T}P_x}$	1	-1	$(-1)^{p_6+1} = \eta_{\mathcal{T}P_x}$
$P_y^{-1}\mathcal{T}^{-1}P_y\mathcal{T}$	$(-1)^{p_7}$	$\eta_{\mathcal{T}P_y}$	1	-1	$(-1)^{p_7+1} = \eta_{\mathcal{T}P_y}$

Projective symmetry groups (PSG)

- Given a mean-field spin-liquid ansatz, how does one calculate the symmetry quantum numbers?
- We can find out PSG phases or matrices for each symmetry operation (space group and time-reversal) - *not gauge-invariant* [Wen, *PRB* 2002; Wang et al, *PRB* 2006]
- We can then combine these phases to get gauge-invariant $\mathbb{Z}_2$ symmetry quantum numbers

PSG for Schwinger bosons

- Fractionalize spin into bosonic partons $\mathbf{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$

$$\begin{aligned} H^b_{MF} &= -\sum_{ij} (Q_{ij} \epsilon_{\alpha\beta} b^\dagger_{i\alpha} b^\dagger_{j\alpha} + h.c.) + \sum_i \lambda_i b^\dagger_{i\alpha} b_{i\alpha}, \\ Q_{ij} &= \langle \epsilon_{\alpha\beta} b_{i\alpha} b_{j\alpha} \rangle, \ \sum_\alpha \langle b^\dagger_{i\alpha} b_{i\alpha} \rangle = 1 \end{aligned}$$

- Local $U(1)$ transformation $b_{\mathbf{r}\alpha} \to e^{i\phi(\mathbf{r})} b_{\mathbf{r}\alpha}$ leaves all physical observables unchanged

- Any Q_{ij} that is invariant under lattice/time-reversal symmetries only upto an additional $U(1)$ phase will give the same physical wave-function

PSG for Schwinger bosons

- For a symmetry operation X, we can choose an associated gauge-transformation G_X such that the mean field ansatz is invariant under $G_X X$

$$G_X : b_{\mathbf{r}\alpha} \to e^{i\phi_X(\mathbf{r})} b_{\mathbf{r}\alpha},\ Q_{ij} \to e^{i(\phi_X(\mathbf{r}_i)+\phi_X(\mathbf{r}_j))} Q_{ij}$$
$$G_X X(Q_{ij}) = G_X(Q_{X[i]X[j]}) = Q_{ij}$$

- Given a set of mean fields Q_{ij}, we can find these phases and hence G_X for that ansatz

- Combine the symmetry operations that are equivalent to the identity on any physical state, then the appropriate (signed) sum of phases will give the corresponding symmetry fractionalization

PSG for Schwinger bosons - example

- Consider the gapped bosonic spin liquid described by Read and Sachdev [Read et al, *PRL*, 1991]

$$Q_{i,i+\hat{x}} \neq Q_{i,i+\hat{y}}, Q_{i,i+\hat{x}+\hat{y}} = Q_{i,i-\hat{x}+\hat{y}} \neq 0, Q_{i,i+2\hat{x}} \neq 0, Q_{i,i+2\hat{y}} = 0$$

$$N = |Q_{i,i+\hat{x}}|^2 - |Q_{i,i+\hat{y}}|^2 \neq 0$$

- Space group generators of a rectangular lattice are translations T_x and T_y, and reflections P_x and P_y

$$\begin{aligned}
\phi_{T_x}(\mathbf{r}) &= \phi_{T_y}(\mathbf{r}) = 0 \\
\phi_{P_x}(\mathbf{r}) &= \pi y + \frac{\pi}{2} \\
\phi_{P_y}(\mathbf{r}) &= \pi y \\
\phi_{\mathcal{T}}(\mathbf{r}) &= 0
\end{aligned}$$

PSG for Schwinger bosons

- Can also solve for an ansatz given a particular set of symmetry quantum numbers
- Algebraic constraints on these phases are imposed by space-group and time-reversal symmetries
- Consider the space-group identity $T_x^{-1} T_y T_x T_y^{-1} = 1$

$$\implies (G_{T_x} T_x)^{-1} (G_{T_y} T_y)(G_{T_x} T_x)(G_{T_y} T_y)^{-1} = (-1)^{p_1}$$

- Can be reduced to the following equation for the phases

$$-\phi_{Tx}\left[T_x(\mathbf{r})\right] + \phi_{Ty}\left[T_x(\mathbf{r})\right] + \phi_{Tx}\left[T_y^{-1} T_x(\mathbf{r})\right] - \phi_{T_y}(\mathbf{r}) = p_1 \pi$$

- Similar relations for other symmetries would give us a set of ϕ_X, and we may be able find an ansatz by solving for $G_X X(Q_{\mathbf{rr'}}) = Q_{\mathbf{rr'}}$

Equivalent fermionic description

- The equivalent description in term of fermions is again a gapped $\mathbb{Z}_2$ spin liquid
- It has pairing on the nearest and the next nearest neighbors, and hopping on the next next nearest neighbor
- The ansatz has π-flux through elementary plaquettes

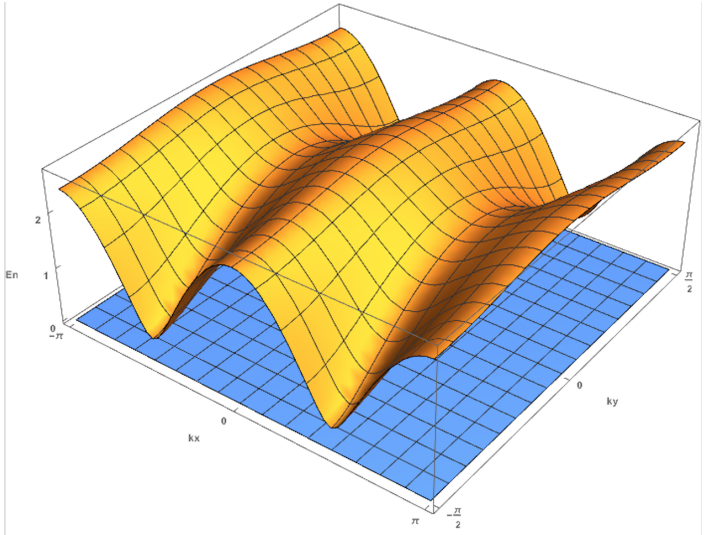


Figure: Dispersion in the doubled unit cell

Confinement transition

- Recall: FL* metal = c fermion Fermi surface + f spin liquid
- Decouple the Kondo term in terms of the auxiliary bosons $B_{1\mathbf{r}} = c^\dagger_{\mathbf{r}\sigma} f_{r\sigma}$ and $B_{2\mathbf{r}} = \epsilon_{\sigma\sigma'} c_{\mathbf{r}\sigma} f_{\mathbf{r}\sigma'}$

$$H_{cf} \sim J_K \sum_{\mathbf{r},\mathbf{r}'} B^\dagger_{1\mathbf{r}} c^\dagger_{\mathbf{r}\sigma} f_{r\sigma} - B^\dagger_{2\mathbf{r}} \epsilon_{\sigma\sigma'} c_{\mathbf{r}\sigma} f_{\mathbf{r}\sigma'} + h.c$$

- Confinement can be induced by a condensation of these bosons
- This confined 'Higgs' state is a superconductor as pairing of f fermions now induces a pairing of c fermions

Confinement transition

- Since the c fermions transform trivially under space group operations, we can derive the projective transformation of B from those of f
- We use this information to construct an effective boson Hamiltonian

$$H_b = \sum_{i,j=1,2} \sum_{\mathbf{r}\mathbf{r}'} B^\dagger_{i\mathbf{r}} T^{ij}_{\mathbf{r}\mathbf{r}'} B_{j\mathbf{r}'} + h.c$$

- The bosons will condense at the minima of the boson dispersion
- We can now find the superconducting order parameter $\Delta^c_{\mathbf{r}\mathbf{r}'} \sim B_r B_{\mathbf{r}'} \Delta^f_{\mathbf{r}\mathbf{r}'}$, where $\Delta^f_{\mathbf{r}\mathbf{r}'}$ is the f pairing in the spin liquid

- Typically the superconductor spontaneously breaks translation symmetry (FFLO state) or time-reversal or both; the form factor is $s + id_{x^2-y^2}$
- It also has an associated density wave, as fermion pairing at momentum $\mathbf{Q}$ automatically induces a charge density order at momentum $2\mathbf{Q}$
- Experiments in cuprates show no evidence of translation symmetry breaking, so we need to work harder!

Confinement transition

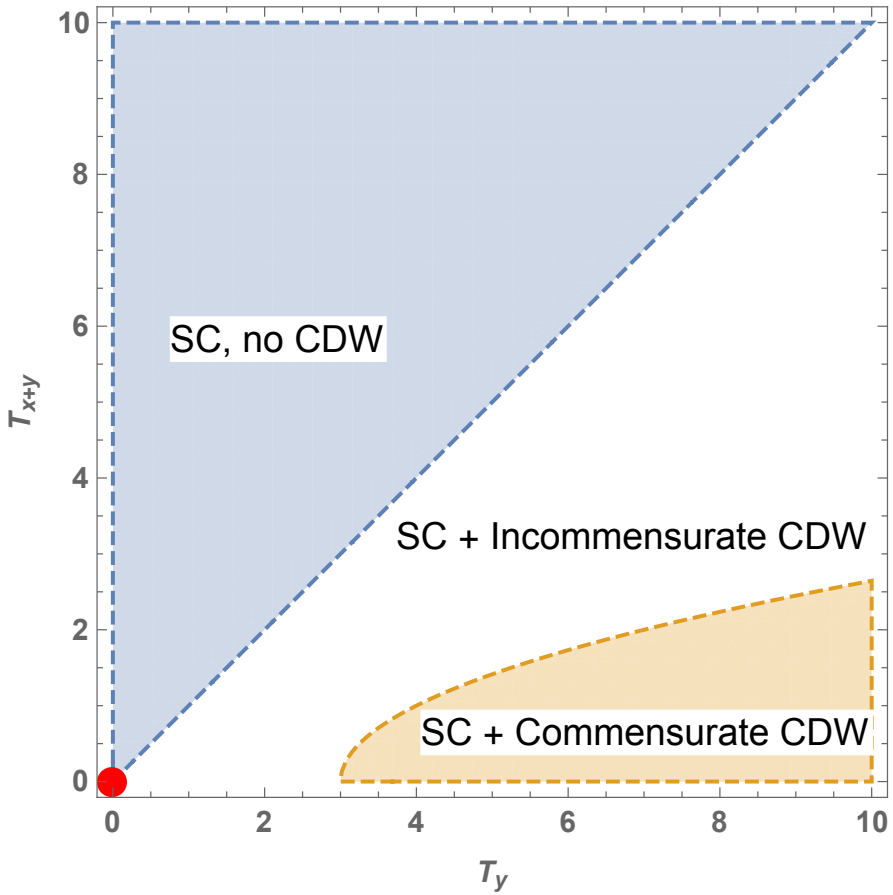


Figure: Phases of the superconductor - phase boundaries are approximate. The dotted red point denotes the phase with unbroken time-reversal $\mathcal{T}$.

Conclusion and outlook

- Described a mapping between bosonic and fermionic spin liquids on a rectangular lattice
- Used it to find the equivalent fermionic description of a gapped $\mathbb{Z}_2$ bosonic spin liquid that can underlie a FL* state
- Described possible confinement transitions out of the FL* metal to superconductors
- Further work required to describe the experimentally observed transition