Superconducting transition out of a FL* metal

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Brief Outline

- Fractionalized Fermi liquids (FL*)
- Experimental evidences for FL*
- Symmetry fractionalization: from bosons to fermions
- Superconducting transition
Conventional Fermi liquid

- Compressible state with a **Fermi surface** in momentum space
- Has gapless long-lived excitations across the Fermi surface
- Satisfies **Luttinger’s Theorem**: Area enclosed by Fermi surface = total density of electrons (mod 2)
Consider an insulating spin-half system with exchange interaction $H = \sum_{r,r'} J_{r,r'} \mathbf{S}_r \cdot \mathbf{S}_{r'}$.

Does the ground state have long range magnetic order in presence of geometric frustration?
Spin liquid

- Neel order is destroyed by quantum fluctuations - no symmetry is broken in the ground state
- Model ground state - resonating valence bond liquid

**Figure:** RVB state - each bond represents a singlet (S. Sachdev, Harvard)
Spin liquid

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Excitations of the $\mathbb{Z}_2$ spin liquid

- Has fractionalized spin half excitations called **spinons** which come in pairs
- Has emergent $\mathbb{Z}_2$ gauge field which mediates interactions between the spinons

**Figure:** Spinon excitation of the RVB state (S. Sachdev, Harvard)
Excitations of the $\mathbb{Z}_2$ spin liquid

- Vortices in the $\mathbb{Z}_2$ gauge field are called visons
- Spinons and visons are mutual semions
- Topological order: states on a cylinder with and without a vison through the hole are topologically degenerate

Figure: Vison excitation of the RVB state (S. Sachdev, Harvard)
Consider the Kondo-lattice problem: a band of mobile fermions interacts with a lattice of localized moments

\[
H = \sum_{k,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{r,r'} J_{r,r'} \mathbf{S}_r \cdot \mathbf{S}_{r'} + J_K \sum_r \frac{1}{2} c_{r\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{r\beta} \cdot \mathbf{S}_r
\]

Figure: Localized moments in the Fermi sea (P. Coleman, Rutgers)
Conventionally, the non-magnetic state is a heavy-fermion metal [Doniach, *Physica B*, 1977]

Local moments hybridize with conduction electrons - leading to formation of heavy quasiparticles

Luttinger’s theorem: Area enclosed by Fermi surface = (density of electrons + density of local moments)(mod 2)

**Figure:** Expansion of the Fermi surface (P. Coleman, Rutgers)
What if there are strong effects of frustrations and quantum fluctuations in 2d?

In $J_K \rightarrow 0$ limit, we have a stable gapped $\mathbb{Z}_2$ spin liquid ground state for the local moments.

Now turn on a small Kondo-coupling: $J_K \neq 0$.

Resulting metallic state is the $\mathbb{Z}_2$ FL*: conduction electron Fermi surface + background spin liquid.

Violates Luttinger’s theorem as it does not count local moments [Senthil et al, PRL, 2003; PRB, 2004]
High temperature superconductors

CuO$_2$ plane

YBa$_2$Cu$_3$O$_{6+x}$

Figure: YBCO (S. Sachdev, Harvard)
Figure: Phase diagram of the cuprates [Shen et al, Science 2005]
Experimental evidence for FL*

- Under hole doping $p$, relative to the band insulator there are $1 + p$ holes per square
- We expect a Fermi surface of size $1 + p$

**Figure:** Antiferromagnets with $p$ holes per square (S. Sachdev, Harvard)
Experimental evidence for FL*  

- Several experiments point to a Fermi liquid like state in the pseudogap regime, with carrier density $p$ instead of $1 + p$  
  2. Drude-like optical conductivity with $\tau^{-1} \sim \omega^2 + T^2$ [Mirzaei et al, PNAS, 2013]  
  3. In plane magnetoresistance obeys Kohler’s rule $\delta \rho/\rho_o \propto H^2T^{-4}$ [M. K. Chan et al, PRL, 2014]
Experimental evidence for FL*

- Metallic with a Fermi surface of size $p$, can qualitatively reproduce results of ARPES [Qi et al, PRB, 2010; Punk et al PNAS, 2015]
- D-form factor density waves with experimentally observed wavevectors is a low temperature instability of the FL* [Chowdhury et al, PRB, 2014]
Objective

How does the FL* metal undergo a confinement transition to a superconductor?

- An attractive insulating Mott state is the gapped $\mathbb{Z}_2$ spin liquid with bosonic spinons $b$, with incommensurate spin correlations and nematic order [Read et al, PRL, 1991]

- Need an equivalent description in terms of fermionic spinons $f$, which can hybridize with the hole-like $c$ fermions

- Pairing correlations between the $f$ spinons (naturally present in a $\mathbb{Z}_2$ spin liquid) will mediate pairing of the $c$ fermions — leading to superconductivity and destruction of topological order.
4 kinds of excitations, $e$, $m$, $\epsilon$ and the trivial local excitation 1

Have the following fusion rules:

$$e \times e = m \times m = \epsilon \times \epsilon = 1$$

$$1 \times 1 = 1, e \times 1 = e, m \times 1 = m, \epsilon \times 1 = \epsilon$$

$$e \times m = \epsilon, e \times \epsilon = m, m \times \epsilon = e$$

In the context of spin liquids, $e$ and $\epsilon$ are bosonic and fermionic spinons $b$ and $f$ respectively, $m$ is the spinless vison, 1 is a local excitation with integer spin.
Low energy quasiparticles in a $\mathbb{Z}_2$ gauge theory


$$H_{TC} = - \sum_p B_p - \sum_s A_s, \text{ with } B_p = \sum_{l \in p} \tau_l^z, \ A_s = \sum_{l \in \text{star}} \tau_l^x$$

- $e$ particles are violations of $A_s = 1$, and $m$ particles of $B_p = 1$, both can be created locally only in pairs
Braiding an $e$ around an $m$ leads to a phase of $-1$: $e$ and $m$ are self bosons but mutual semions

$$|\xi\rangle \rightarrow (\prod_{l \in C} \tau_l^z) |\xi\rangle = \left( \prod_{p \text{ inside } C} B_p \right) |\xi\rangle = - |\xi\rangle$$

$\epsilon$ can be thought of as a bound state of $e$ and $m$ - it is a fermion
Mapping bosons to fermions

Symmetry fractionalization and symmetry quantum numbers

- \( T_x T_y T_x^{-1} T_y^{-1} = 1 \) on any physical state (with even no. of anyons)

- Consider a state |\( \xi \)\rangle with two well-separated e particles

- Assume it is possible factorize the action of the symmetry on the two e particles locally, i.e, \( T_x |\xi\rangle = T_x(1) T_x(2) |\xi\rangle \)

  \[
  \implies |\xi\rangle = \prod_{i=1,2} T_x(i) T_y(i) [T_x(i)]^{-1} [T_y(i)]^{-1} |\xi\rangle
  \]

- Each e particle can pick up a phase: symmetries act projectively on anyons

  \[
  T_x(i) T_y(i) [T_x(i)]^{-1} [T_y(i)]^{-1} = e^{i\phi_i}
  \]
Merging two $e$ particles gives a trivial 1 particle, so $e^{i\phi_i} = \pm 1$

This phase is the same for all $e$ particles, and cannot change in a $\mathbb{Z}_2$ spin liquid phase without breaking translation symmetry.

Fractionalization pattern is a universal gauge-invariant property of $\mathbb{Z}_2$ spin liquids with translation symmetry.

Similar construction can be repeated for all space group and time-reversal symmetry - this gives us a classification of $\mathbb{Z}_2$ spin liquids independent of parton construction [Essin et al, PRB, 2013]
Assume we know the symmetry fractionalizations of the bosonic spinons $b$ and visons $v$.

The fermionic spinon $f$ can be thought of as a bound state of $b$ and $v$.

Naively, we expect that $e^{i\phi_f} = e^{i\phi_b} e^{i\phi_v}$

Not true when the bosonic and vison strings cross during the symmetry operation - extra twist factor of -1

$e^{i\phi_f} = e^{i\phi_t} e^{i\phi_b} e^{i\phi_v}$

Figure: $2\pi$-rotation - example of a fusion rule with an extra sign

[Y. M. Lu et al, arXiv:1403.0575]
### Mapping bosons to fermions

#### How to fuse the boson and the vison?

<table>
<thead>
<tr>
<th>Commutation relation</th>
<th>Bosonic PSG</th>
<th>Fermionic PSG</th>
<th>Vison PSG</th>
<th>Twist factor</th>
<th>Relation</th>
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<td>$(-1)^{p_1+1} = \eta T_xT_y$</td>
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<td>$(1)^{p_7} + 1 = \eta TP_y$</td>
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</table>
Given a mean-field spin-liquid ansatz, how does one calculate the symmetry quantum numbers?

We can find out PSG phases or matrices for each symmetry operation (space group and time-reversal) - not gauge-invariant [Wen, PRB 2002; Wang et al, PRB 2006]

We can then combine these phases to get gauge-invariant $\mathbb{Z}_2$ symmetry quantum numbers
Fractionalize spin into bosonic partons $S_i = \frac{1}{2} b_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} b_{i\beta}$

\[
H_{MF}^b = -\sum_{ij} (Q_{ij} \epsilon_{\alpha\beta} b_{i\alpha}^\dagger b_{j\alpha}^\dagger + \text{h.c.}) + \sum_i \lambda_i b_{i\alpha}^\dagger b_{i\alpha},
\]

\[
Q_{ij} = \langle \epsilon_{\alpha\beta} b_{i\alpha} b_{j\alpha} \rangle, \quad \sum_\alpha \langle b_{i\alpha}^\dagger b_{i\alpha} \rangle = 1
\]

Local $U(1)$ transformation $b_{r\alpha} \rightarrow e^{i\phi(r)} b_{r\alpha}$ leaves all physical observables unchanged

Any $Q_{ij}$ that is invariant under lattice/time-reversal symmetries only upto an additional $U(1)$ phase will give the same physical wave-function
For a symmetry operation $X$, we can choose an associated gauge-transformation $G_X$ such that the mean field ansatz is invariant under $G_X X$

$$G_X : b_{r\alpha} \to e^{i\phi_X(r)} b_{r\alpha}, \quad Q_{ij} \to e^{i(\phi_X(r_i)+\phi_X(r_j))} Q_{ij}$$

$$G_X X (Q_{ij}) = G_X (Q_{X[i] X[j]}) = Q_{ij}$$

Given a set of mean fields $Q_{ij}$, we can find these phases and hence $G_X$ for that ansatz.

Combine the symmetry operations that are equivalent to the identity on any physical state, then the appropriate (signed) sum of phases will give the corresponding symmetry fractionalization.
Consider the gapped bosonic spin liquid described by Read and Sachdev [Read et al, PRL, 1991]

\[ Q_{i,i+\hat{x}} \neq Q_{i,i+\hat{y}}, Q_{i,i+\hat{x}+\hat{y}} = Q_{i,i-\hat{x}+\hat{y}} \neq 0, Q_{i,i+2\hat{x}} \neq 0, Q_{i,i+2\hat{y}} = 0 \]

\[ N = |Q_{i,i+\hat{x}}|^2 - |Q_{i,i+\hat{y}}|^2 \neq 0 \]

Space group generators of a rectangular lattice are translations \( T_x \) and \( T_y \), and reflections \( P_x \) and \( P_y \)

\[ \phi_{T_x}(r) = \phi_{T_y}(r) = 0 \]
\[ \phi_{P_x}(r) = \pi y + \frac{\pi}{2} \]
\[ \phi_{P_y}(r) = \pi y \]
\[ \phi_T(r) = 0 \]
Can also solve for an ansatz given a particular set of symmetry quantum numbers

Algebraic constraints on these phases are imposed by space-group and time-reversal symmetries

Consider the space-group identity $T_x^{-1}T_yT_xT_y^{-1} = 1$

$$\implies (G_{T_x}T_x)^{-1}(G_{T_y}T_y)(G_{T_x}T_x)(G_{T_y}T_y)^{-1} = (-1)^{p_1}$$

Can be reduced to the following equation for the phases

$$-\phi_{T_x}[T_x(r)] + \phi_{T_y}[T_x(r)] + \phi_{T_x}[T_y^{-1}T_x(r)] - \phi_{T_y}(r) = p_1\pi$$

Similar relations for other symmetries would give us a set of $\phi_X$, and we may be able find an ansatz by solving for $G_XX(Q_{rr'}) = Q_{rr'}$
The equivalent description in term of fermions is again a gapped $\mathbb{Z}_2$ spin liquid. It has pairing on the nearest and the next nearest neighbors, and hopping on the next next nearest neighbor. The ansatz has $\pi$-flux through elementary plaquettes.

**Figure:** Dispersion in the doubled unit cell
Recall: FL* metal = $c$ fermion Fermi surface + $f$ spin liquid

Decouple the Kondo term in terms of the auxiliary bosons $B_{1r} = c^\dagger_{r\sigma} f_{r\sigma}$ and $B_{2r} = \epsilon_{\sigma\sigma'} c_{r\sigma} f_{r\sigma'}$

$$H_{cf} \sim J_K \sum_{r,r'} B_{1r}^\dagger c_{r\sigma}^\dagger f_{r\sigma} - B_{2r}^\dagger \epsilon_{\sigma\sigma'} c_{r\sigma} f_{r\sigma'} + h.c$$

Confinement can be induced by a condensation of these bosons

This confined ‘Higgs’ state is a superconductor as pairing of $f$ fermions now induces a pairing of $c$ fermions
Transition to the superconductor

Confinement transition

- Since the \( c \) fermions transform trivially under space group operations, we can derive the projective transformation of \( B \) from those of \( f \)

- We use this information to construct an effective boson Hamiltonian

\[
H_b = \sum_{i,j=1,2} \sum_{rr'} B_{ir}^\dagger T_{rr'}^{ij} B_{jr'} + \text{h.c}
\]

- The bosons will condense at the minima of the boson dispersion

- We can now find the superconducting order parameter

\[
\Delta_{rr'}^c \sim B_r B_{r'} \Delta_{rr'}^f, \text{ where } \Delta_{rr'}^f \text{ is the } f \text{ pairing in the spin liquid}
\]
Typically the superconductor spontaneously breaks translation symmetry (FFLO state) or time-reversal or both; the form factor is $s + id_{x^2-y^2}$

- It also has an associated density wave, as fermion pairing at momentum $Q$ automatically induces a charge density order at momentum $2Q$

- Experiments in cuprates show no evidence of translation symmetry breaking, so we need to work harder!
**Figure**: Phases of the superconductor - phase boundaries are approximate. The dotted red point denotes the phase with unbroken time-reversal $\mathcal{T}$. 
Conclusion and outlook

- Described a mapping between bosonic and fermionic spin liquids on a rectangular lattice
- Used it to find the equivalent fermionic description of a gapped $\mathbb{Z}_2$ bosonic spin liquid that can underlie a FL* state
- Described possible confinement transitions out of the FL* metal to superconductors
- Further work required to describe the experimentally observed transition