Probing excitations in insulators via injection of spin-currents

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Chatterjee, Sachdev, Physical Review B **92**, 165113 (2015)

Brief plan

- Motivation - spin liquid ground states of Mott insulators
- Setup and formalism to calculate spin current
- Application to antiferromagnetic insulators
- Results for insulators without magnetic order

- One electron per unit cell - band theory predicts a metallic state ($d \geq 2$)
- Strong electron-electron repulsion drives the system to an insulating state
- $H_{Hubbard} = -\sum_{i,j} t_{ij} c_i^\dagger c_j + h.c + \sum_i U n_i(n_i - 1)/2$
- Perturbation theory on the Hubbard model yields an effective Heisenberg Hamiltonian

$$H \approx J_H \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j$$

- Do the spins always order at low enough temperatures?

Ground states with no magnetic order

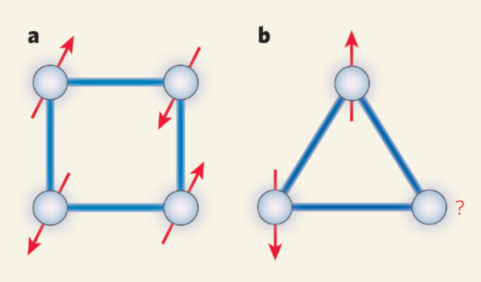
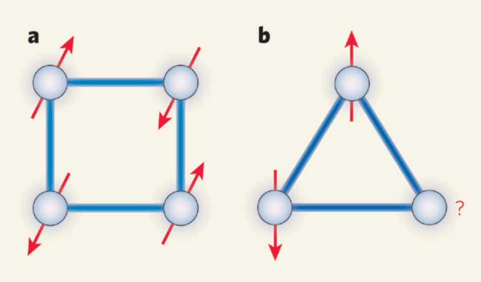


Figure: a. Neel order, b. Frustration (Nature, v456 n7224)

- Possible scenarios when we might expect no long range magnetic order:
  1. Small spins (like S = 1/2) $\implies$ large quantum fluctuations
  2. Geometric frustration
  3. Low dimension

Ground states with no magnetic order



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- Are the disordered ground states just like thermal paramagnets, or do these describe different quantum phases of matter?

- Triangular lattice organic salts are electrical insulators, but show behavior similar to metals

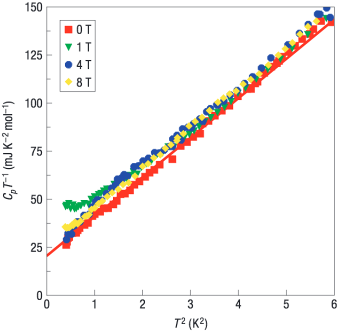


Figure: Low T specific heat of κET, [S Yamashita et al, *Nature Physics*, 2008]

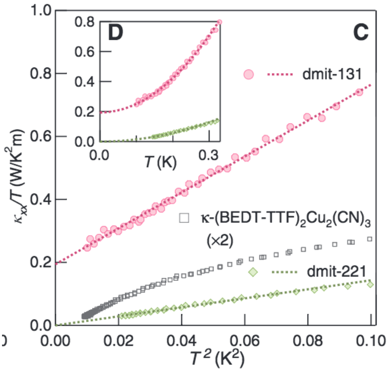


Figure: Low T thermal conductivity of dmit, [M Yamashita et al, *Science*, 2010]

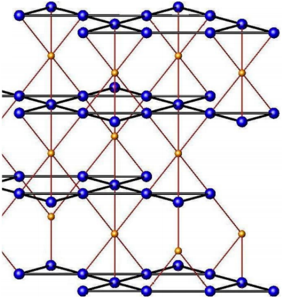
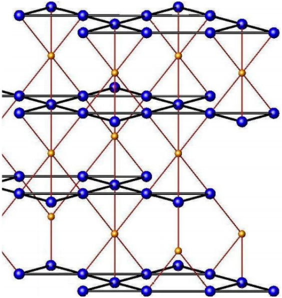


Figure: Layered Kagome lattice formed by Cu^{2+} with $S = \frac{1}{2}$

- Magnetic susceptibility measurements show no sign of magnetic order down to $50\,mK$, which is 4 orders of magnitude below $J_H \approx 200K$

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- Inelastic neutron scattering on Herbertsmithite single crystals [Han et al, *Nature*, 2014]
- No sharp features in the structure factor as we would expect in an ordered antiferromagnet

Alternative scenario

- Are there charge neutral mobile fractionalized spin half excitations (spinons) in Mott insulators?

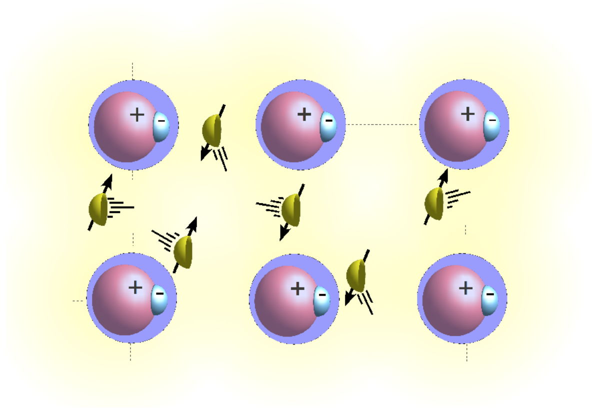

Figure: Caricature of a spin liquid (T. Senthil, MIT)

Spin liquid ground state

- Neel order is destroyed by quantum fluctuations - no symmetry is broken in the ground state
- Model ground state - resonating valence bond liquid [Anderson, *Mat. Res. Bulletin*, 1973]

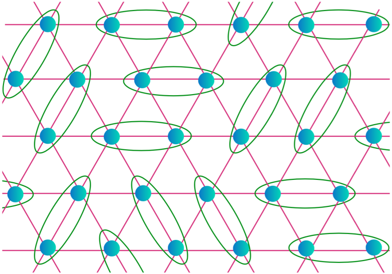

Figure: RVB state - each bond represents a singlet (S. Sachdev, Harvard)

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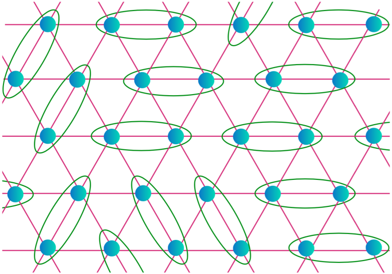

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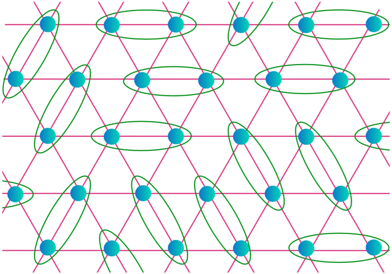

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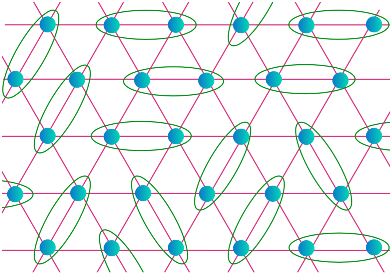


Figure: RVB state - each bond represents a singlet (S. Sachdev, Harvard)

Spin liquid ground state

- Has fractionalized spin half excitations and emergent gauge fields in the deconfined phase
- States have topological order - characterized by locally indistinguishable ground states which are degenerate on a cylinder/torus

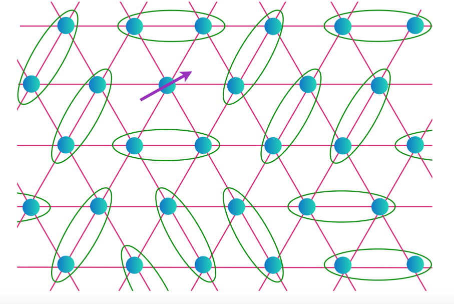


Figure: Spinon excitation of the RVB state (S. Sachdev, Harvard)

- Schwinger boson mean field approach - the parton construction with bosonic spins

$$\vec{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}, \text{ with } \sum_\alpha b^\dagger_{i\alpha} b_{i\alpha} = 1$$

- Mean field decoupling of the Heisenberg Hamiltonian in the spin singlet channel leads to

$$H = -J \sum_{ij} Q^*_{ij} \epsilon_{\alpha\beta} b_{i\alpha} b_{j\beta} + h.c + \sum_i \lambda_i (b^\dagger_{i\alpha} b_{i\alpha} - 1)$$

  $Q_{ij} = \epsilon_{\alpha\beta} \langle b_{i\alpha} b_{j\beta} \rangle / 2$, and λ_i are the mean-fields
- Emergent gauge field arises from phase fluctuations of the spinon-pairing field Q_{ij} and the Lagrange multipliers λ_i that enforces single occupancy on average
- Low energy effective theory has dynamic gauge fields coupled to spinons

- Large N (spinon-flavor) theory predict a gapped $\mathbb{Z}_2$ spin liquid [Sachdev, *PRB*, 1992]
- Projected wave-function studies predict a gapless U(1) spin liquid [Ran et al, *PRL*, 2007]
- DMRG calculations provide evidence for a gapped spin liquid ground state, with gap ≈ 0.1 J [Depenbrock et al, *PRL*, 2012]
- Inelastic neutron scattering on Herbertsmithite single crystals has not observed a gap
- Site-specific NMR measurements show evidence of a spin gap ≈ 0.05 J [Fu et al, *Science*, 2015]

What is the true nature of the ground state and the low-energy excitations?

- Experiments till this point:
  1. Thermodynamic measurements - magnetic susceptibility and specific heat
  2. Thermal conductivity
  3. NMR and μSR
  4. Inelastic neutron scattering

What other experiments can tell us about the nature of excitations above the ground state?

Spin transport measurements might be helpful in resolving some of these puzzles

- Couple metal with non-equilibrium distribution of spin to an insulating spin-system at the boundary

$$H = J \sum_j \vec{S}_e \cdot \vec{S}_j \, \delta^d(\vec{x} - \vec{X}_j)$$

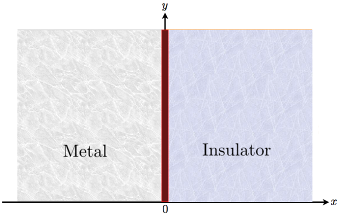


Figure: Geometry of $d-$dimensional metal coupled to an insulating spin system at $d-1$-dimensional boundary

- Use a longitudinal electric current in the metal to set up a spin accumulation at the boundary

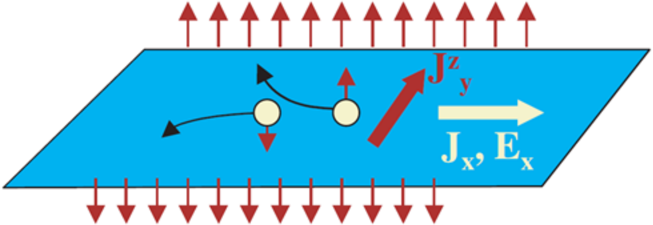


Figure: Spin Hall effect (G. Vignale, University of Missouri)

- Model the spin polarization of the boundary as different chemical potentials for up and down spins

$$n_\uparrow(\xi_{\vec{k}}) = n_F(\xi_{\vec{k}} - V),\ n_\downarrow(\xi_{\vec{k}}) = n_F(\xi_{\vec{k}})$$

$$\text{with } \xi_{\vec{k}} = \epsilon_{\vec{k}} - \mu_\downarrow \text{ and } V = \mu_\uparrow - \mu_\downarrow$$

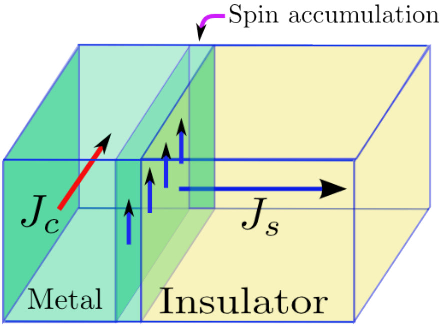


Figure: Injecting spin currents, via spin hall effect

- Spin transport across the insulator will set up a charge current via the inverse spin hall effect

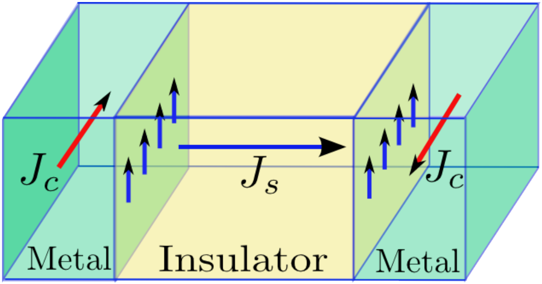


Figure: Detecting spin currents, via the inverse spin hall effect

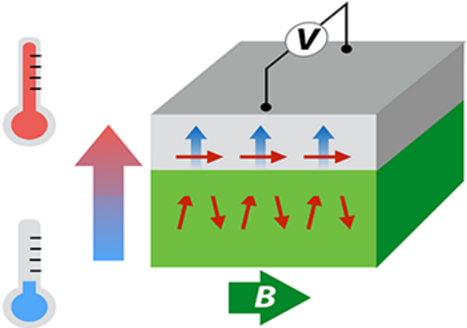
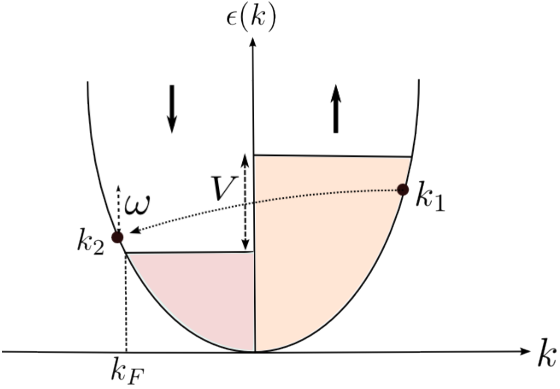


Figure: Thermal generation and detection of spin current from AF Cr_2O_3 to Pt, via inverse spin hall effect [Seki et al, *PRL*, 2015]

- Use Fermi's golden rule to calculate spin current due to spin-flip scattering at the boundary



Spin Current

- $$\begin{aligned} I_{spin,\uparrow} &= \frac{\pi J^2 A_\perp}{2m_e} \int_{k_{1x}>0} \frac{d^d k_1}{(2\pi)^d} \int_{q_x > k_{1x}} \frac{d^d q}{(2\pi)^d} \times \\ & n_\uparrow(\xi_{\vec{k}_1})(1 - n_\downarrow(\xi_{\vec{k}_1 - \vec{q}}))\ q_x\ S_{-+}\left(\vec{q}_\perp, \omega = \frac{2\vec{k}\cdot\vec{q} - \vec{q}^2}{2m_e}\right) \end{aligned}$$

  where the spin-structure factor $S_{-+}(\vec{q},\omega)$ is given by

  $$S_{-+}(\vec{q},\omega) = \frac{1}{A_\perp} \sum_{l,j} e^{-i\vec{q}\cdot(\vec{X}_l - \vec{X}_j)} \int_{-\infty}^{\infty} dt\ e^{i\omega t} \langle S_l^-(t) S_j^+(0)\rangle$$

- $I_{spin} = I_{spin,\uparrow} - I_{spin,\downarrow}$

- The spin current simplifies when $S_{-+}(\vec{q}_\perp, \omega)$ is significant for small $|\vec{q}_\perp|, \omega$

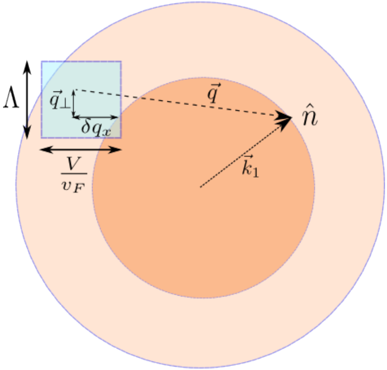


Figure: A scattering event (gap exaggerated)

- The spin current simplifies when $S_{-+}(\vec{q}_\perp,\omega)$ is significant for small $|\vec{q}_\perp|,\omega$

$$\boxed{I_{spin} \overset{T\to 0}{=} \frac{\pi J^2 A_\perp \nu(\epsilon_F)}{2}\int_0^V \frac{d\omega}{2\pi}\int \frac{d^{d-1}q_\perp}{(2\pi)^{d-1}}(V-\omega)S_{-+}(\vec{q}_\perp,\omega)}$$

- If $S_{-+}(\vec{q}_\perp,\omega)$ is peaked about momenta $\{\vec{Q}_\perp\}$ (well-isolated), the only change is a constant angular factor $f_{ang}(k_F/Q_\perp)$

Antiferromagnets

- Square lattice antiferromagnet with ordering wave-vector $\vec{Q}_\perp = (\pi, \pi)$
- Low energy excitations are spin waves with $\omega(\vec{q}) = v_s|\vec{q}|$

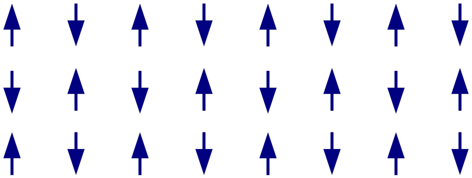


Figure: Square lattice AF at wavevector $\vec{Q}_\perp = (\pi, \pi)$

- Neel order pointing perpendicular to the spin-quantization axis in the metal
- Formerly analyzed for elastic spin-flip scattering [Takei et al, *PRB*, 2014]

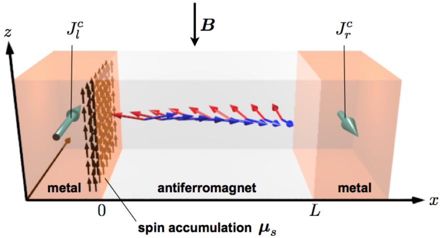


Figure: Spin current carried by a dynamically precessing Neel texture

- We calculate the elastic contribution assuming an average magnetic moment, and also find the inelastic contributions due to propagating spin wave modes

$$I_{spin} \stackrel{T\to 0}{=} \frac{\pi J^2 A_\perp \nu(\epsilon_F) V}{4} \left(f_{ang} + const.\,(1 + f_{ang}) V^3 \right)$$

- The first term is the elastic contribution that can be obtained from a Landauer formalism, the second term represents the excess spin current carried by magnons

- SU(2) spin-rotation symmetry is unbroken, but translational symmetry is spontaneously broken
- The dominant excitations at low energy are gapped triplons

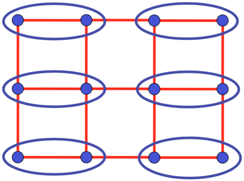
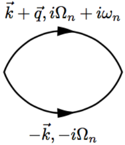


Figure: VBS state on a square lattice (S. Sachdev, Harvard)

$$I_{spin} \overset{T\to 0}{\propto} (V-\Delta_T)^{d/2+1}\,\Theta(V-\Delta_T)$$

- Each spin flip scattering excites two spinons that share the momentum



- Continuum field theory calculation using quadratic spinon bands for a model ground state with a spinon gap of Δ_s

$$I_{spin} \overset{T\to 0}{\propto} (V-2\Delta_s)^3\,\Theta(V-2\Delta_s)$$

- For a generic low energy dispersion $\epsilon(\vec{k}) = \Delta_s + v_\alpha |\vec{k}|^\alpha$ and dimensionality d of the system, we can use scaling and phase space restrictions to figure out the exponent above the threshold

$$I_{spin} \overset{T \to 0}{\propto} (V - 2\Delta_s)^{1+\frac{2(d-1)}{\alpha}} \, \Theta(V - 2\Delta_s)$$

- For a gapped spin liquid, this result should hold to a very good approximation for $T \ll \Delta_s$

- Using a Schwinger boson mean field approach on the Kagome lattice - two inequivalent choices of the mean field Q's for ground states [Sachdev, *PRB*, 1992]
- Both the $Q_1 = Q_2$ and $Q_1 = -Q_2$ states have gapped spinons with quadratic dispersion near the band minima

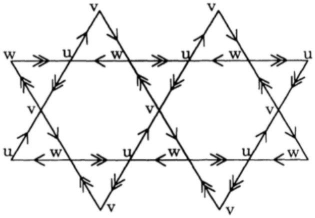
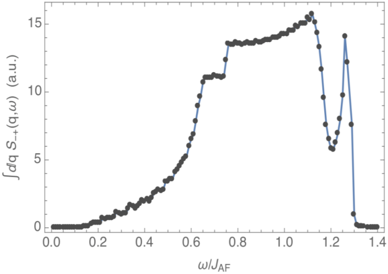
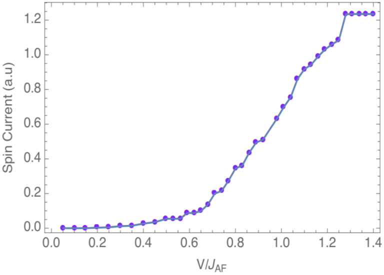


Figure: $Q_{ij} = Q_1$ for single arrows and Q_2 for double arrows

- Numerical calculations for the spin-current were carried out for the $Q_1 = Q_2$ state, a candidate ground state for Herbertsmithite [Punk et al, *Nature Physics*, 2014]





- Spin currents may be used as a gateway to probe the nature of possible exotic states in Mott insulators
- For antiferromagnets with gapless excitations, the spin current is calculated taking into account both elastic and inelastic scattering processes to check the formalism
- For spin liquids with a gap, the spin current is zero below a threshold, and rises as a function of the voltage with a power law that depends on the spinon *dispersion* and the *dimensionality* of the system
- The spin current is also able to distinguish between competing non-magnetic ground states - *spin liquids* and *VBS*

Thank you for your attention!