Transport in hole-doped cuprates:

Spin density wave order, topological order and Fermi surface reconstruction

Shubhayu Chatterjee
Harvard University

IIT Kanpur
December 16, 2016
In collaboration with:

Andreas Eberlein, Harvard University
Subir Sachdev, Harvard/Perimeter
Erez Berg, Weizmann/U. Chicago
Yoni Schattner, Weizmann Inst.

S. Chatterjee, A. Eberlein and S. Sachdev (to appear)
Structure of cuprate superconductors

High temperature superconductors

$\text{CuO}_2$ plane

$YBa_2Cu_3O_{6+x}$

$T_c = 95 \text{ K}$
Pseudogap metal: Fermi arcs

Conventional Fermi liquid: Large hole Fermi surface of size $1 + p$.

Figure credits: K. Fujita et al, Nature Physics 12, 150–156 (2016)
M. Plate et al, PRL. 95, 077001 (2005)
Pseudogap Metal:

Behaves like a Fermi liquid, with a Fermi surface of size $p$ instead of $1 + p$.

Hall effect experiments show that it is also present at high magnetic fields and low temperatures.

Figure credits: K. Fujita et al, Nature Physics 12, 150–156 (2016)
Evidences of metallic behavior in PG phase

- Optical conductivity \( \sim 1/(\omega + 1/\tau) \), with \( 1/\tau \sim \omega^2 + T^2 \)

  
  Mirzaei et al, PNAS 110, 5774 (2013)

- Magnetoresistance \( \sim \tau^{-1}(1 + aH^2\tau^2) \) follows Kohler’s rule for Fermi liquids

  
  Chan et al, PRL 113, 177005 (2014)

- T independent Hall coefficient corresponding to a carrier density of \( p \) in both higher temperature PG and in low T at high magnetic fields

  
Is there a quantum critical point (QCP) at optimal doping under the superconducting dome?

What is the nature of the associated phase transition? Symmetry-breaking or topological?

Figure credits: K. Fujita et al, Nature Physics 12, 150–156 (2016)
If I were a magician…

Superconductivity

Image credits: CNBC.com, Dreamstime.com
Low T Hall effect measurements in YBCO

$\eta_H = V / e R_H$

Fermi liquid (FL) with carrier density $1 + p$

Evidence for a "topological" metal with Fermi surface of size $p$?

How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order

- $\langle \phi \rangle \neq 0$
  - Metal with electron and hole pockets

- $\langle \phi \rangle = 0$
  - Metal with "large" Fermi surface

Image credits: S. Sachdev, Harvard
How does the Fermi surface reconstruct?

**Possibility 1: Symmetry breaking: Spin density wave (SDW) order**

\[
H_{SDW} = H_c + H_\theta + H_Y
\]

\[
H_c = -\sum_{i,j} t_{i,j} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i
\]

\[
H_\theta = -\sum_{i,j} J_{i,j} \cos(\theta_i - \theta_j) + 4\Delta \sum_i N_i^2, \ [\theta_i, N_j] = i\delta_{i,j}
\]

\[
H_Y = -\lambda \sum_i (-1)^{x_i+y_i} (e^{-i\theta_i} c_i^\dagger c_i + h.c.)
\]

However, long range symmetry breaking seems unlikely in the cuprates as correlation lengths of such order parameters are quite small.
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Electron and/or hole Fermi pockets form in “local” SDW order, but quantum fluctuations destroy long-range SDW order.

\[ \langle \phi \rangle = 0 \]

Algebraic Charge liquid (ACL) or Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and pocket Fermi surfaces.

\[ \langle \phi \rangle = 0 \]

Metal with “large” Fermi surface.

Metal with electron and hole pockets.

Image credits: S. Sachdev, Harvard
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

**Z\(_2\) fractionalized XY order coupled to electrons**

\[
H_c = - \sum_{i,j} t_{ij} c_{i}^{\dagger} c_{j} - \mu \sum_{i} c_{i}^{\dagger} c_{i}
\]

\[
H_{\theta,Z_2} = - \sum_{i,j} J_{ij} \mu_{i,j}^{z} \cos((\theta_i - \theta_j)/2) + 4\Delta \sum_{i} N_{i}^{2}
\]

**Z\(_2\) gauge theory Hamiltonian**

\[
- g \sum_{<ij>} \mu_{ij}^{x} - K \sum \left[ \prod \mu_{ij}^{z} \right]
\]

\[
H_{Y} = - \lambda \sum_{i} (-1)^{x_{i}+y_{i}} (e^{-i\theta_{i}} c_{i}^{\dagger} c_{i} + h.c.)
\]
Detour: $\mathbb{Z}_2$ gauge theory

$\mathbb{Z}_2$ electromagnetism: A simpler version of the familiar U(1) EM

$$H_{\mathbb{Z}_2} = -g \sum_{<ij>} \mu_{ij}^x - K \sum_{\square} \prod_{<ij> \in \square} \mu_{ij}^z$$

$\mathbb{Z}_2$ EM dictionary

$$\mu_{ij}^z = e^{iA_{ij}}, A_{ij} = 0, \pi$$
$$\mu_{ij}^x = e^{i\pi E_{ij}}, E_{ij} = 0, 1$$

$\mathbb{Z}_2$ Magnetic Flux

$$\prod_{\square} \mu_{ij}^z = e^{i \int_{\square} A_{ij}} = e^{iB}$$

$\mathbb{Z}_2$ gauge transformations

$$G_r = \prod_{<ij> \in +r} \mu_{ij}^x, [G_r, H_{\mathbb{Z}_2}] = 0 \forall r$$

Physical Hilbert space is given by $G_r = 1 \forall r$


Image credits: S. Gazit, UC Berkeley
Detour: $\mathbb{Z}_2$ gauge theory

$\mathbb{Z}_2$ electromagnetism: A simpler version of the familiar $U(1)$ EM

$\mathbb{Z}_2$ gauge constraints with electric charges

$\mathbb{Z}_2$ Gauss Law

With ‘$\mathbb{Z}_2$ matter fields’

$$\prod_{<ij> \in +r} \mu_{ij}^x = e^{i\pi \sum_{<ij> \in +r} E_{ij}} = e^{i\pi \Delta \cdot E} = \rho_{\mathbb{Z}_2}$$

Image credits: S. Gazit, UC Berkeley
Detour: $\mathbb{Z}_2$ gauge theory

$\mathbb{Z}_2$ electromagnetism: A simpler version of the familiar U(1) EM

Confinement transition in $\mathbb{Z}_2$ gauge theory

$$H_{\mathbb{Z}_2} = -g \sum_{<ij>} \mu_{i,j}^x - K \sum_{\square} \left[ \prod_{<i,j> \in \square} \mu_{i,j}^z \right]$$

$\mathbb{Z}_2$ Gauss Law: $$\prod_{<i> \in +_r} \mu_{i,j}^x = \rho_{\mathbb{Z}_2}$$

$$V(L) \sim gL$$

$E_{vortex} \sim |g - g_c|^\nu$

$E_{string} \sim |g - g_c|^\nu$

$\prod_{\square} |B_{\square} = 0\rangle$

$\prod_{<i,j>} |\mu_{i,j}^x = 1\rangle$

Image credits: S. Gazit, UC Berkeley
Detour: $\mathbb{Z}_2$ gauge theory

$\mathbb{Z}_2$ electromagnetism: A simpler version of the familiar $U(1)$ EM

Topological degeneracy in deconfined phase

\[
\chi = \prod_{i,j} \mu_{ij}^x, \\
Z = \prod_{i,j} \mu_{ij}^z, \\
\{X, Z\} = 0
\]

$|G\rangle$ and $|G_2\rangle = \chi |G\rangle$ are distinct ground states

$2^N$ degeneracy on a Riemann surface of genus $N$

Image credits: Sid Parameswaran, UC Irvine
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

**$Z_2$ fractionalized XY order coupled to electrons**

$$H_c = - \sum_{i,j} t_{i,j} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$H_{\theta,Z_2} = - \sum_{i,j} J_{i,j} \mu_{i,j}^z \cos((\theta_i - \theta_j)/2) + 4\Delta \sum_i N_i^2$$

**$Z_2$ gauge theory Hamiltonian**

$$H_Y = -\lambda \sum_i (-1)^{x_i+y_i} (e^{-i\theta_i} c_i^\dagger c_i + h.c.)$$

Gauge transformations:

$$e^{i\theta_i/2} \rightarrow \epsilon_i e^{i\theta_i/2}$$

$$\mu_{i,j}^z \rightarrow \epsilon_i \mu_{i,j}^z \epsilon_j$$

$$G_r = e^{2i\pi N_r} \prod_{<i,j> \in +_r} \mu_{i,j}^x = 1 \text{ in physical Hilbert space}$$
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Define canonical fermionic operators: Spinless chargons

\[ \psi_+ = e^{i\theta/2} c^\uparrow \]
\[ \psi_- = e^{-i\theta/2} c^\downarrow \]

The chargons see constant antiferromagnetic order

\[ H_Y = -\lambda \sum_i (-1)^{x_i+y_i} (\psi_i^\dagger \psi_i - + h.c.) \]

If we can realize a phase where the chargons are deconfined, we can have Fermi pockets of U(1) charged fermions

Carry U(1) EM charge of an electron + $Z_2$ gauge charge
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

No long range XY order

Deconfined chargons

Destroy XY order by proliferating vortices in the XY field ($\theta$)

$$\psi_+ = e^{i\theta/2} c_\uparrow$$
$$\psi_- = e^{-i\theta/2} c_\downarrow$$

Problem: Chargons are not single valued around $2\pi$ vortices in $\theta$

Chargons are confined: FS reconstruction and XY ordering coincide
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

- No long range XY order
- Deconfined chargons

Destroy XY order by proliferating vortices in the XY field ($\theta$)

Solution: Chargons are single valued around $4\pi$ vortices in $\theta$

Chargons deconfined: FS reconstruction distinct from XY ordering
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

No long range XY order

Deconfined chargons

Without a $\mathbb{Z}_2$ vortex, $2\pi$ vortices in $\Theta$ have a branch cut and therefore cost extensive energy

$2\pi$ vortex in $\Theta$ needs to tie with a $\mathbb{Z}_2$ vortex for proliferation

Make $\mathbb{Z}_2$ vortices very energy expensive: Take K to $\infty$

$$- \sum_{i,j} J_{i,j} \mu_i \mu_j \cos\left(\frac{\theta_i - \theta_j}{2}\right)$$
How does the Fermi surface reconstruct?

1: Symmetry Breaking SDW, 2: Topological order

\( K \)

\( \Delta \)

(C) ACL/FL*
\( \mathbb{Z}_2 \) topological order

\( \langle e^{i\theta} \rangle = 0 \)

\( \langle e^{i\theta} \rangle \neq 0 \)

Proliferation of double vortices
Proliferation of single vortices

(B) SDW metal

(A) Fermi liquid
How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order

(C) ACL/FL*
$\mathbb{Z}_2$ topological order

\[ \langle e^{i\theta} \rangle = 0 \]
\[ \langle e^{i\theta} \rangle \neq 0 \]

Fe based superconductors

Proliferation of double vortices

Proliferation of single vortices

(B) SDW metal

(A) Fermi liquid
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)
How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Violation of Luttinger’s theorem

• Luttinger’s theorem relates the volume of the Fermi surface to the density of fermions

\[
\frac{V_{FS}}{4\pi^2} = n \equiv (1 + p)(\text{mod } 2)
\]

M. Oshikawa, PRL 84, 3370 (2000)

• Topological metal violates Luttinger’s theorem

\[
\frac{V_{FS}}{4\pi^2} = p(\text{mod } 2)
\]

Paramekanti et al, PRB 70, 245118 (2004)
Hall effect data can be well-explained by commensurate (Neel) order/incommensurate (spiral) order/the topological metal (ACL/FL*)

Other probes to further substantiate/distinguish different QCPs?

J. G. Storey, EPL, **113** (2016) 27003
A. Eberlein et al, PRL, **117** (2016), 187001
Thermal transport with reconstructed FS

Regime 1: Clean limit

**D-wave superconductivity: Universal thermal conductivity**

\[
\frac{\kappa}{T} = \frac{k_B^2}{3} \frac{v_F^2 + v_{\Delta}^2}{v_F v_{\Delta}}
\]

Durst and Lee, PRB 62, 1720 (2000)

**D-wave superconductivity + Neel order: Nodal collision gaps out the charge carriers beyond a critical Neel amplitude \( A_c \)**

\[
\frac{\kappa}{T} = \frac{k_B^2}{3 v_F v_{\Delta}} \left( \sqrt{1 - \alpha^2} v_F^2 + \frac{1}{\sqrt{1 - \alpha^2}} v_{\Delta}^2 \right), \quad \alpha = \frac{A}{A_c} < 1
\]

Different threshold behavior from CDW, preserves \( C_4 \) symmetry

Durst and Sachdev, PRB 80, 054518 (2009)
Regime 1: Clean limit

D-wave superconductivity (dSC) + spiral order (iAF):

Distinguish Neel and spiral orders

Spiral order co-existing with d-wave superconductivity has a much steeper drop \((\text{why?})\) than Neel order + dSC
Thermal transport with reconstructed FS

Regime 2: Dirty limit

Inverse scattering lifetime $\gg$ SC amplitude

Distinguish Neel and spiral orders

Behaves nearly like a magnetic metal— the drops are different for cAF and iAF for the same doping range.
Summary

Discussed possible scenarios that lead to FS reconstruction in the hole-doped cuprates

Presented a concrete model with FS reconstruction without symmetry breaking but with $Z_2$ topological order

Spectroscopic and Hall data agree well with the topological metal or field-induced AF order

Presented computations of thermal conductivity as a function of doping, which can serve as an additional check
Thank you for your attention!
Extra slide: Fermi surface reconstruction

A. Eberlein et al, PRL, 117 (2016), 187001