Transport in hole-doped cuprates:

Spin density wave order, topological order and Fermi surface reconstruction

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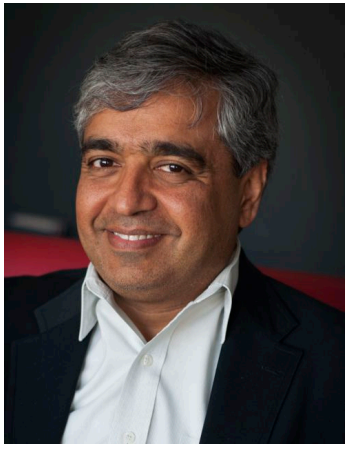


Subir Sachdev,
Harvard/Perimeter

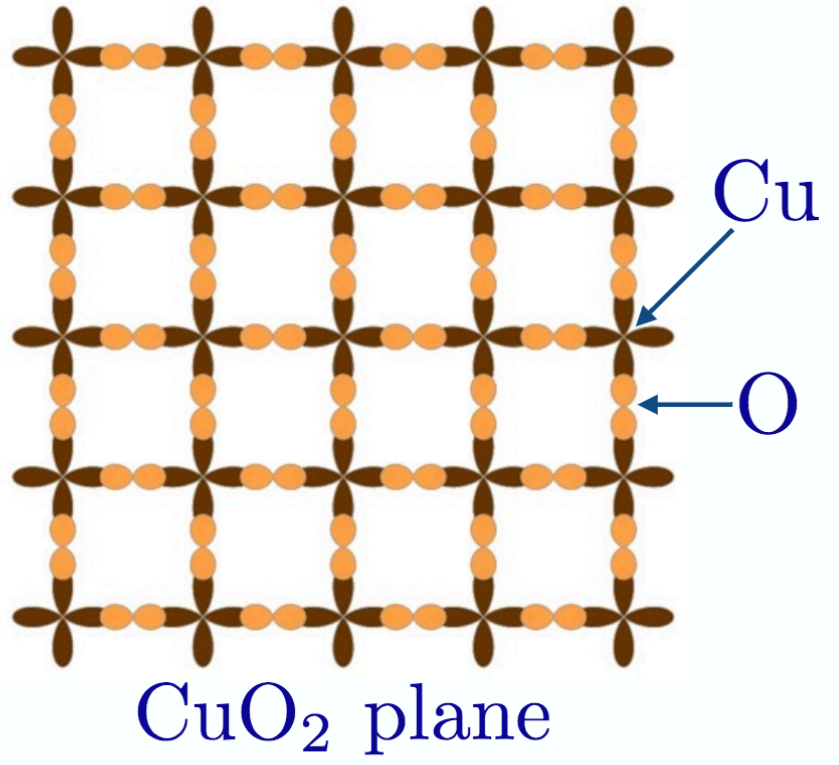
Erez Berg,
Weizmann/U. Chicago

Yoni Schattner,
Weizmann Inst.

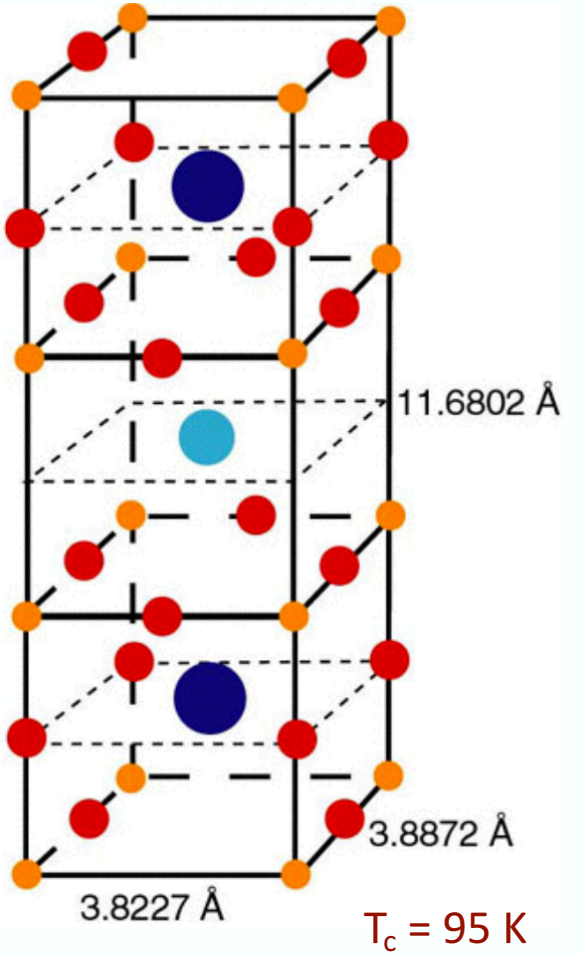
S. Sachdev, E. Berg. *S. Chatterjee* and Y. Schattner, **Phys. Rev. B** 94, 115147, 2016;
S. Chatterjee, A. Eberlein and S. Sachdev (to appear)

Structure of cuprate superconductors

High temperature superconductors



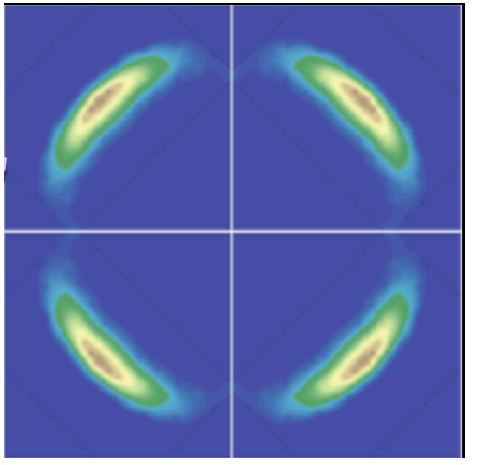
CuO_2 plane

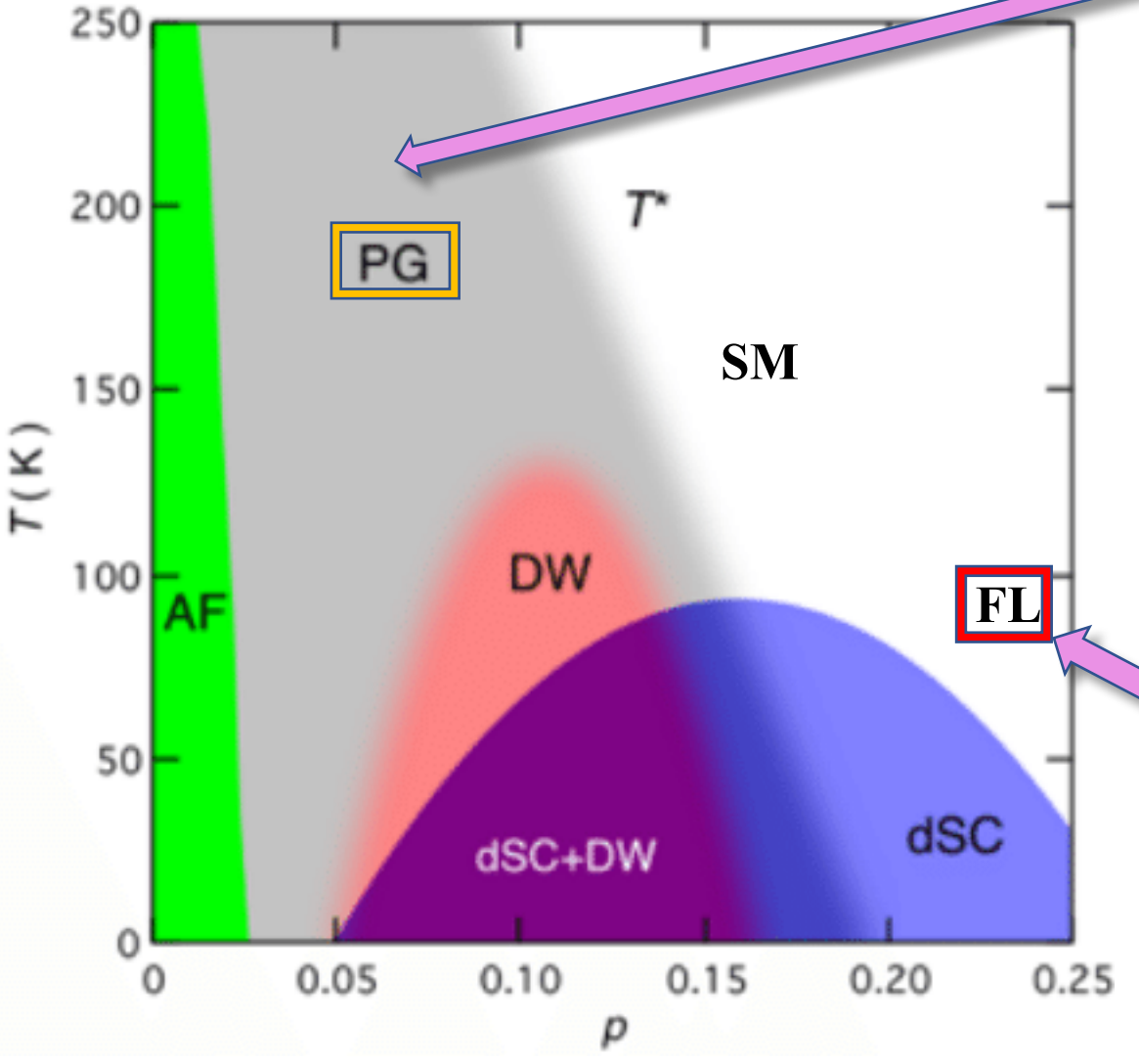


T_c = 95 K

$YBa_2Cu_3O_{6+x}$

Pseudogap metal:
Fermi arcs





Conventional Fermi liquid:
Large hole Fermi surface of size 1 + p.

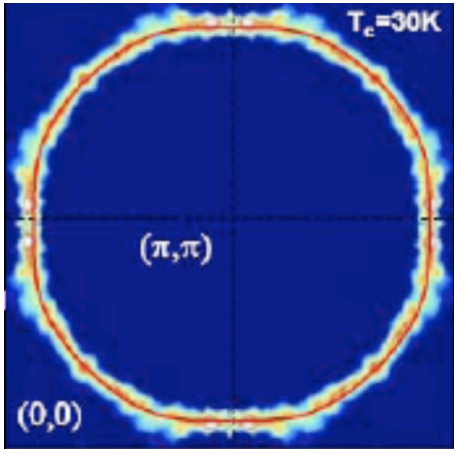
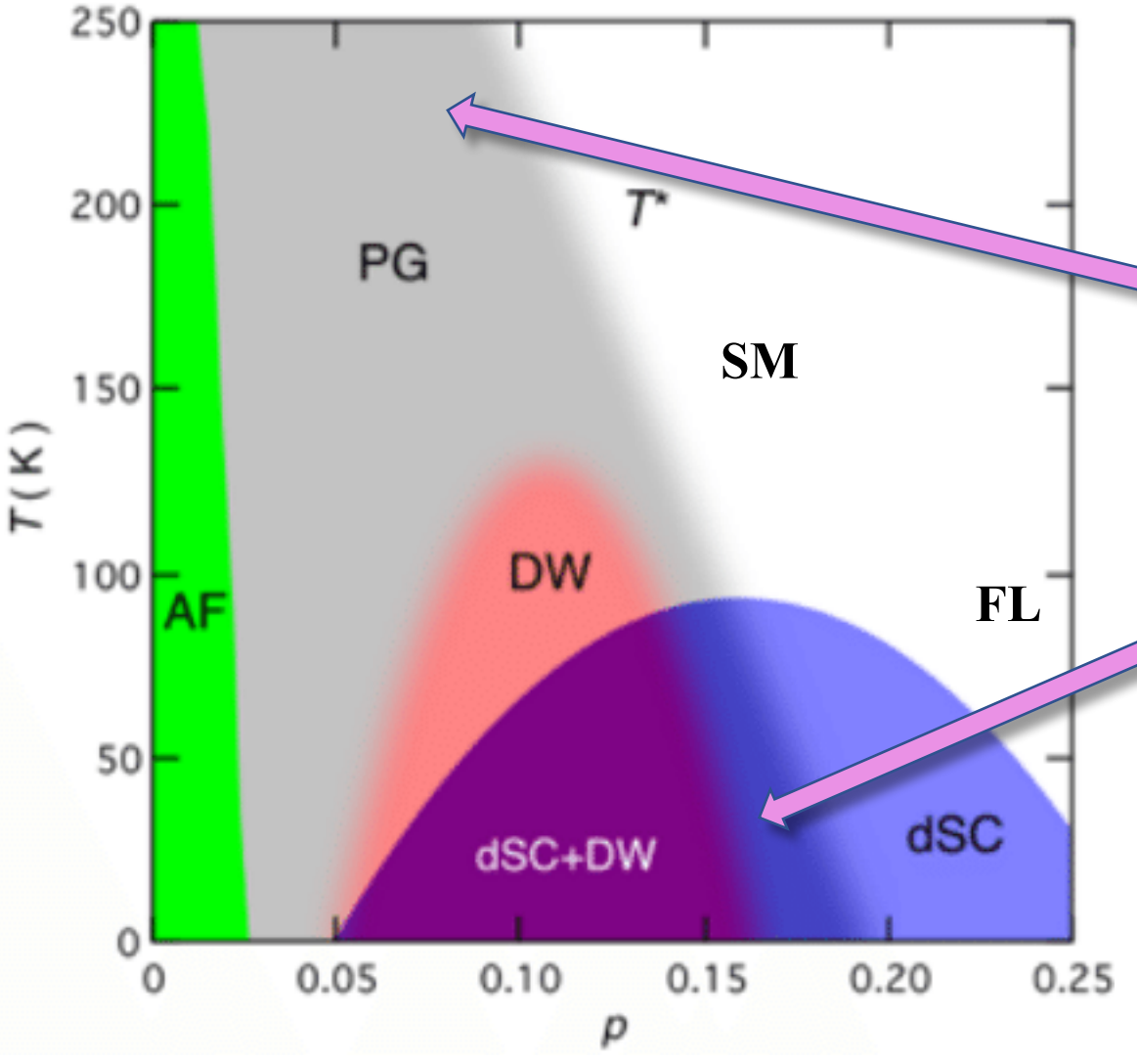


Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)
M. Plate *et al*, PRL. 95, 077001 (2005)



Pseudogap Metal:

Behaves like a Fermi liquid, with a Fermi surface of size p instead of $1 + p$.

Hall effect experiments show that it is also present at high magnetic fields and low temperatures.

Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)
C. Proust *et al*, Nature **531**, 210 (2016).

Evidences of metallic behavior in PG phase

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$, with $1/\tau \sim \omega^2 + T^2$

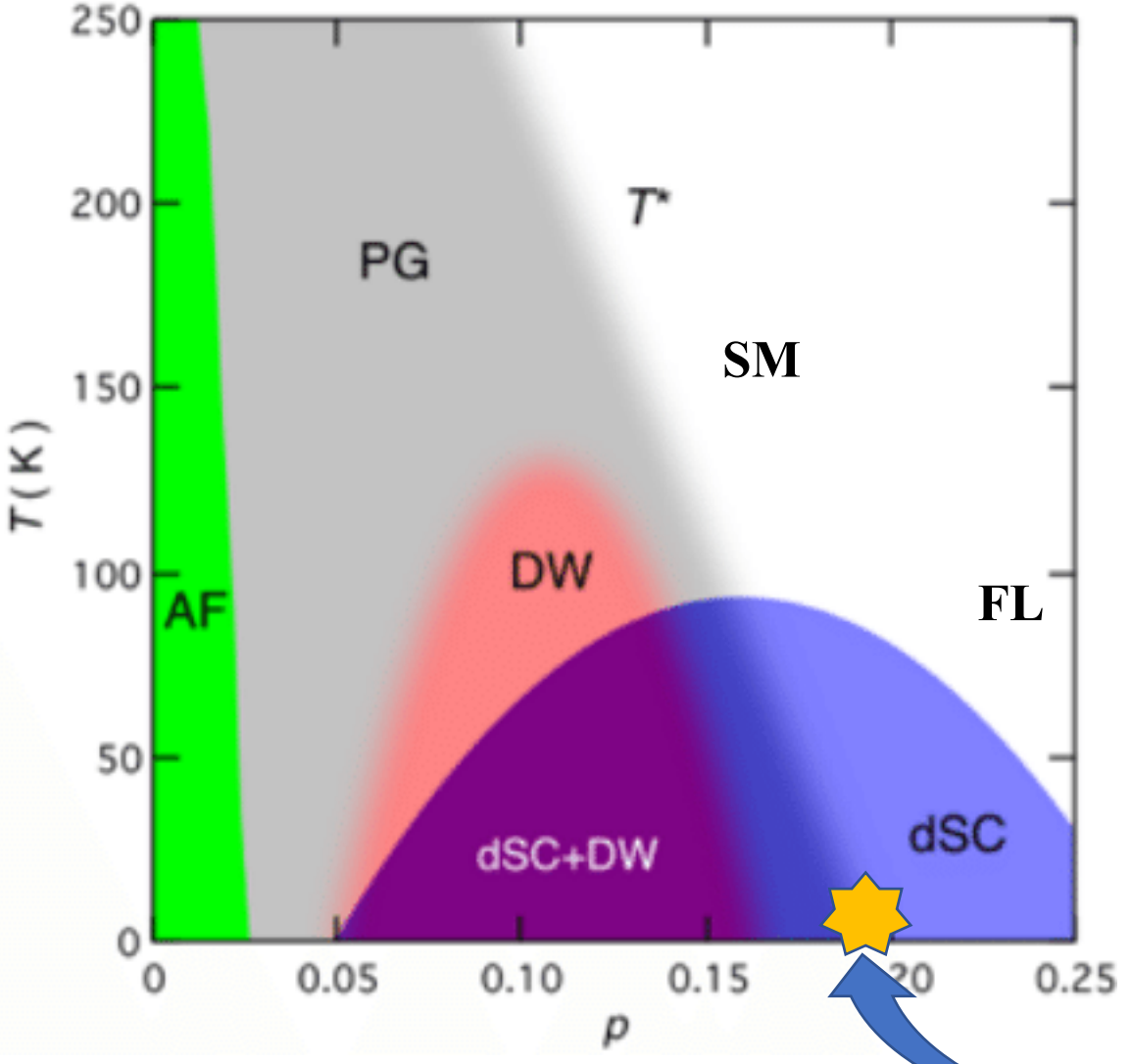
  Mirzaei *et al*, PNAS **110**, 5774 (2013)

- Magnetoresistance $\sim \tau^{-1}(1 + aH^2\tau^2)$ follows Kohler's rule for Fermi liquids

  Chan *et al*, PRL **113**, 177005 (2014)

- T independent Hall coefficient corresponding to a carrier density of *p* in both higher temperature PG and in low T at high magnetic fields

  Ando *et al*, PRL **92**, 197001 (2004), Badoux *et al*, Nature **531**, 210 (2016)



Is there a quantum critical point (QCP) at optimal doping under the superconducting dome?

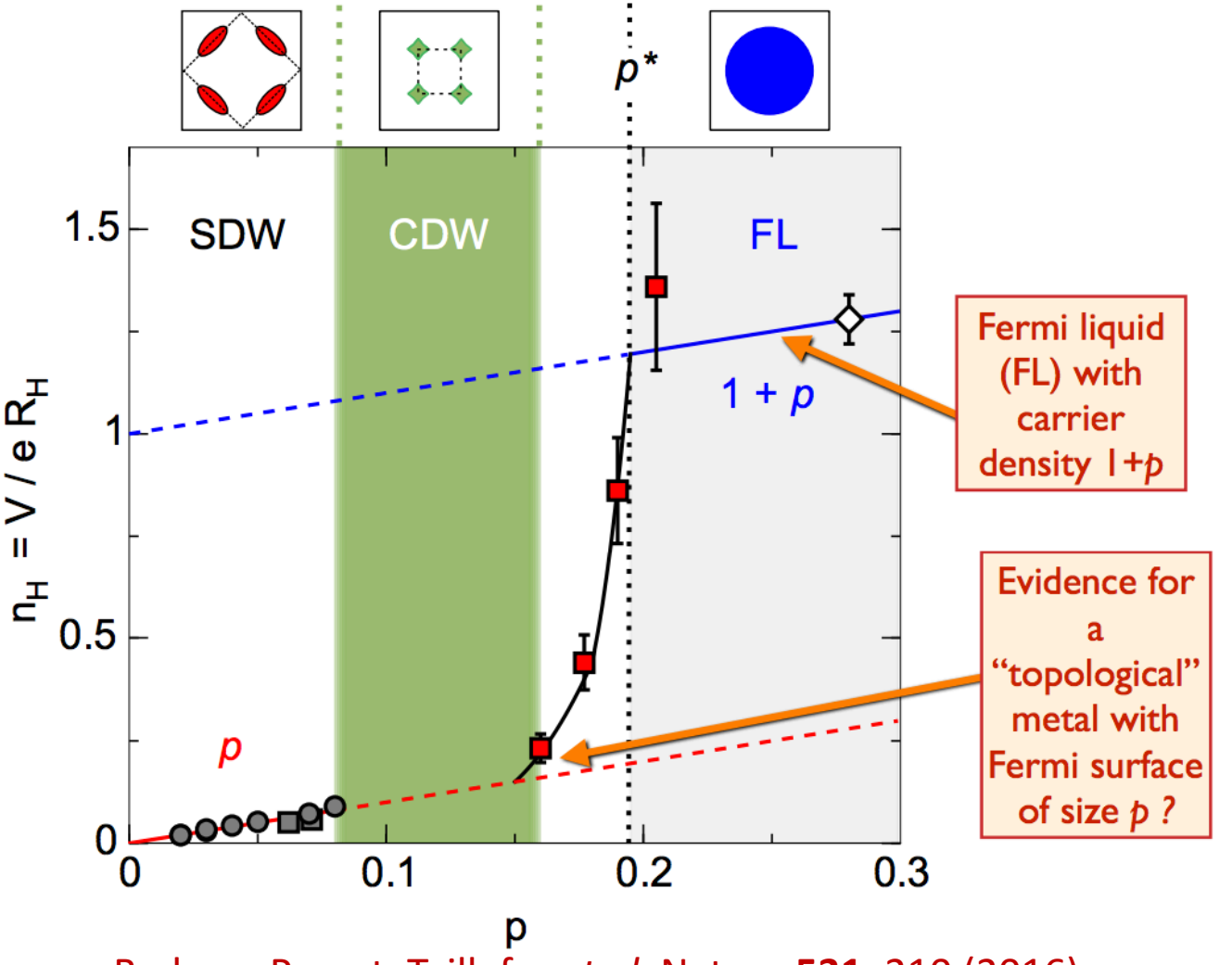
What is the nature of the associated phase transition? Symmetry-breaking or topological?

Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)

If I were a magician…

Image credits: CNBC.com, Dreamstime.com

Low T Hall effect measurements in YBCO



Badoux, Proust, Taillefer *et al*, Nature **531**, 210 (2016)

How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order

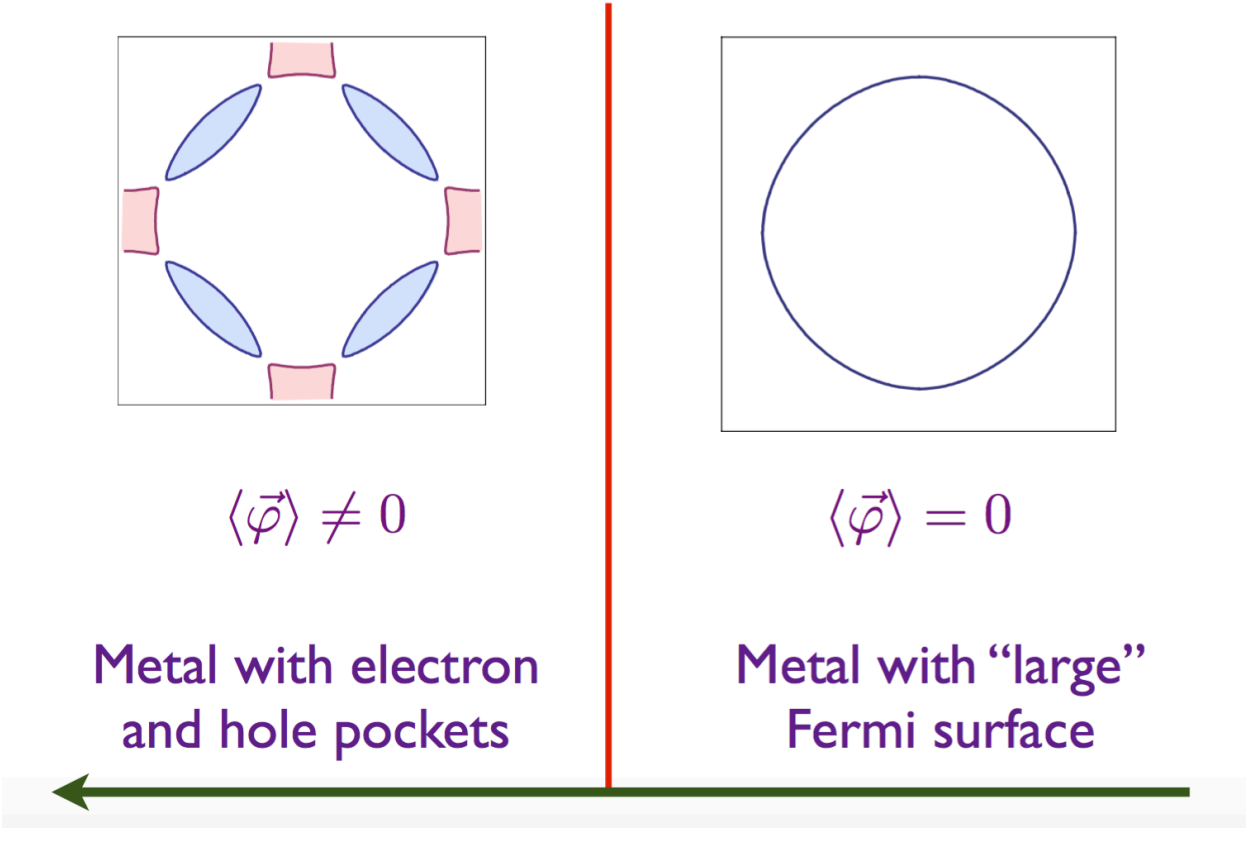


Image credits: S. Sachdev, Harvard

How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order

Hertz Theory

$$H_{SDW} = H_c + H_\theta + H_Y$$

$$H_c = -\sum_{i,j} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

XY order parameter

$$H_\theta = -\sum_{i,j} J_{ij} \cos(\theta_i - \theta_j) + 4\Delta \sum_i N_i^2, \; [\theta_i, N_j] = i\delta_{ij}$$

$$H_Y = -\lambda \sum_i (-1)^{x_i + y_i} (e^{-i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow} + h.c.)$$

However, long range symmetry breaking seems unlikely in the cuprates as correlation lengths of such order parameters are quite small

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

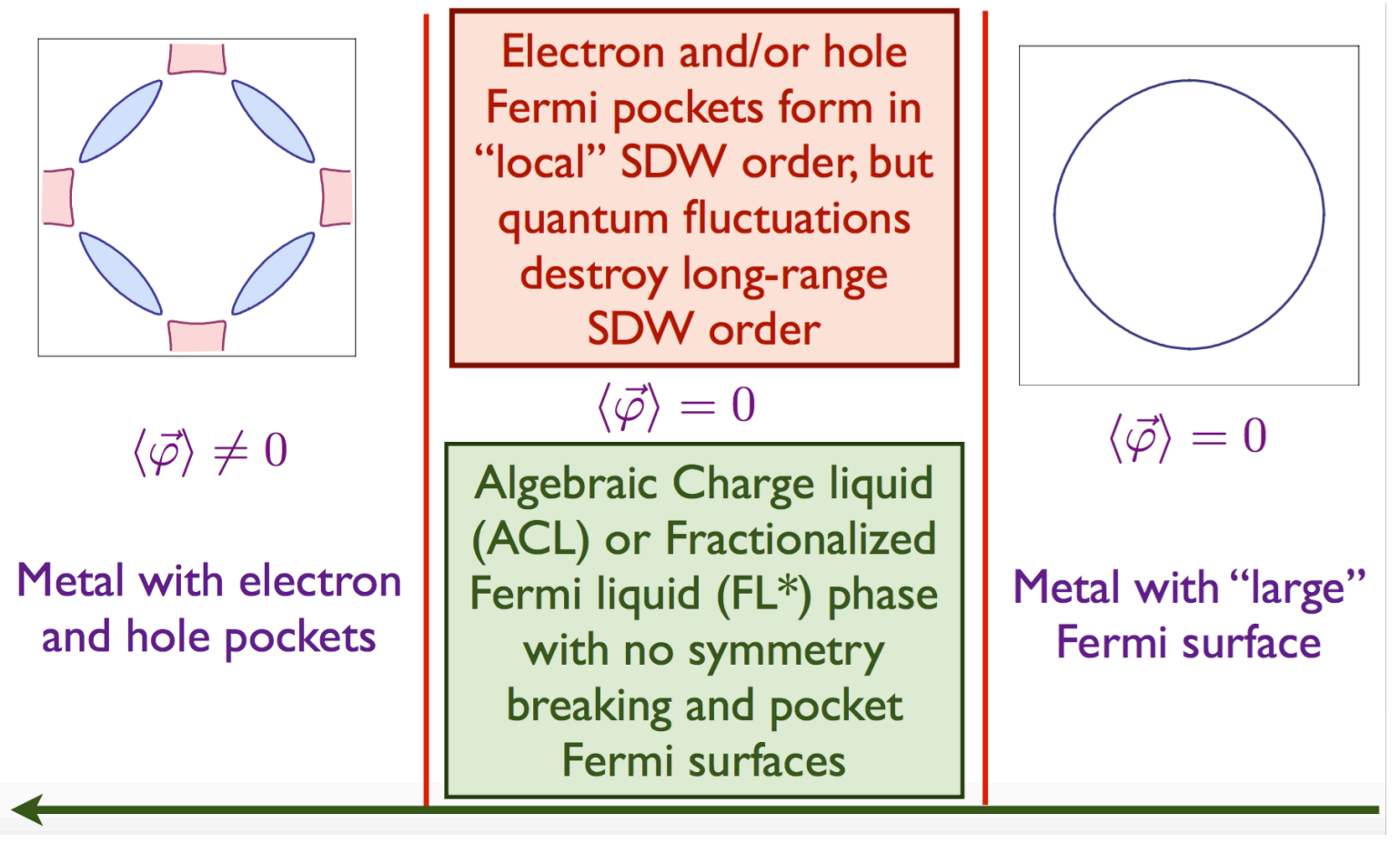


Image credits: S. Sachdev, Harvard

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Z_2 fractionalized XY order coupled to electrons

$$H_c = -\sum_{i,j} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$H_{\theta,\mathbb{Z}_2} = -\sum_{i,j} J_{ij} \mu_{ij}^z \cos((\theta_i - \theta_j)/2) + 4\Delta \sum_i N_i^2$$

Z_2 gauge theory Hamiltonian

$$-g \sum_{<ij>} \mu_{ij}^x - K \sum_{\square} \left[\prod_{\square} \mu_{ij}^z \right]$$

$$H_Y = -\lambda \sum_i (-1)^{x_i + y_i} (e^{-i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow} + h.c.)$$

Detour: Z_2 gauge theory

Z_2 electromagnetism: A simpler version of the familiar U(1) EM

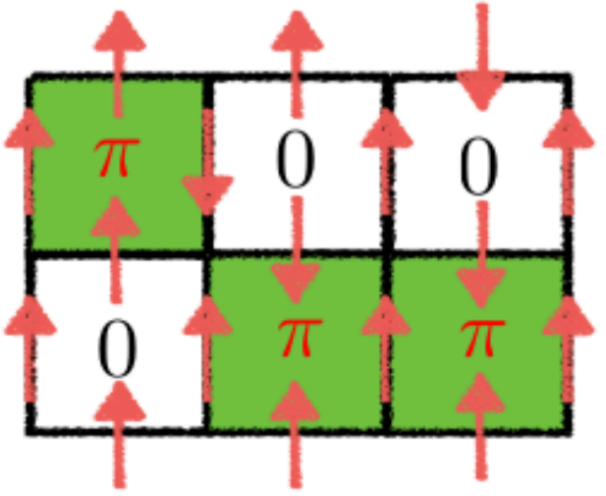
$$H_{\mathbb{Z}_2} = -g \sum_{<ij>} \mu^x_{ij} - K \sum_{\square} \left[\prod_{<ij> \in \square} \mu^z_{ij} \right]$$

Z_2 EM dictionary

$$\mu^z_{ij} = e^{iA_{ij}}, A_{ij} = 0, \pi$$

$$\mu^x_{ij} = e^{i\pi E_{ij}}, E_{ij} = 0, 1$$

F. Wegner (1971),
J. Kogut (1979)



Z_2 Magnetic Flux

$$\left[\prod_{\square} \mu^z_{ij} \right] = e^{i \int_{\square} A_{ij}} = e^{iB}$$

Z_2 gauge transformations

$$G_r = \prod_{<ij> \in +_r} \mu^x_{ij}, \; [G_r, H_{\mathbb{Z}_2}] = 0 \; \forall r$$

Physical Hilbert space is given by $G_r = 1 \; \forall r$

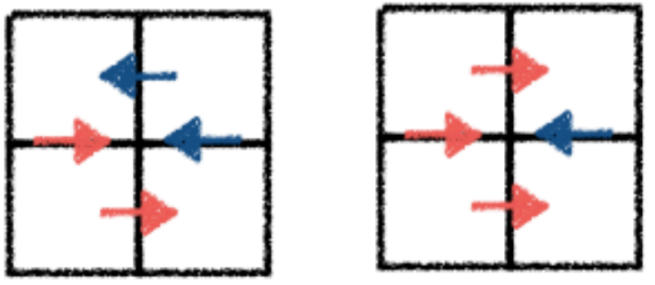
Image credits: S. Gazit, UC Berkeley

Detour: Z_2 gauge theory

Z_2 electromagnetism: A simpler version of the familiar U(1) EM

Z_2 gauge constraints with electric charges

Z_2 Gauss Law



✓ ✗

$$\prod_{<ij>\in+_r} \mu^x_{ij} = e^{i\pi \sum_{<ij>\in+_r} E_{ij}} = e^{i\pi \Delta\cdot E} = \rho_{\mathbb{Z}_2}$$

With 'Z_2 matter fields'

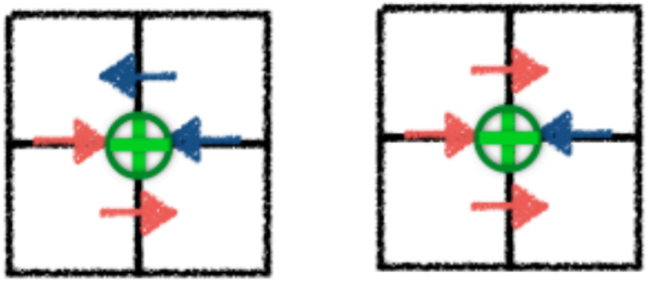


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Image credits: S. Gazit, UC Berkeley

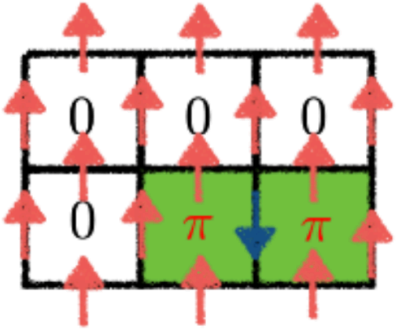
Detour: Z_2 gauge theory

Z_2 electromagnetism: A simpler version of the familiar U(1) EM

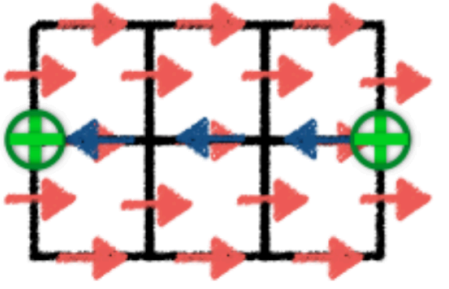
Confinement transition in Z_2 gauge theory

$$H_{\mathbb{Z}_2} = -g \sum_{<ij>} \mu^x_{ij} - K \sum_{\square} \left[\prod_{<ij> \in \square} \mu^z_{ij} \right]$$

$\mathbb{Z}_2$ Gauss Law: $\prod_{<ij> \in +_r} \mu^x_{ij} = \rho_{\mathbb{Z}_2}$



$V(L) \sim gL$



$z = 1$

3D Ising

$E_{vortex} \sim |g - g_c|^\nu$

$E_{string} \sim |g - g_c|^\nu$

deconfined g_c confined

$|GS\rangle = \prod_{\square} |B_\square = 0\rangle$

$|GS\rangle = \prod_{<ij>} |\mu^x_{ij} = 1\rangle$

Image credits: S. Gazit, UC Berkeley

Detour: Z_2 gauge theory

Z_2 electromagnetism: A simpler version of the familiar U(1) EM

Topological degeneracy in deconfined phase

$$\mathcal{X} = \prod_{\equiv} \mu^x_{ij},$$

$$\mathcal{Z} = \prod_{o} \mu^z_{ij},$$

$$\{X, Z\} = 0$$

$|G\rangle$ and $|G_2\rangle = \chi|G\rangle$ are distinct ground states

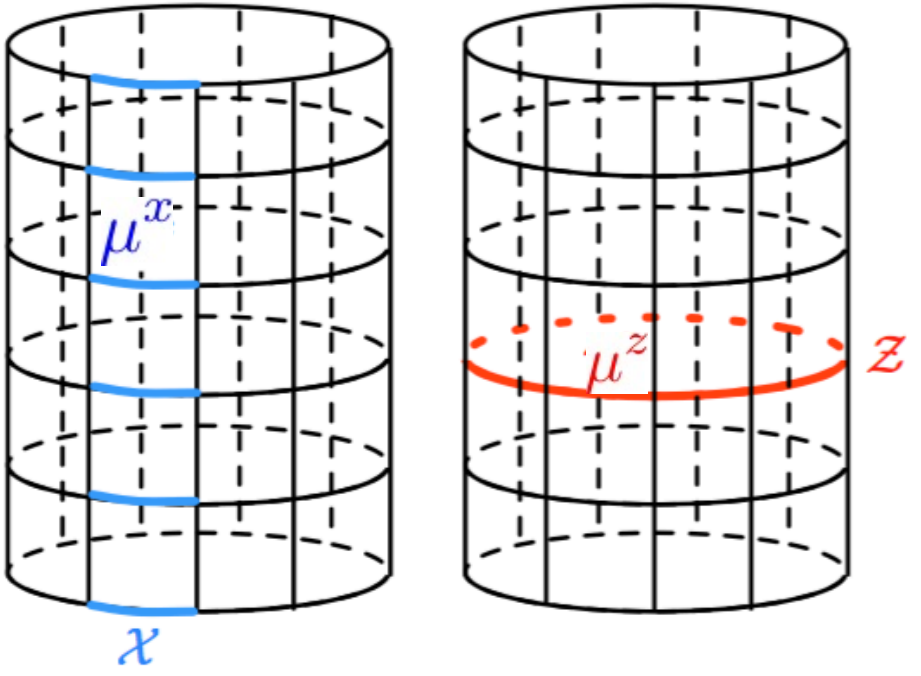


Image credits: Sid Parameswaran, UC Irvine

2^N degeneracy on a Riemann surface of genus N

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Z_2 fractionalized XY order coupled to electrons

$$H_c = -\sum_{i,j} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$H_{\theta,\mathbb{Z}_2} = -\sum_{i,j} J_{ij} \mu_{ij}^z \cos((\theta_i - \theta_j)/2) + 4\Delta \sum_i N_i^2$$

Z_2 gauge theory Hamiltonian

$$-g \sum_{<ij>} \mu_{ij}^x - K \sum_{\square} \left[\prod_{\square} \mu_{ij}^z \right]$$

$$H_Y = -\lambda \sum_i (-1)^{x_i + y_i} (e^{-i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow} + h.c.)$$

Gauge transformations: $e^{i\theta_i/2} \to \epsilon_i e^{i\theta_i/2}, \mu_{ij}^z \to \epsilon_i \mu_{ij}^z \epsilon_j$

$$G_r = e^{2i\pi N_r} \prod_{<ij> \in +_r} \mu_{ij}^x = 1 \text{ in physical Hilbert space}$$

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Define canonical fermionic operators: Spinless chargons

$$\psi_+ = e^{i\theta/2} c_\uparrow$$
$$\psi_- = e^{-i\theta/2} c_\downarrow$$

Carry U(1) EM charge of an electron + Z_2 gauge charge

The chargons see constant antiferromagnetic order

$$H_Y = -\lambda \sum_i (-1)^{x_i + y_i} (\psi^\dagger_{i+} \psi_{i-} + h.c.)$$

If we can realize a phase where the chargons are deconfined, we can have Fermi pockets of U(1) charged fermions

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

No long range XY order

Deconfined chargons

Destroy XY order by proliferating vortices in the XY field ($\boldsymbol{\theta}$)

$$\psi_+ = e^{i\theta/2} c_\uparrow$$
$$\psi_- = e^{-i\theta/2} c_\downarrow$$

Problem: Chargons are not single valued around 2π vortices in $\boldsymbol{\theta}$

Chargons are confined: FS reconstruction and XY ordering coincide

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

No long range XY order

Deconfined chargons

Destroy XY order by proliferating vortices in the XY field ($\boldsymbol{\theta}$)

$$\psi_+ = e^{i\theta/2} c_\uparrow$$
$$\psi_- = e^{-i\theta/2} c_\downarrow$$

Solution: Chargons are single valued around 4π vortices in $\boldsymbol{\theta}$

Chargons deconfined: FS reconstruction distinct from XY ordering

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Basic ingredients required:

No long range XY order

Deconfined chargons

$$-\sum_{i,j} J_{ij}\mu^z_{ij}\cos((\theta_i-\theta_j)/2)$$

Without a Z_2 vortex, 2π vortices in θ have a branch cut and therefore cost extensive energy

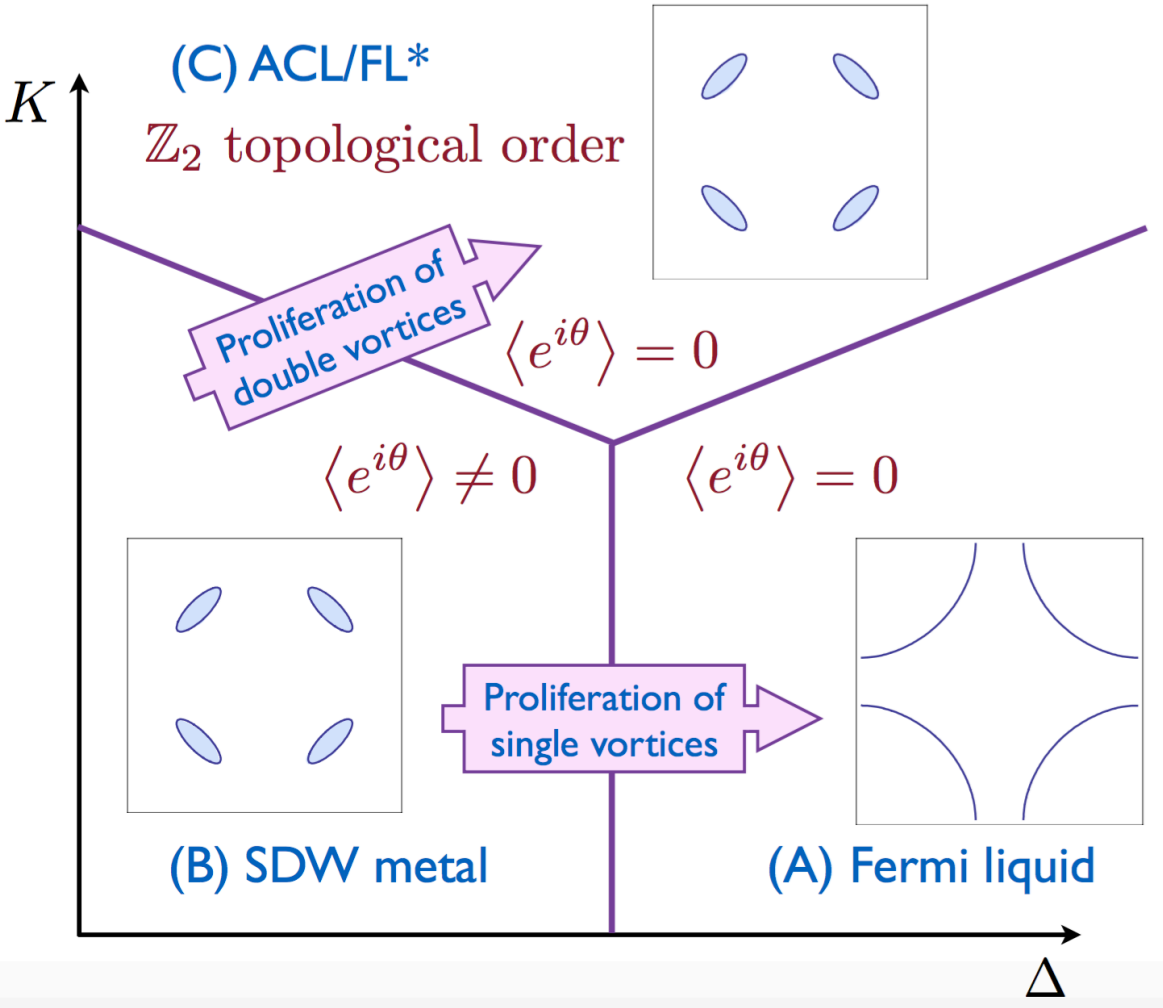
2π vortex in θ needs to tie with a Z_2 vortex for proliferation

$$H_{\mathbb{Z}_2} \ni -K\sum_{\square}\left[\prod_{\square}\mu^z_{ij}\right]$$

Make Z_2 vortices very energy expensive: Take K to ∞

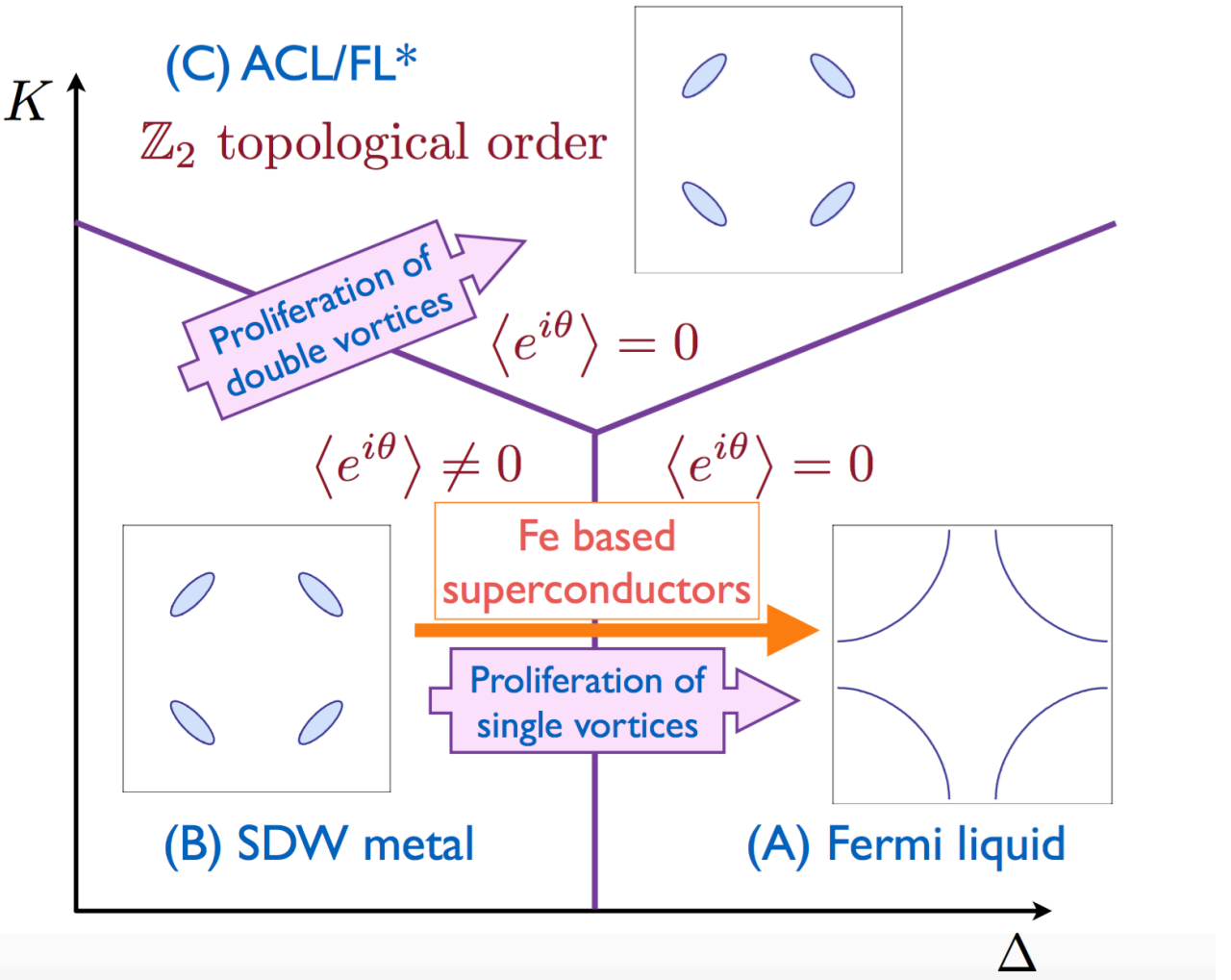
How does the Fermi surface reconstruct?

1: Symmetry Breaking SDW, 2: Topological order



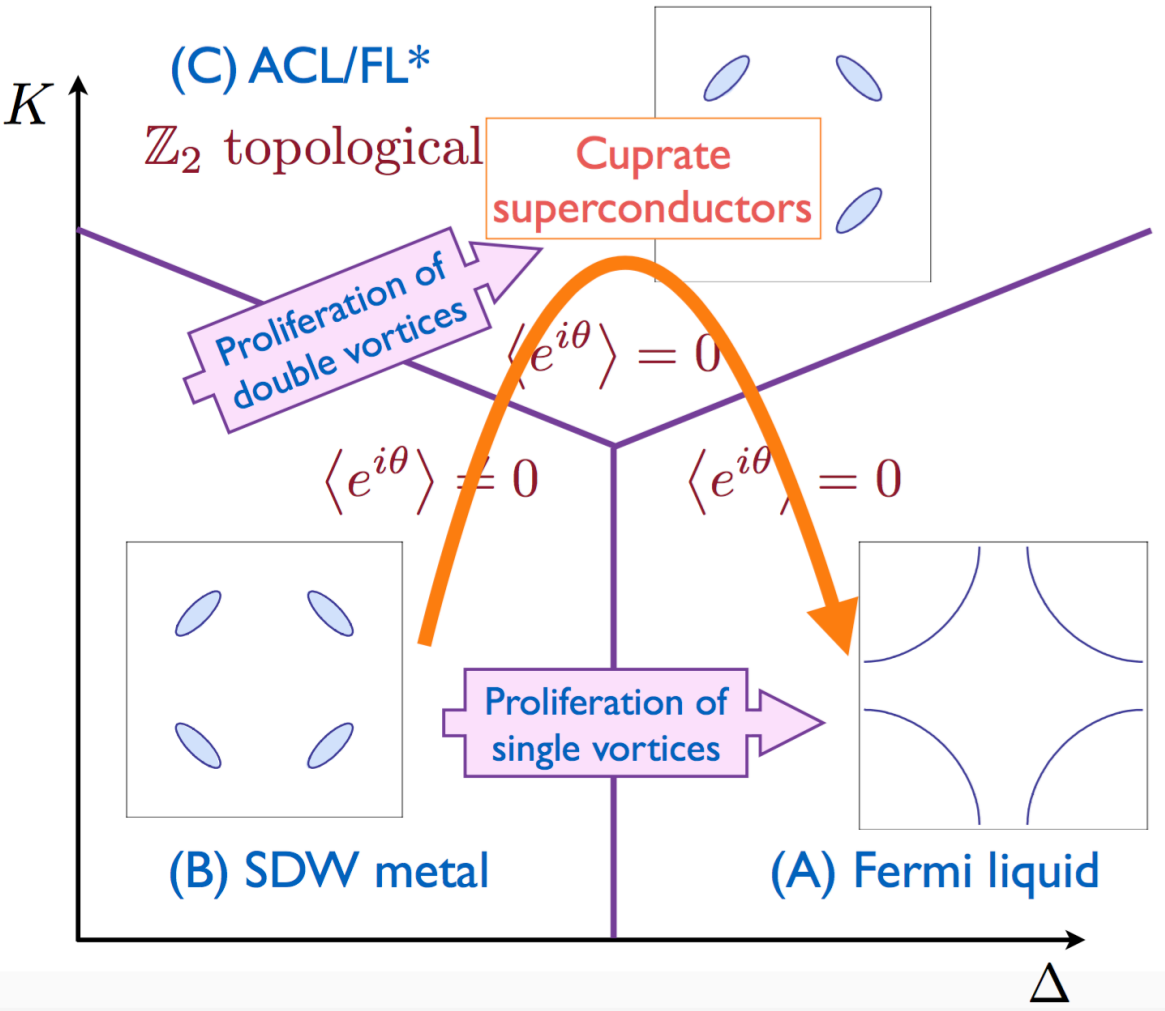
How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order



How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)



How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)

Violation of Luttinger's theorem

- Luttinger's theorem relates the volume of the Fermi surface to the density of fermions

$$\frac{V_{FS}}{4\pi^2} = n \equiv (1+p)(\mathrm{mod}\ 2)$$

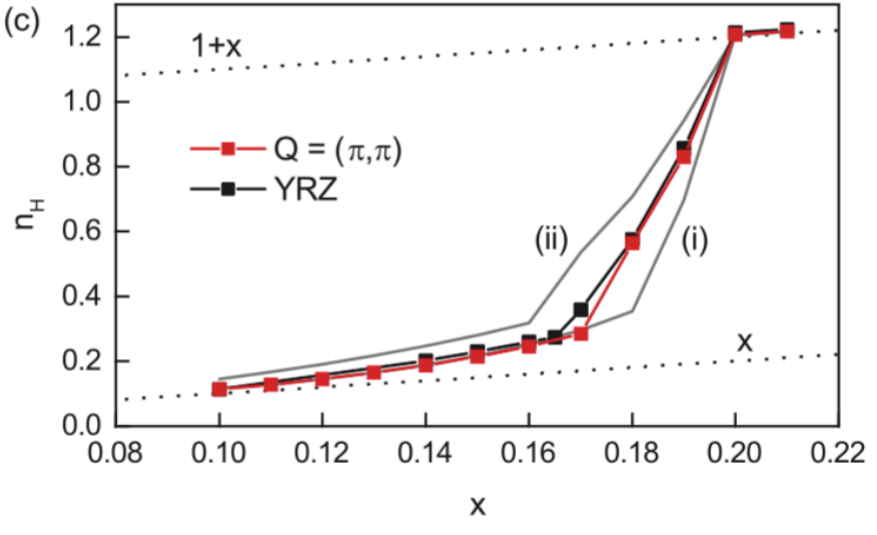
M. Oshikawa, PRL **84**, 3370 (2000)

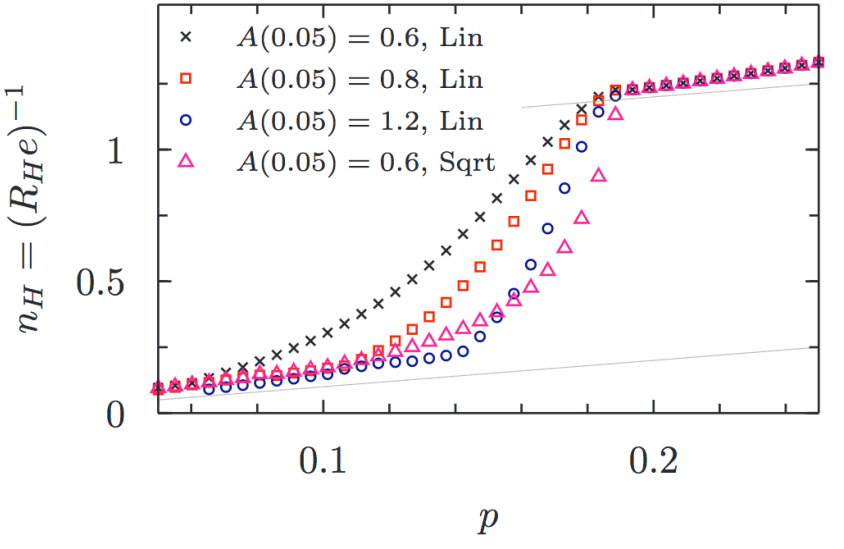
- Topological metal violates Luttinger's theorem

$$\frac{V_{FS}}{4\pi^2} = p(\mathrm{mod}\ 2)$$

T. Senthil *et al*, PRL **90**, 216403 (2003)
Paramekanti *et al*, PRB **70**, 245118 (2004)

Charge transport with reconstructed FS





J. G. Storey, EPL, **113** (2016) 27003

A. Eberlein *et al*, PRL, **117** (2016), 187001

Hall effect data can be well-explained by commensurate (Neel) order/ incommensurate (spiral) order/ the topological metal (ACL/FL*)

Other probes to further substantiate/distinguish different QCPs?

Thermal transport with reconstructed FS

Regime 1: Clean limit

D-wave superconductivity: Universal thermal conductivity

$$\frac{\kappa}{T} = \frac{k_B^2}{3} \frac{v_F^2 + v_\Delta^2}{v_F v_\Delta}$$

Durst and Lee, PRB **62**, 1720 (2000)

D-wave superconductivity + Neel order: Nodal collision gaps out the charge carriers beyond a critical Neel amplitude A_c

$$\frac{\kappa}{T} = \frac{k_B^2}{3 v_F v_\Delta} \left(\sqrt{1-\alpha^2}\, v_F^2 + \frac{1}{\sqrt{1-\alpha^2}}\, v_\Delta^2 \right), \quad \alpha = A/A_c < 1$$

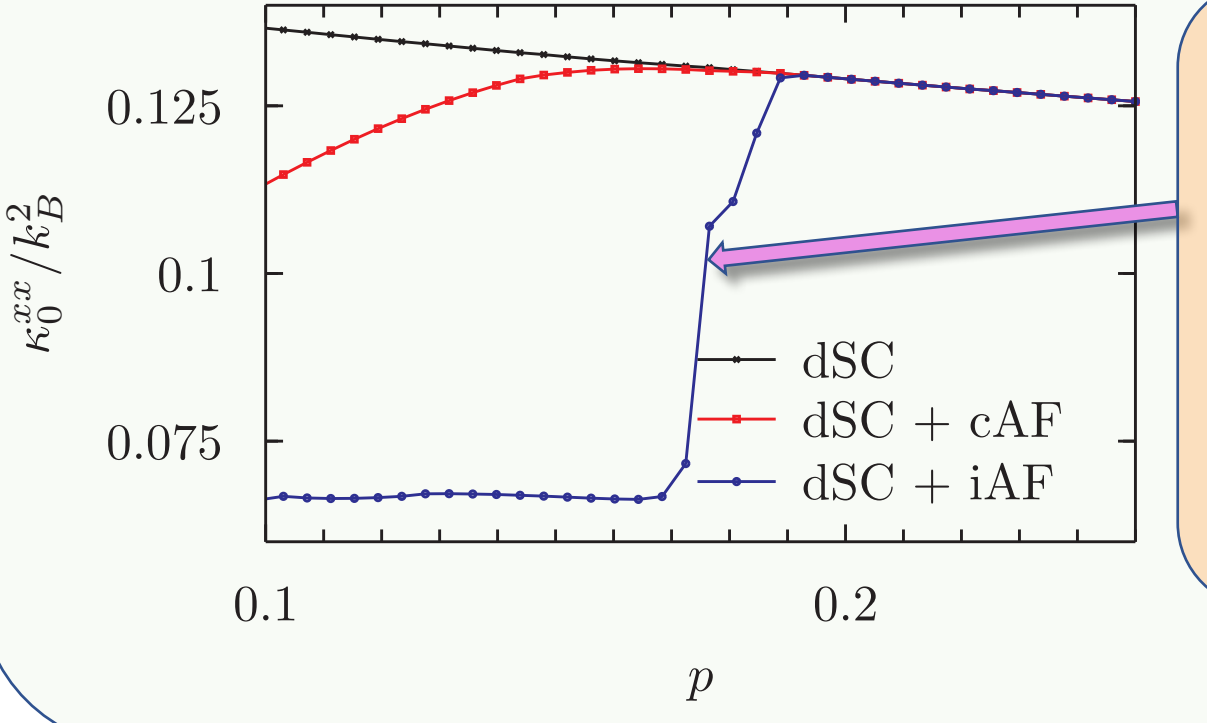
Different threshold behavior from CDW, preserves C_4 symmetry

Durst and Sachdev, PRB **80**, 054518 (2009)

Thermal transport with reconstructed FS

Regime 1: Clean limit

D-wave superconductivity (dSC) + spiral order (iAF):



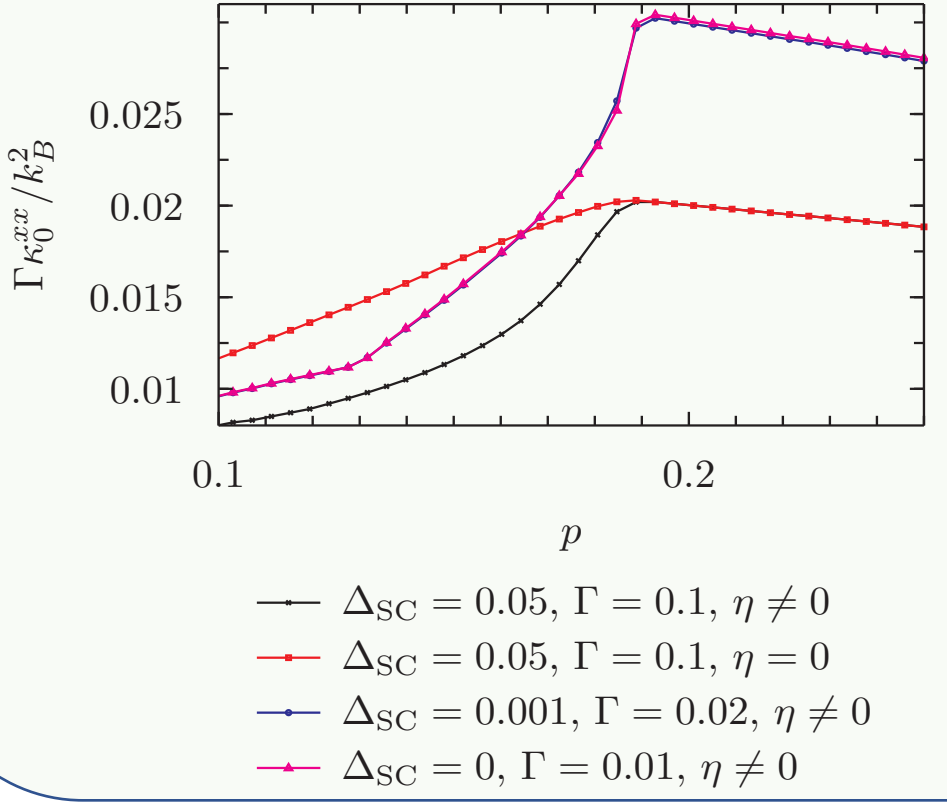
Distinguish Neel and spiral orders

Spiral order co-existing with d-wave superconductivity has a much steeper drop (why?) than Neel order + dSC

Thermal transport with reconstructed FS

Regime 2: Dirty limit

Inverse scattering lifetime >> SC amplitude



Distinguish Neel and spiral orders

Behaves nearly like a magnetic metal– the drops are different for cAF and iAF for the same doping range.

Summary

Discussed possible scenarios that lead to FS reconstruction in the hole-doped cuprates

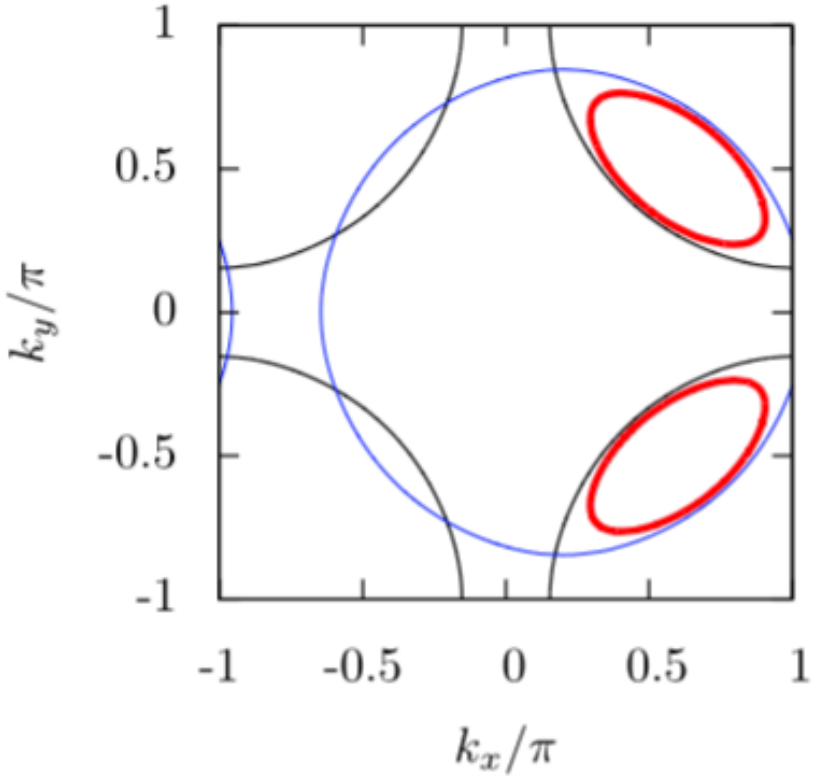
Presented a concrete model with FS reconstruction without symmetry breaking but with Z_2 topological order

Spectroscopic and Hall data agree well with the topological metal or field-induced AF order

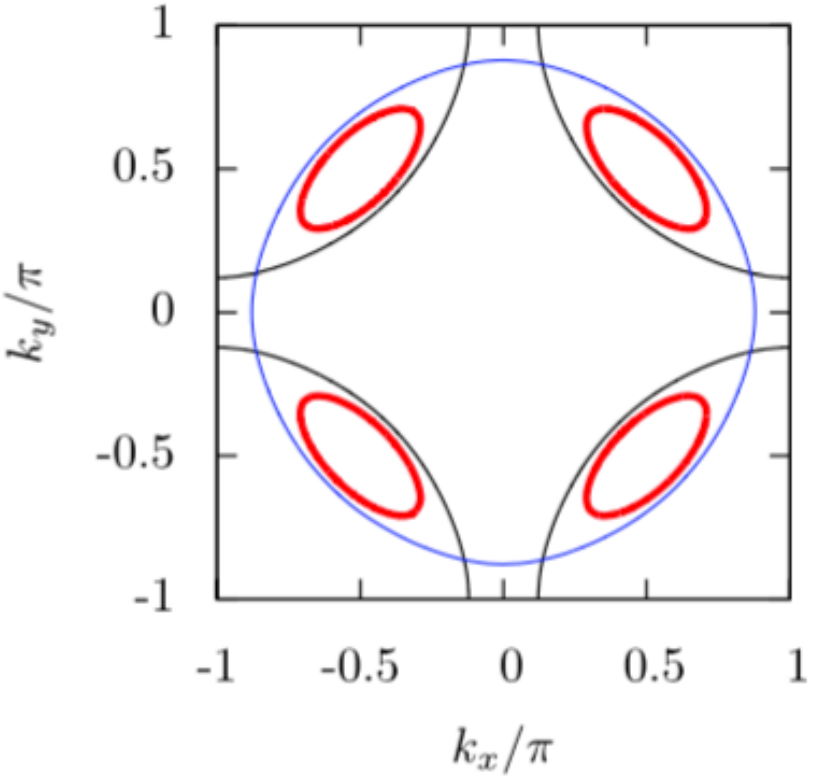
Presented computations of thermal conductivity as a function of doping, which can serve as an additional check

Thank you for your attention!

Extra slide: Fermi surface reconstruction



Spiral Order



Neel Order

A. Eberlein *et al*, PRL, **117** (2016), 187001