Z_2 topological order near the Néel state of the square lattice antiferromagnet

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HARVARD UNIVERSITY

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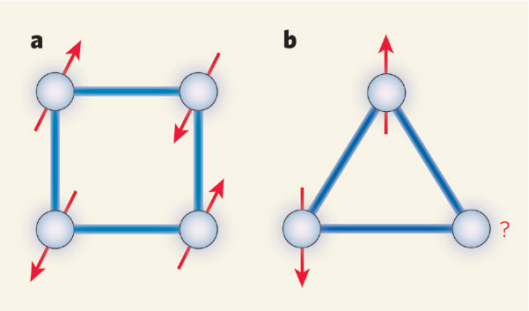
Harvard University

Subir Sachdev,
Harvard/Perimeter

S. Chatterjee, S. Sachdev and Mathias S. Scheurer, **Phys. Rev. Lett.** 119, 227007, 2017

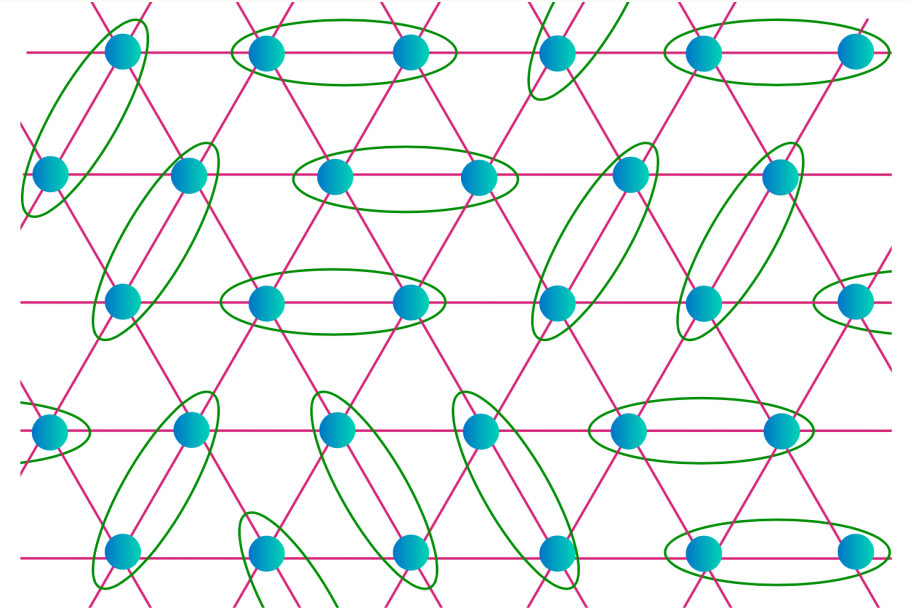
Topological order in quantum magnets: Spin liquids

- Traditional quantum magnetism: Ordered ground states with broken symmetry (Eg: Antiferromagnet)



- Geometric frustration: Additional possibilities
- Spin liquids: Mott insulators with no broken symmetry due to quantum fluctuations

Topological order in quantum magnets: Spin liquids



Anderson 1973, 1987; Kivelson, Rokhsar 1987; Wen 1991, Sachdev 1992, and a lot of others. Figure: S. Sachdev (Harvard)

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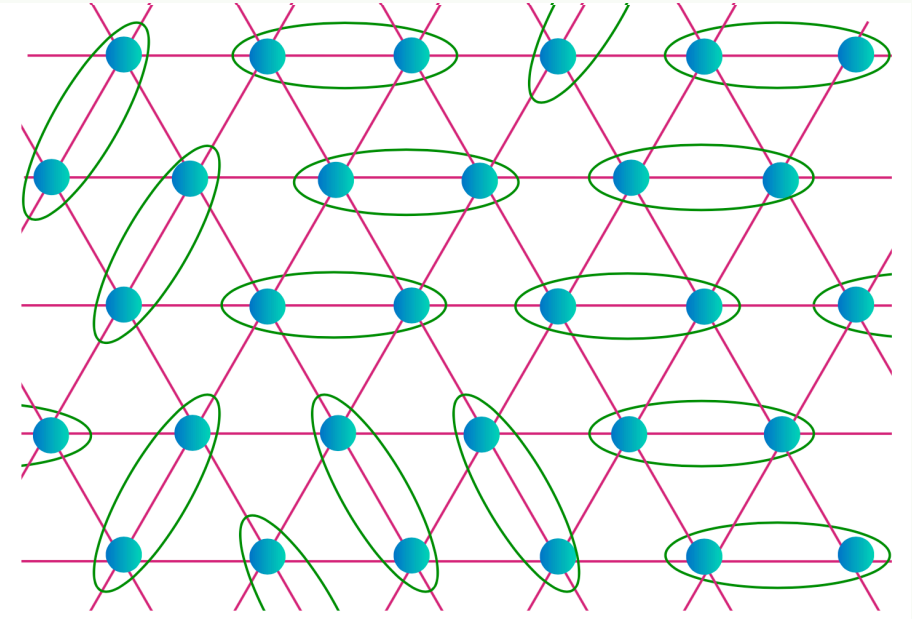

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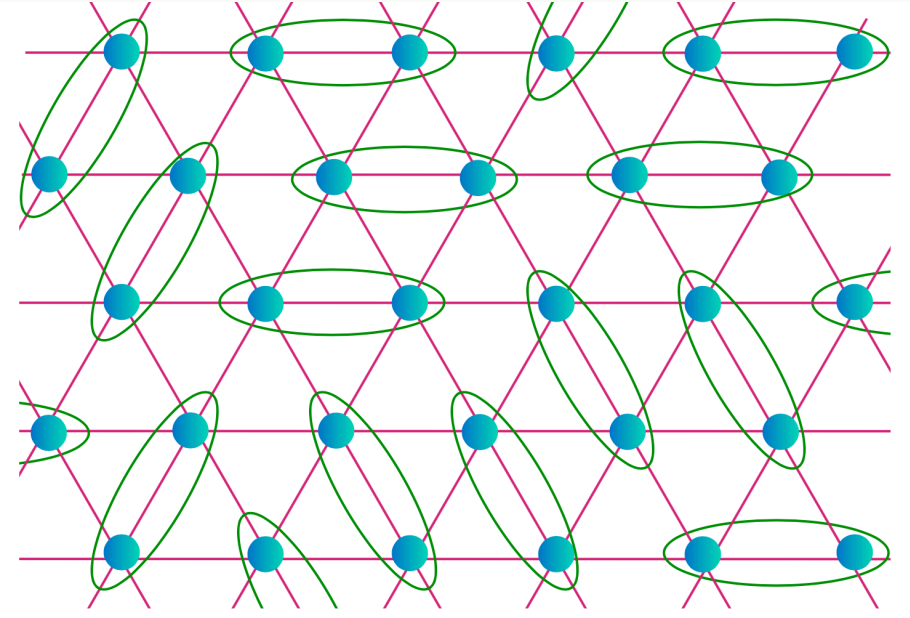

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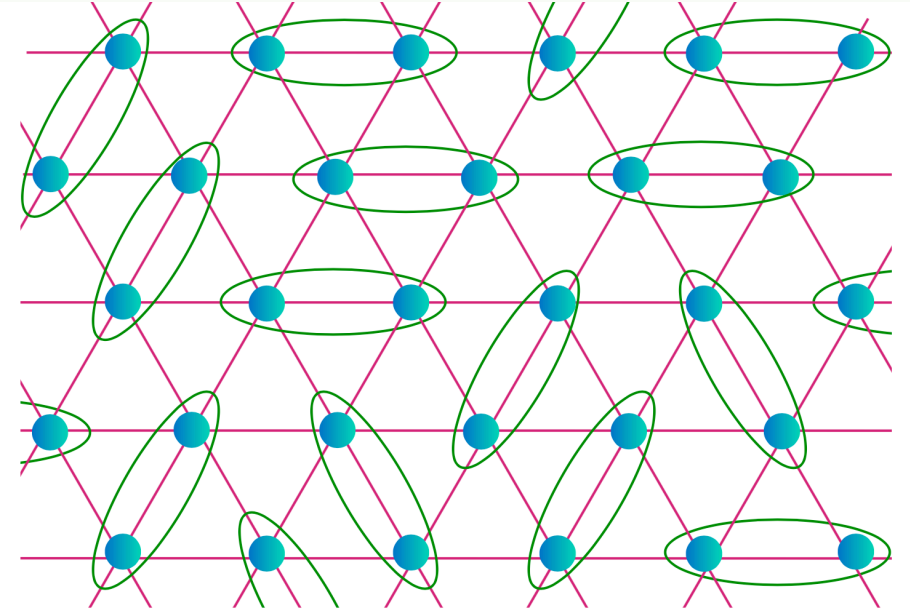

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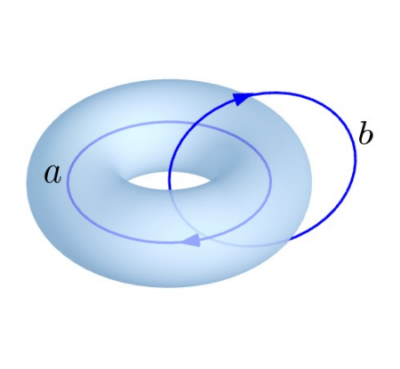
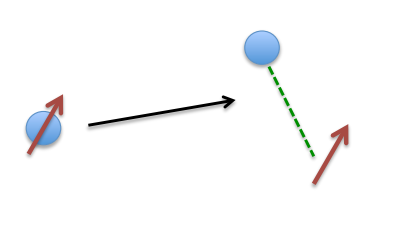
Topological order in quantum magnets: Spin liquids

- Long range quantum entanglement in ground state
- G.s. degeneracy on a torus, emergent gauge fields [Z_2 or U(1)] (topological order)



- Excitations which carry fractional quantum number (S = ½ spinons)



- Classified based on symmetry (point group, time-reversal, etc) and topology

Figures: Barkeshli *et al*, arXiv: 1410.4540, E. Berg (U. Chicago)

Topological order in quantum magnets: Spin liquids

Problem: Classification schemes that rely exclusively on symmetry and topology lead to too many such phases

than $SU(2)$. Thus there are at most $272 - 48 - 4 - 24 = 196$ different mean-field Z_2 spin liquids that can be constructed form u_{ij}.

Wen, PRB **65**, 165113 (2002) Fermionic partons

Bosonic partons Yang and Wang, PRB **94**, 035160 (2016)

Essin and Hermele, PRB **87**, 139905 (2013) Symmetry fractionalization

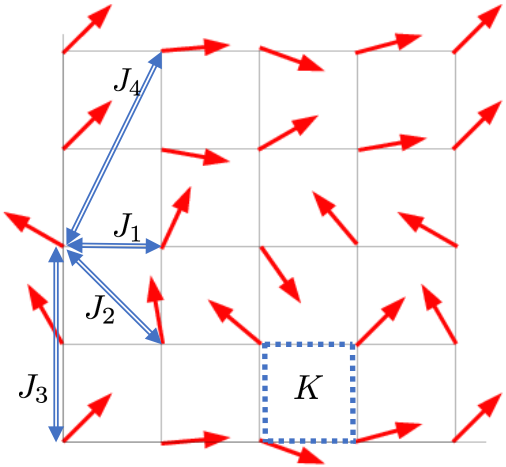
Schwinger boson	
$(p_1, p_2, p_3, p_4, p_7, p_8)$	Label
(0,0,1,0,0,1)	$Z_2[0,0]$
(0,0,1,0,1,1)	$Z_2[0,\pi]$
(1,0,1,0,1,0)	$Z_2[\pi,0]\mathcal{R}$
(1,0,1,0,1,1)	$Z_2[\pi,0]\mathcal{I}$
(1,1,0,1,0,0)	$Z_2[\pi,\pi]\mathcal{R}$
(1,1,0,1,0,1)	$Z_2[\pi,\pi]\mathcal{I}$

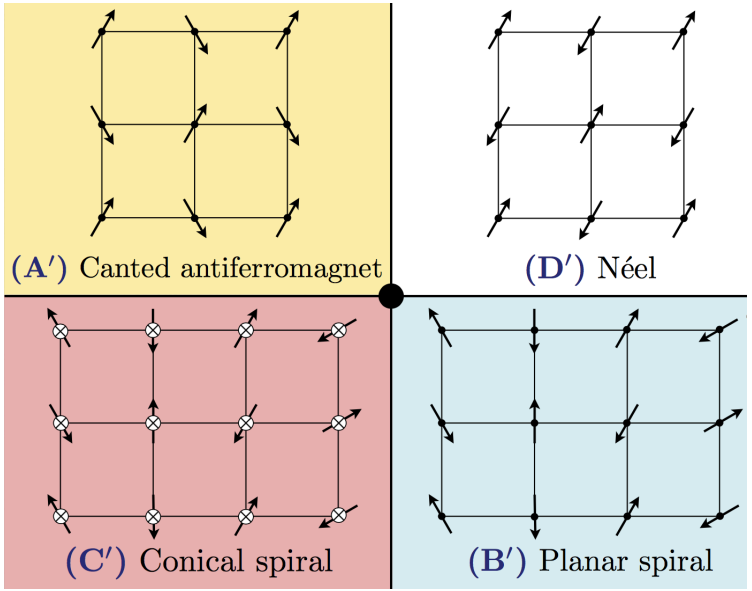
the classification when point group symmetries are present. For square lattice space group, time reversal and SO(3) spin rotation symmetries, we find $2\,098\,176 \approx 2^{21}$ distinct symmetry classes. Our symmetry classification is not complete, as we exclude, by assumption, permutation of the different types of anyons by symmetry operations. We give an explicit construction of symmetry classes for

This talk: Which of these phases are likely to occur in nature?

Hints from classical phase diagram

Specialize to square lattice AF: Ordered phases proximate to Neel phase

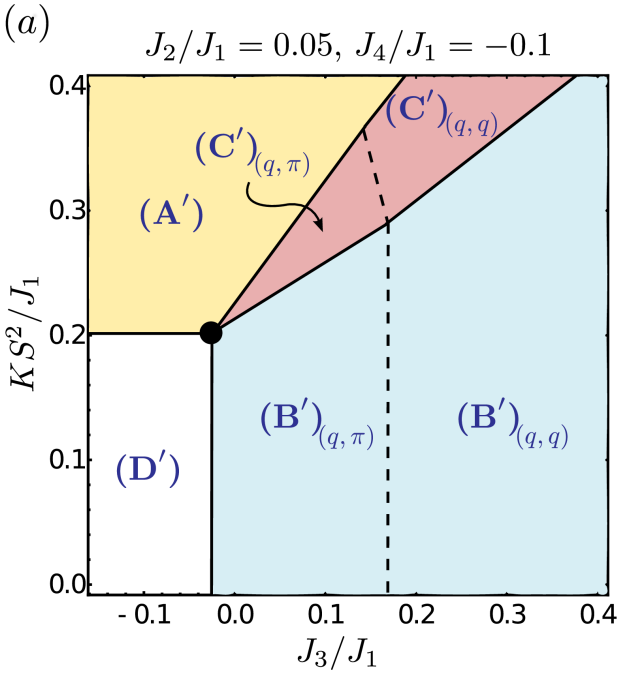


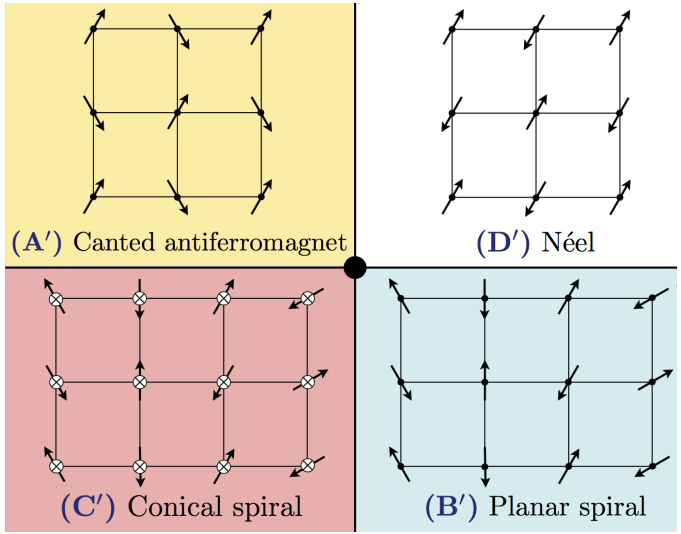


Heisenberg exchanges J_1, J_2, J_3 and J_4 and ring exchange K

Hints from classical phase diagram

Square lattice AF with Heisenberg exchanges J_1, J_2, J_3 and J_4 and ring exchange K

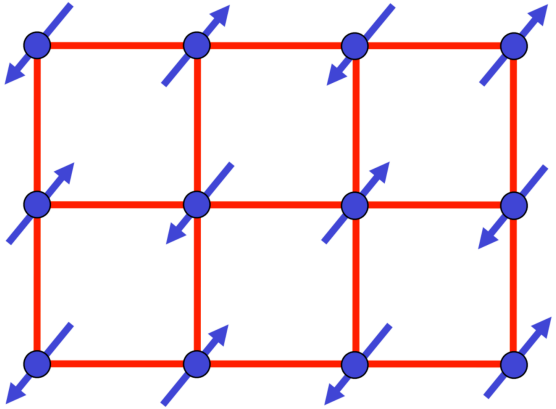




Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

$$\mathbf{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \text{ with } \alpha, \beta = \uparrow, \downarrow, \ |z_\alpha|^2 = 1$$



$$S = \frac{1}{2g} \int d^2 r dt \ (\partial_\mu \mathbf{n})^2$$
$$\rightarrow \frac{1}{2g} \int d^2 r dt \ |(\partial_\mu - i a_\mu) z_\alpha|^2$$

The CP^1 theory an has emergent U(1) gauge field a_μ

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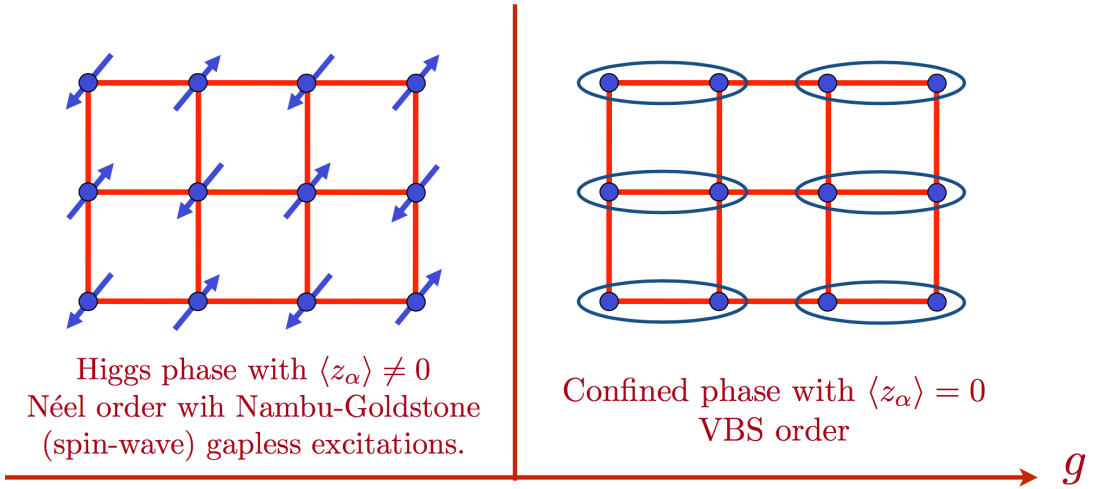
For S = ½ , additional Berry phase term for the U(1) gauge field



Read and Sachdev, PRL **62**, 1694 (1989)

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For Z_2 topological order, need to condense Higgs fields with charge 2 under emergent U(1) gauge field

Simplest candidates: Spin rotation invariant long-wavelength spinon pairs:

$$P \sim \varepsilon_{\alpha\beta} z_\alpha \partial_t z_\beta \quad , \quad Q_a \sim \varepsilon_{\alpha\beta} z_\alpha \partial_a z_\beta \text{ with } a = x, y$$

Gauge invariance + Symmetry

$$\mathcal{L} = \frac{1}{g} |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_1 |P|^2 + s_2 |Q_a|^2$$

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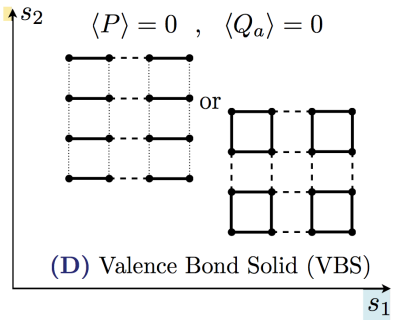
Phase diagram at large g with $\langle z_\alpha \rangle = 0$



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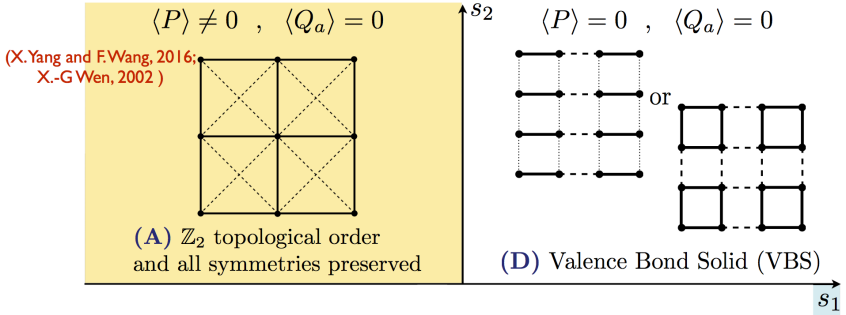

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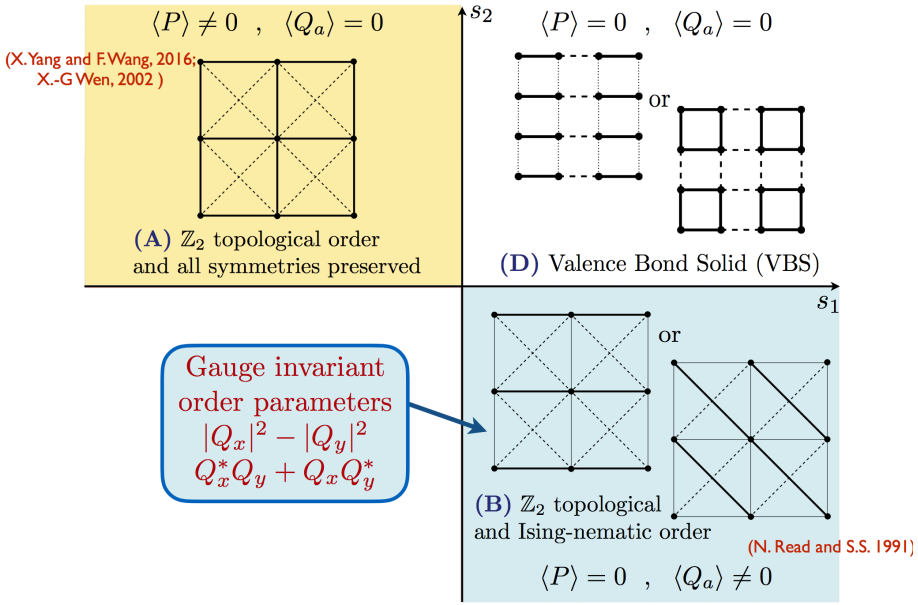

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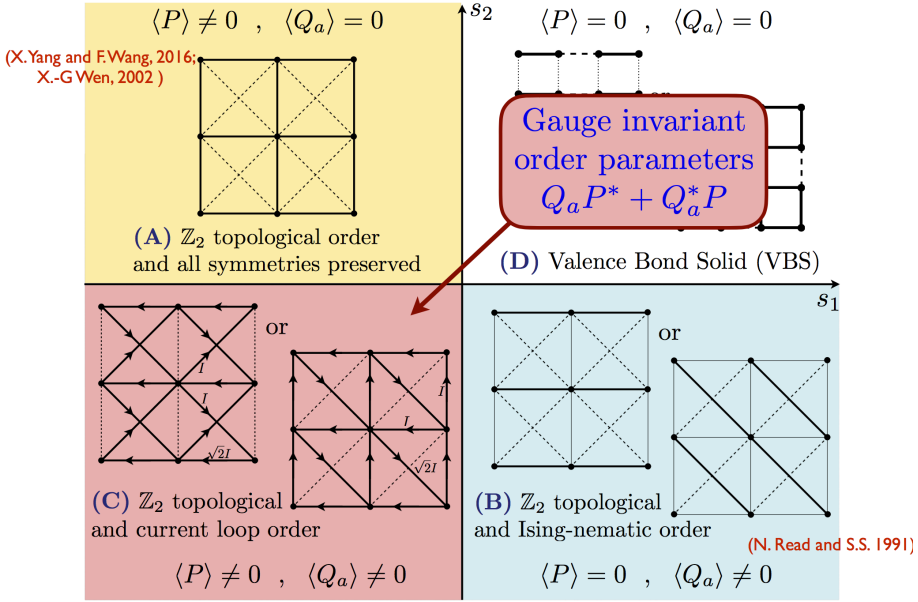
Three phases with Z_2 topological order



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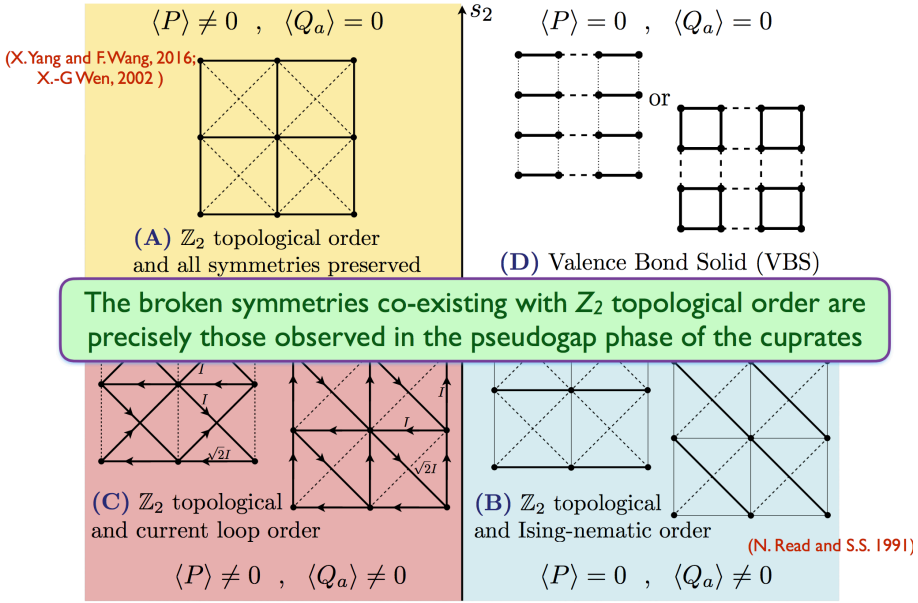
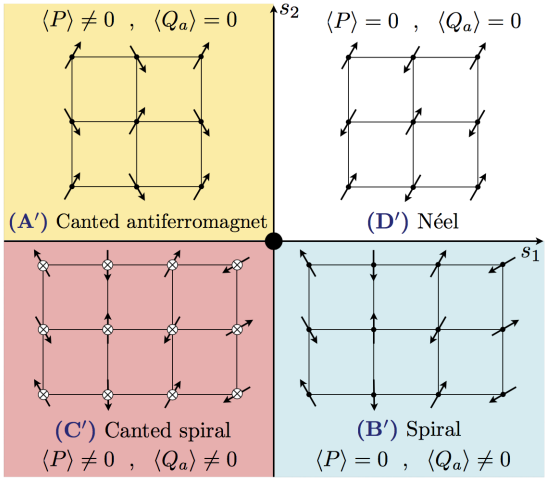


The broken symmetries co-existing with Z_2 topological order are precisely those observed in the pseudogap phase of the cuprates

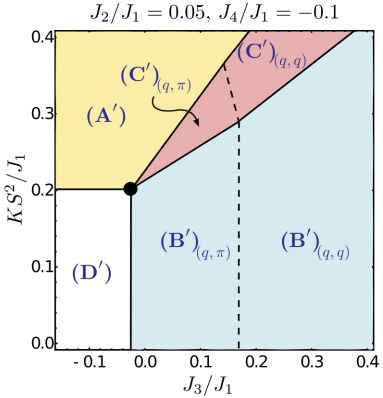
Add topological order: CP^1 theory

Describes ordered phases at small g: break translation and spin-rotation symmetries, and have no topological order.

Phase diagram of $\mathbb{CP}^1$ model at small g, coupled to Higgs fields P and Q_a $(a = x, y)$. All phases have $\langle z_\alpha \rangle \neq 0$



Classical phase diagram of square lattice antiferromagnet with near-neighbor exchanges J_1, J_2, J_3, J_4 and ring-exchange K



Same ordered phases recovered from the CP^1 model!

Summary and Outlook

Discussed topologically ordered states that lie energetically proximate to the Néel antiferromagnet – constraints on exotic phases which are most likely to appear in material candidates

The theory for insulators can be generalized to doped spin liquids: SU(2) gauge theory of metals with Z_2 topological order

Talk tomorrow: A. Georges, R04.00005

Topologically ordered phases close to the Néel state naturally possess the desired broken symmetries

Possible explanation of the concurrent appearance of anti-nodal gap and discrete broken symmetries in the hole-doped cuprates

Thank you for your attention!