$Z_2$ topological order near the Néel state of the square lattice antiferromagnet

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_S. Chatterjee, S. Sachdev and Mathias S. Scheurer, Phys. Rev. Lett. 119, 227007, 2017_
• **Traditional quantum magnetism**: Ordered ground states with broken symmetry (E.g., Antiferromagnet)

![Diagram showing lattice structures](image)

• Geometric frustration: **Additional possibilities**

• **Spin liquids**: Mott insulators with no broken symmetry due to quantum fluctuations
Topological order in quantum magnets: Spin liquids

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Topological order in quantum magnets: Spin liquids

• Long range quantum entanglement in ground state

• G.s. degeneracy on a torus, emergent gauge fields [ $Z_2$ or $U(1)$] (topological order)

• Excitations which carry fractional quantum number ($S = \frac{1}{2}$ spinons)

• Classified based on symmetry (point group, time-reversal, etc) and topology

Topological order in quantum magnets: Spin liquids

Problem: Classification schemes that rely exclusively on symmetry and topology lead to too many such phases

than $SU(2)$. Thus there are at most $272 - 48 - 4 - 24 = 196$ different mean-field $Z_2$ spin liquids that can be constructed form $u_{ij}$.

Wen, PRB 65, 165113 (2002)

Yang and Wang, PRB 94, 035160 (2016)

Essin and Hermele, PRB 87, 139905 (2013)

the classification when point group symmetries are present. For square lattice space group, time reversal and SO(3) spin rotation symmetries, we find $2098176 \approx 2^{21}$ distinct symmetry classes. Our symmetry classification is not complete, as we exclude, by assumption, permutation of the different types of anyons by symmetry operations. We give an explicit construction of symmetry classes for

This talk: Which of these phases are likely to occur in nature?
Specialize to square lattice AF: Ordered phases proximate to Neel phase

Heisenberg exchanges $J_1$, $J_2$, $J_3$ and $J_4$ and ring exchange $K$
Hints from classical phase diagram

Square lattice AF with Heisenberg exchanges $J_1$, $J_2$, $J_3$ and $J_4$ and ring exchange $K$

(a) $J_2/J_1 = 0.05$, $J_4/J_1 = -0.1$

- $(C'_{(q,\pi)})$
- $(C'_{(q,q)})$
- $(A')$
- $(B'_{(q,\pi)})$
- $(B'_{(q,q)})$
- $(D')$

$(A')$ Canted antiferromagnet
$(D')$ Néel
$(C')$ Conical spiral
$(B')$ Planar spiral
Add topological order: CP\textsuperscript{1} theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

\[ n = z_{\alpha}^* \bar{\sigma}_{\alpha \beta} z_{\beta} \text{ with } \alpha, \beta = \uparrow, \downarrow, \quad |z_\alpha|^2 = 1 \]

\[ S = \frac{1}{2g} \int d^2r dt (\partial_\mu n)^2 \rightarrow \frac{1}{2g} \int d^2r dt \left| (\partial_\mu - ia_\mu) z_\alpha \right|^2 \]

The CP\textsuperscript{1} theory an has emergent U(1) gauge field \( a_\mu \)
For $S = \frac{1}{2}$, additional Berry phase term for the U(1) gauge field.

Add topological order: CP$^1$ theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Higgs phase with $\langle z_\alpha \rangle \neq 0$ Neél order with Nambu-Goldstone (spin-wave) gapless excitations.

Confined phase with $\langle z_\alpha \rangle = 0$ VBS order

Read and Sachdev, PRL 62, 1694 (1989)
For $Z_2$ topological order, need to condense Higgs fields with charge 2 under emergent U(1) gauge field.

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Simplest candidates: Spin rotation invariant long-wavelength spinon pairs:

$$P \sim \epsilon_{\alpha\beta} z_\alpha \partial_t z_\beta, \quad Q_a \sim \epsilon_{\alpha\beta} z_\alpha \partial_a z_\beta \text{ with } a = x, y$$

Gauge invariance + Symmetry

$$\mathcal{L} = \frac{1}{g} |(\partial_\mu - ia_\mu) z_\alpha|^2 + s_1 |P|^2 + s_2 |Q_a|^2$$
Add topological order: CP\(^1\) theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large \( g \) with \( \langle z_\alpha \rangle = 0 \)
Add topological order: CP\(^1\) theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large \(g\) with \(\langle z_\alpha \rangle = 0\)

(A) \(\mathbb{Z}_2\) topological order and all symmetries preserved

(D) Valence Bond Solid (VBS)
Add topological order: CP$^1$ theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large $g$ with $\langle z_\alpha \rangle = 0$

(A) $\mathbb{Z}_2$ topological order and all symmetries preserved

(Gauge invariant order parameters

$|Q_x|^2 - |Q_y|^2$

$Q_x^* Q_y + Q_x Q_y^*$

(D) Valence Bond Solid (VBS)

\[
\begin{aligned}
\langle P \rangle = 0, & \quad \langle Q_a \rangle = 0 \\
\langle P \rangle \neq 0, & \quad \langle Q_a \rangle = 0
\end{aligned}
\]
Add topological order: CP$^1$ theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large $g$ with $\langle z_\alpha \rangle = 0$

Three phases with $Z_2$ topological order

(A) $Z_2$ topological order and all symmetries preserved

$\langle P \rangle \neq 0$, $\langle Q_a \rangle = 0$

$\langle P \rangle = 0$, $\langle Q_a \rangle = 0$

(D) Valence Bond Solid (VBS)

Gauge invariant order parameters $Q_a P^* + Q_a^* P$

(C) $Z_2$ topological and current loop order

$\langle P \rangle \neq 0$, $\langle Q_a \rangle \neq 0$

(B) $Z_2$ topological and Ising-nematic order

$\langle P \rangle = 0$, $\langle Q_a \rangle \neq 0$

(N. Read and S.S. 1991)
Add topological order: CP$^1$ theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large $g$ with $\langle z_\alpha \rangle = 0$

Three phases with $Z_2$ topological order

(A) $Z_2$ topological order and all symmetries preserved

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$\langle P \rangle = 0$, $\langle Q_\alpha \rangle \neq 0$

(C) $Z_2$ topological and current loop order

$\langle P \rangle \neq 0$, $\langle Q_\alpha \rangle \neq 0$

(D) Valence Bond Solid (VBS)

$\langle P \rangle = 0$, $\langle Q_\alpha \rangle = 0$

The broken symmetries co-existing with $Z_2$ topological order are precisely those observed in the pseudogap phase of the cuprates

(X. Yang and F. Wang, 2016
X. G. Wen, 2002)

(N. Read and S.S. 1991)
Describes ordered phases at small $g$: break translation and spin-rotation symmetries, and have no topological order.

Same ordered phases recovered from the CP$^1$ model!
Summary and Outlook

Discussed topologically ordered states that lie energetically proximate to the Néel antiferromagnet – constraints on exotic phases which are most likely to appear in material candidates.

The theory for insulators can be generalized to doped spin liquids: SU(2) gauge theory of metals with $Z_2$ topological order.

Talk tomorrow: A. Georges, R04.00005

Topologically ordered phases close to the Néel state naturally possess the desired broken symmetries.

Possible explanation of the concurrent appearance of anti-nodal gap and discrete broken symmetries in the hole-doped cuprates.
Thank you for your attention!