Transport and symmetry breaking in strongly correlated systems with topological order

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Thesis Defense

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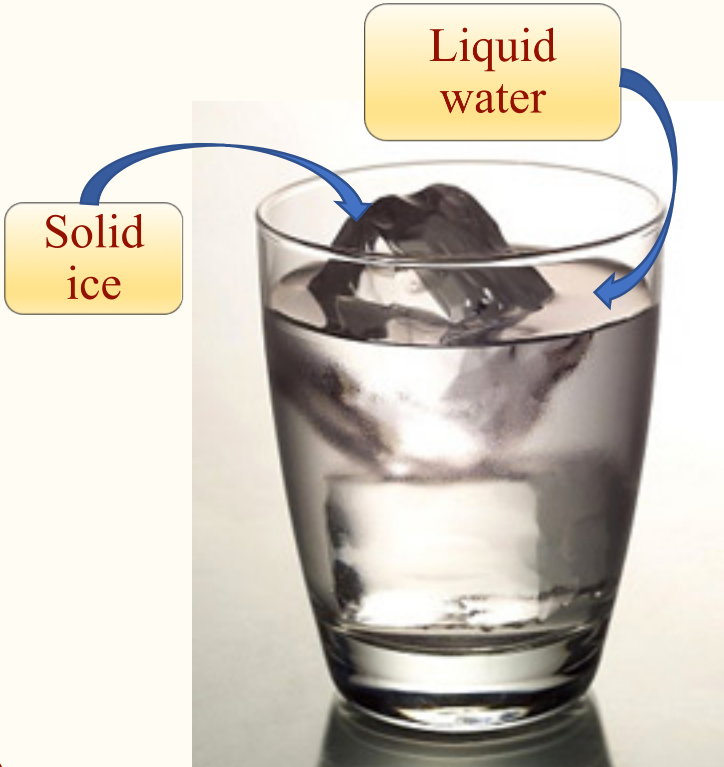
HARVARD UNIVERSITY

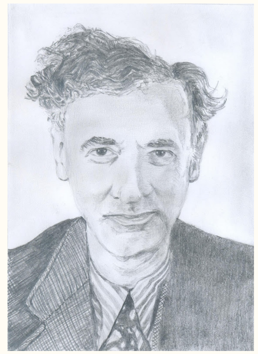
Conventional phases of matter

- Paradigm of symmetry breaking and local order parameters



At room temperature, this magnet attracts iron pins.

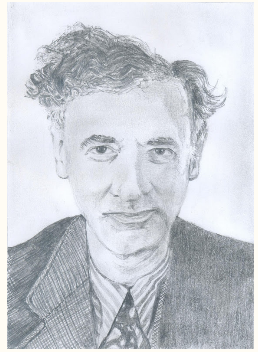
When heated, it stops doing so



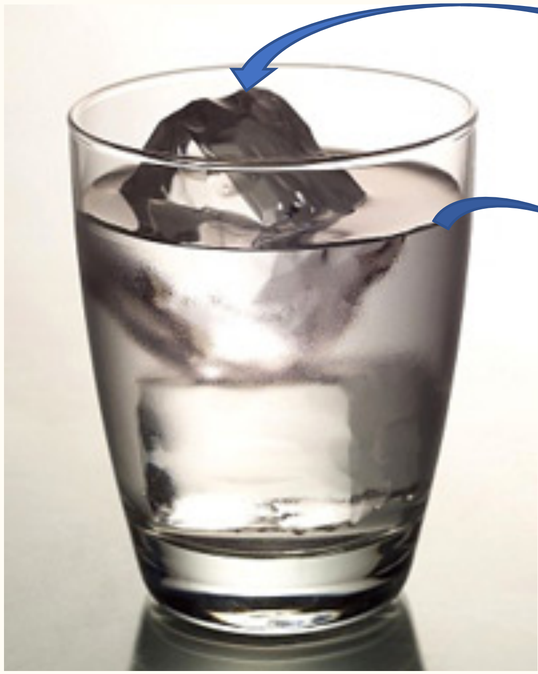
Lev. D. Landau

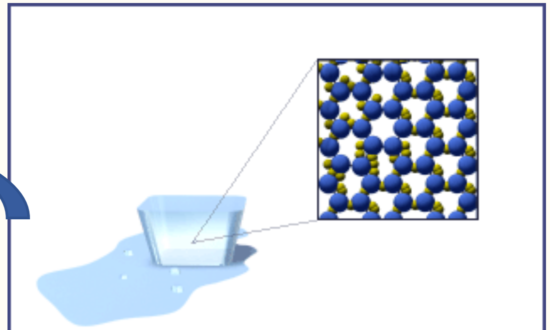
Conventional phases of matter

- Paradigm of symmetry breaking and local order parameters

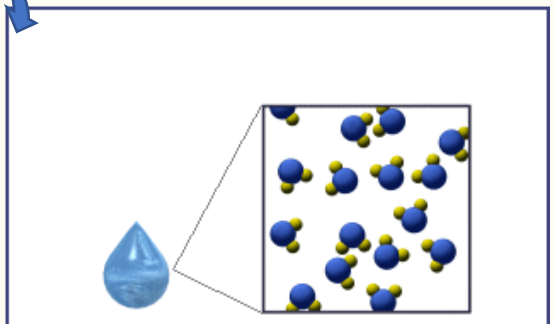


Lev. D. Landau





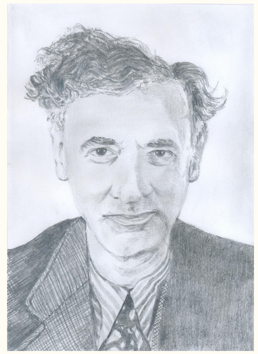
Density is non-uniform



Density is uniform

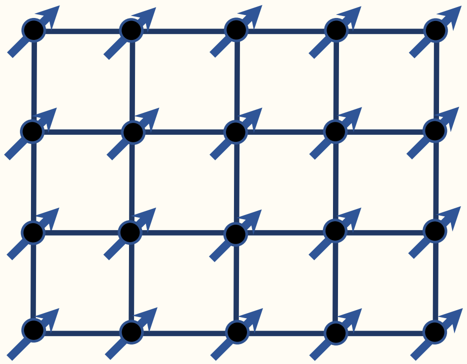
Conventional phases of matter

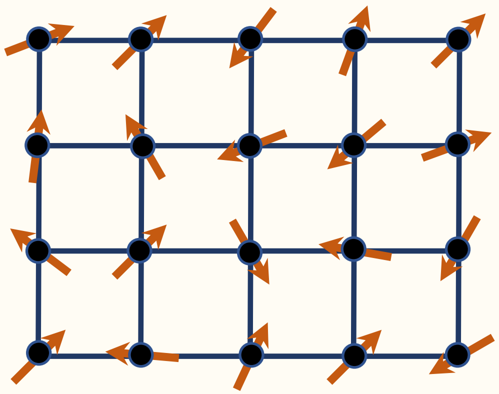
- Paradigm of symmetry breaking and local order parameters



Lev. D. Landau

At room temperature, tiny magnetic moments inside the magnet are aligned

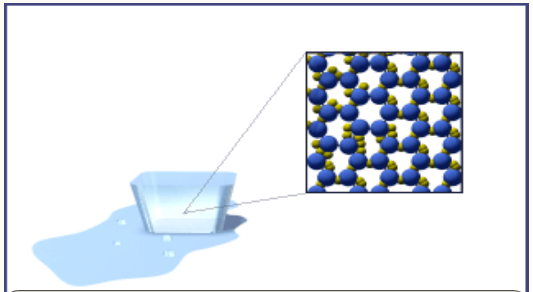




When heated, the moments point in random directions, and the net moment is zero

Conventional phases of matter

- Paradigm of symmetry breaking and local order parameters



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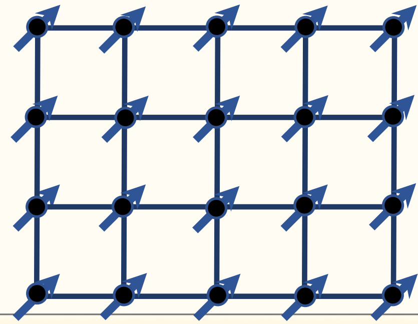
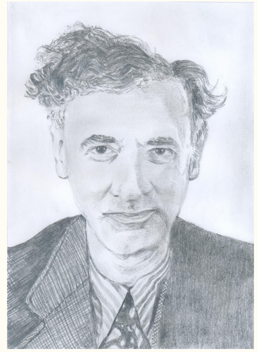
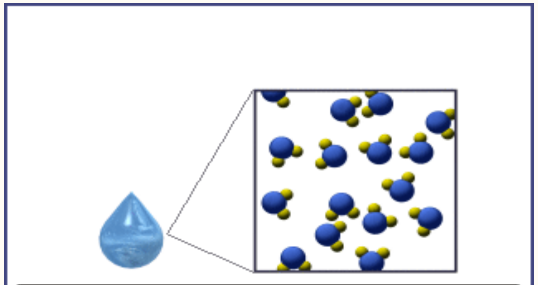


Moments are aligned



Lev. D. Landau



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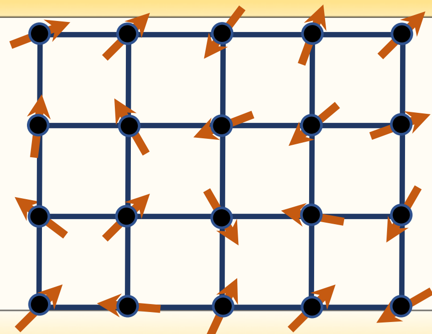


Moments point random

Translation symmetry

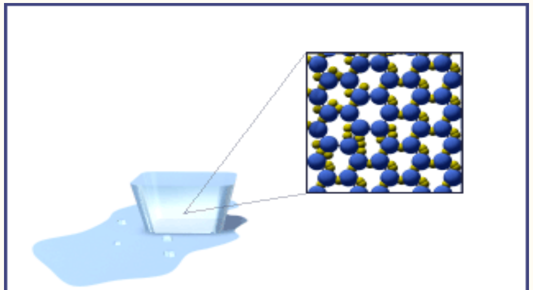
Rotation symmetry of moments

Common theme at low temperature:

- Symmetry is broken

Conventional phases of matter

- Paradigm of symmetry breaking and local order parameters



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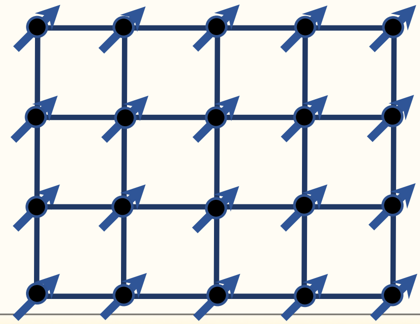

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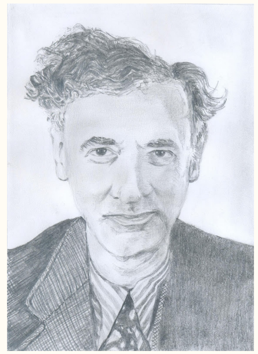
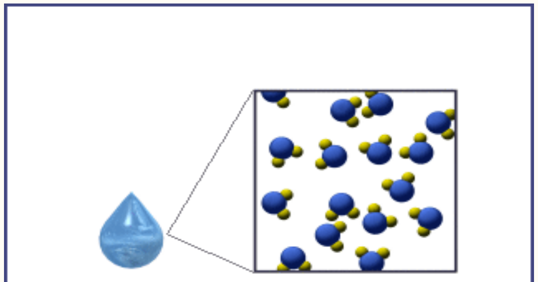


Lev. D. Landau



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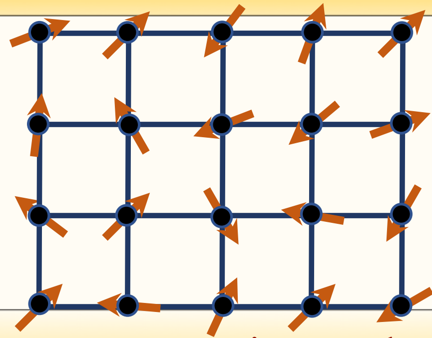


Moments point random

Common theme at low temperature:

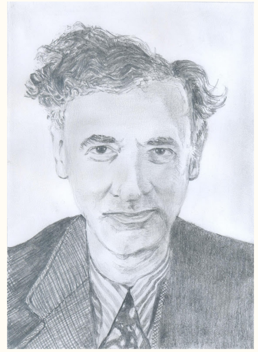
- Symmetry is broken
- Non-zero local order parameter

Order parameter: Density

Order parameter: Magnetization

Conventional phases of matter

- Fermi liquid theory for metals

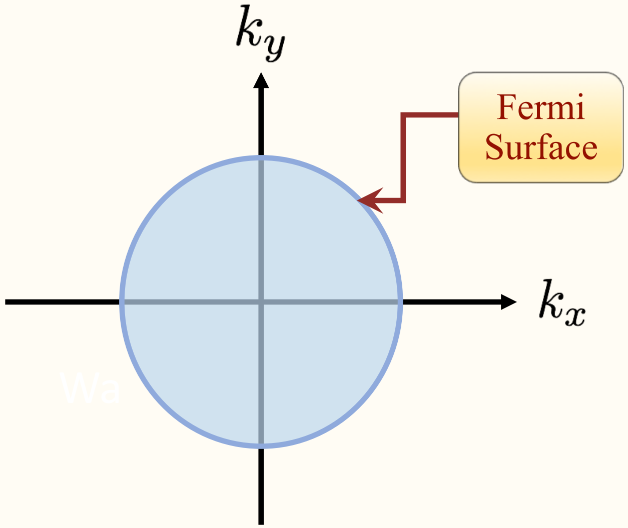


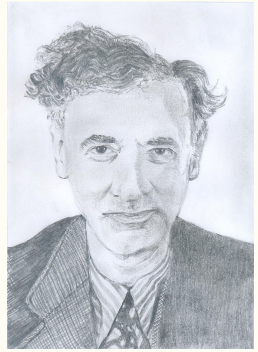
Lev. D. Landau

- Shiny, good conductors of heat and electricity
- Lots of highly mobile electrons which can easily move around

Conventional phases of matter

- Fermi liquid theory for metals





Lev. D. Landau

- Gapless excitations close to the Fermi surface (even with interactions)
- Volume of Fermi surface = density of electrons [Luttinger's Theorem]

Luttinger PR **119** 1153, 1960, Oshikawa PRL **84**, 3370 (2000)

Conventional phases of matter

- Fermi surface *reconstructs* if an electronic order parameter break translation discrete translation symmetry of the lattice

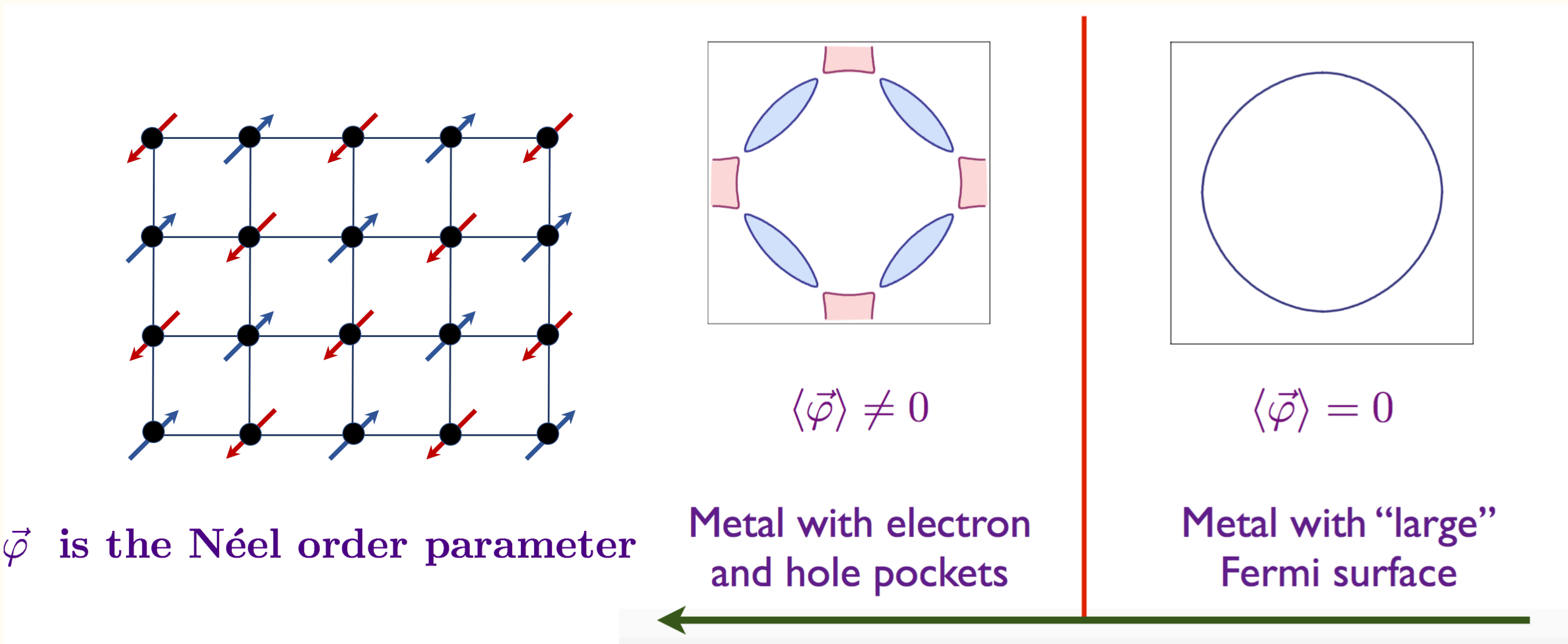


Figure Credits: S. Sachdev

Questions beyond conventional phases

- Can distinct phases occur without a local order parameter?
- What are the nature of emergent quasiparticles in these phases?
- Does Fermi surface reconstruction need to coincide with breaking of lattice translation symmetry?

  In other words, is it possible to violate Luttinger's Theorem?

Questions beyond conventional phases

- Can distinct phases occur without a local order parameter?
- What are the nature of emergent quasiparticles in these phases?
- Does Fermi surface reconstruction need to coincide with breaking of lattice translation symmetry?
  In other words, is it possible to violate Luttinger's Theorem?

- **Yes**, via *topological order*!
- The electron can split into independent charge and spin degrees of freedom (*fractionalization*).
- Luttinger's theorem is violated in a precise fashion.

Topological order/fractionalization

Relevance to pseudogap metal phase of cuprates

Bosonic charge carriers

- Classification
  PRB 2016
- Transport properties and instabilities
  PRB 2016

Fermionic charge carriers

- FS reconstruction
  PRB 2016
  PRB 2017
- Transport
- Discrete symmetry breaking
  PRB 2017, PRL 2017
- Comparison with numerical studies
  arXiv:1707.06602, PNAS 2018

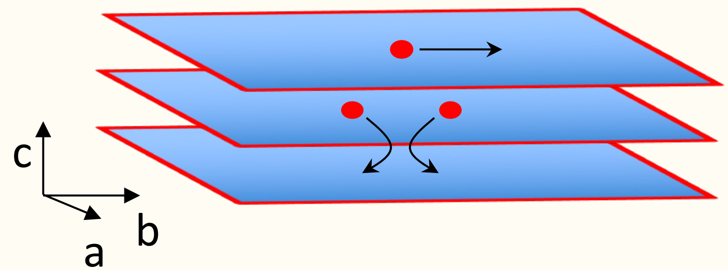
Experimental signatures of quantum-disordered magnets

Spin currents

PRB 2015

Thermal transport

arXiv:1708:02584



Topological order/fractionalization

- Described in terms of emergent gauge fields.

Topological order/fractionalization

- Described in terms of emergent gauge fields.
- What are gauge fields? Redundancies in our description of nature.

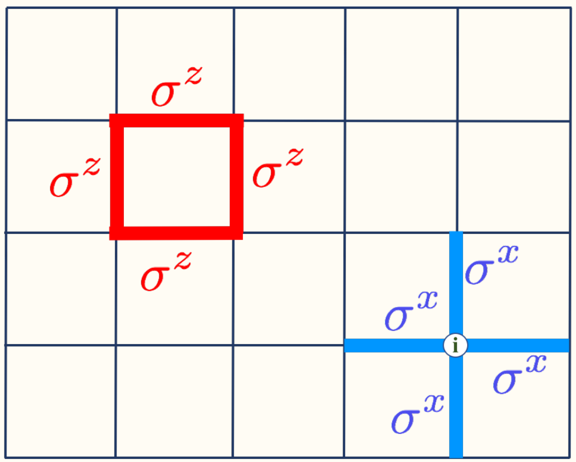
Topological order/fractionalization

- Described in terms of emergent gauge fields.
- Wegner's Z_2 gauge theory: $$H_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} \sigma^z_\ell - g \sum_{\ell} \sigma^x_\ell$$

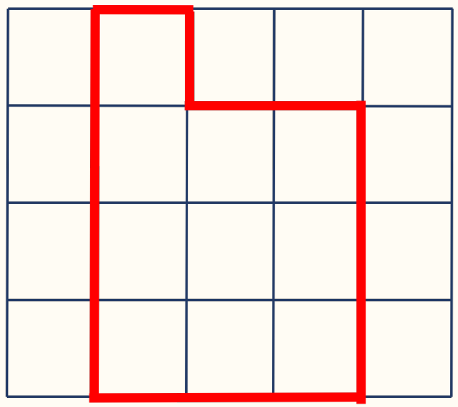
$$G_i = \prod_{\ell \in +(i)} \sigma^x_\ell \; , \; [G_i, H_{\mathbb{Z}_2}] = 0$$

$$G_i = 1 \text{ on any physical state}$$

Wegner, J. Math Phys. **12**, 2259 (1971)



$$\prod_{\ell \in \square} \sigma^z_\ell = e^{i b_\square} \quad G_i = \prod_{\ell \in +(i)} \sigma^x_\ell = e^{i\pi \Delta \cdot e(i)}$$



$$W_c = \prod_{\ell \in C} \sigma^z_\ell$$

$W_c \sim$ Perimeter Law — $W_c \sim$ Area Law

Deconfining Phase — g_c — Confining Phase — g

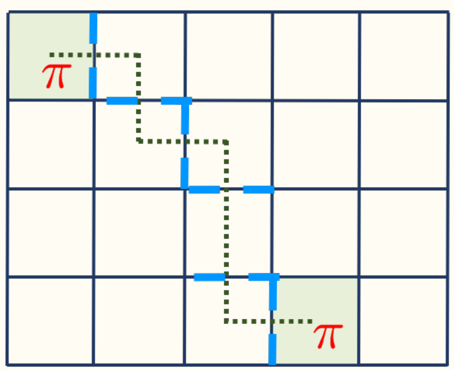
Topological order/fractionalization

- Wegner's Z_2 gauge theory $$H_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} \sigma^z_\ell - g \sum_\ell \sigma^x_\ell$$

$$G_i = \prod_{\ell \in +(i)} \sigma^x_\ell \;, \; [G_i, H_{\mathbb{Z}_2}] = 0 \qquad G_i = 1 \text{ on any physical state}$$

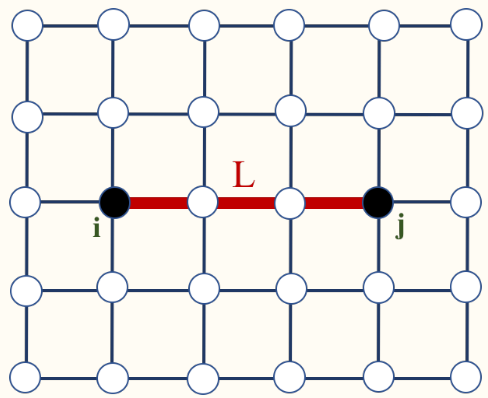
Gapped Z_2 fluxes (visons):

- Created locally in pairs
- Connected by invisible string



$E_{flux} \sim |g - g_c|^{\nu}$

$\Delta E(L) = T_{string} L$



$T_{string} \sim |g - g_c|^{\nu'}$

Test electric charges confined

g_c — 3d Ising criticality — g

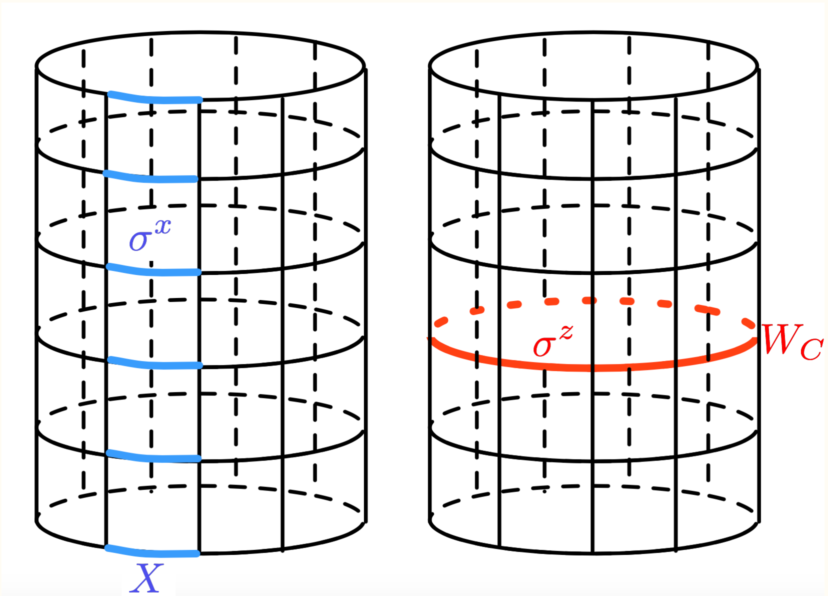
Deconfining Phase (Topological)

Confining Phase (Non-topological)

Topological order/fractionalization

- Wegner's Z_2 gauge theory
- Deconfined phase has topological ground state degeneracy (eg. On a torus), confined phase has unique ground state.

X creates a flux/vison through the hole of the cylinder



Wilson loop W_c detects the flux

|G> and |G'> = X|G> are distinct ground states

Topological order/fractionalization

- Symmetry enrichment of topological phases
- Deconfined gauge charges can carry an additional fractional symmetry quantum numbers.
- In theories of fluctuating quantum magnetism, there are S = ½ spinon excitations in Z_2 spin liquids with disordered ground states.

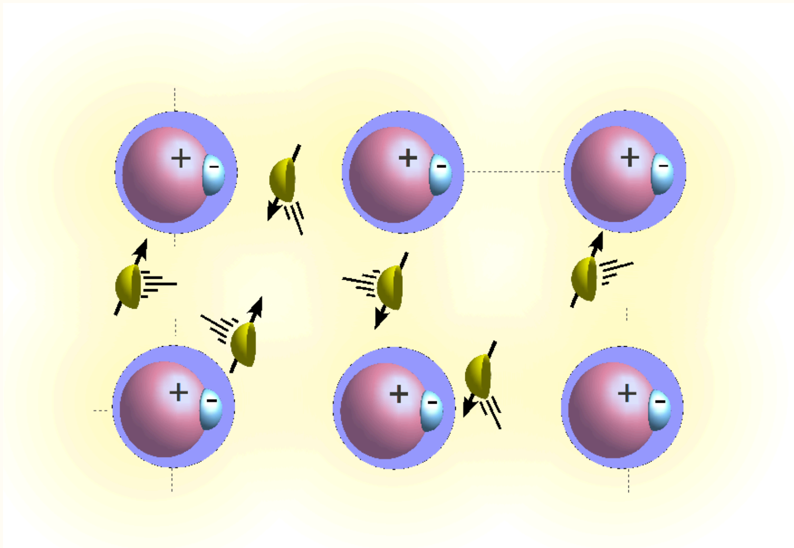
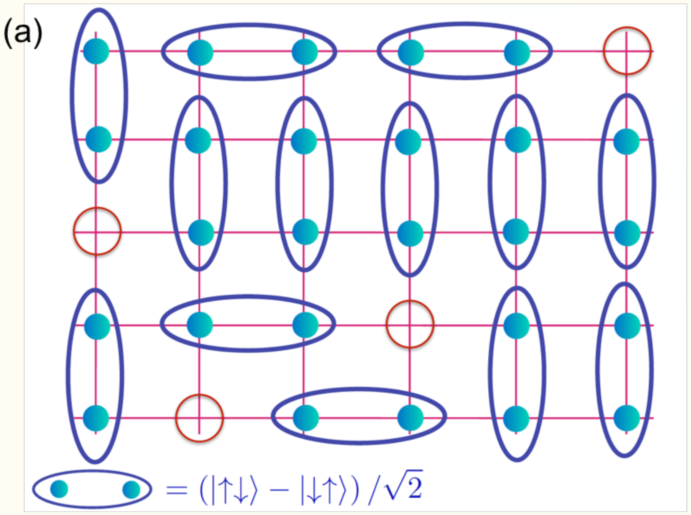


Figure Credits: T. Senthil

Topological order/fractionalization

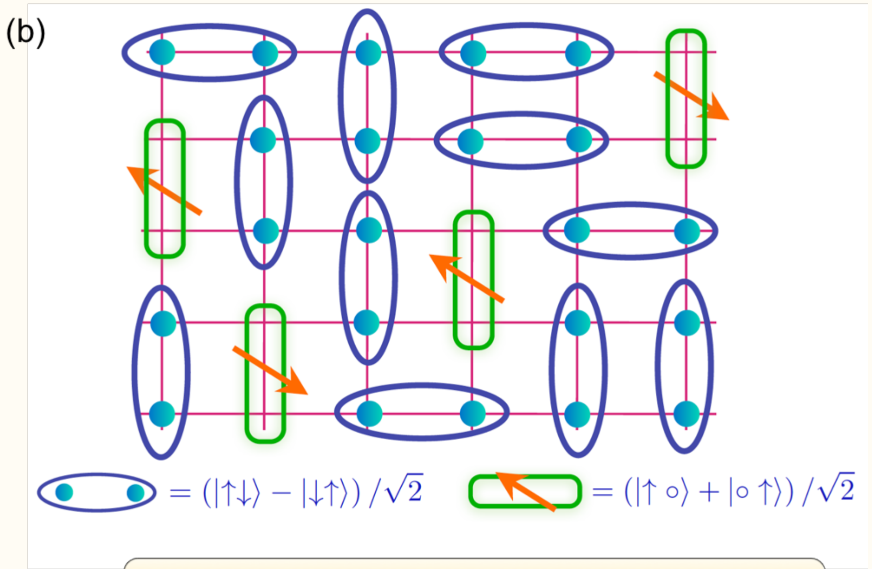
- A Z_2 topologically ordered phase with a Fermi surface can *violate Luttinger's theorem*!

Senthil et al, PRL **90**, 216403 (2003)



Doped *holons* with density p

Spinless charge carriers (*chargons*)



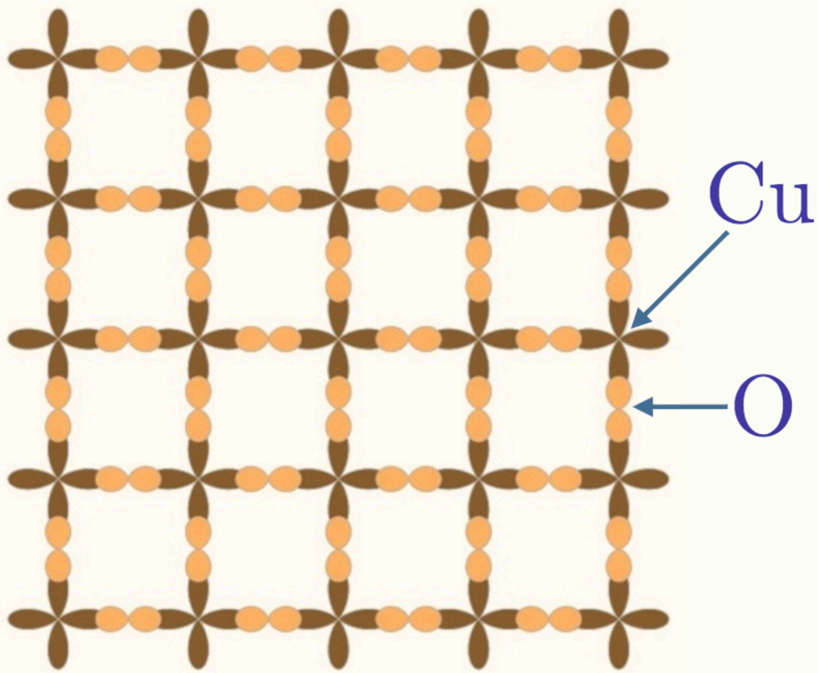
Holon-spinon bound states: FL*

Spinful charge carriers (dimers) form Fermi surface of size p. Luttinger count is 1+p

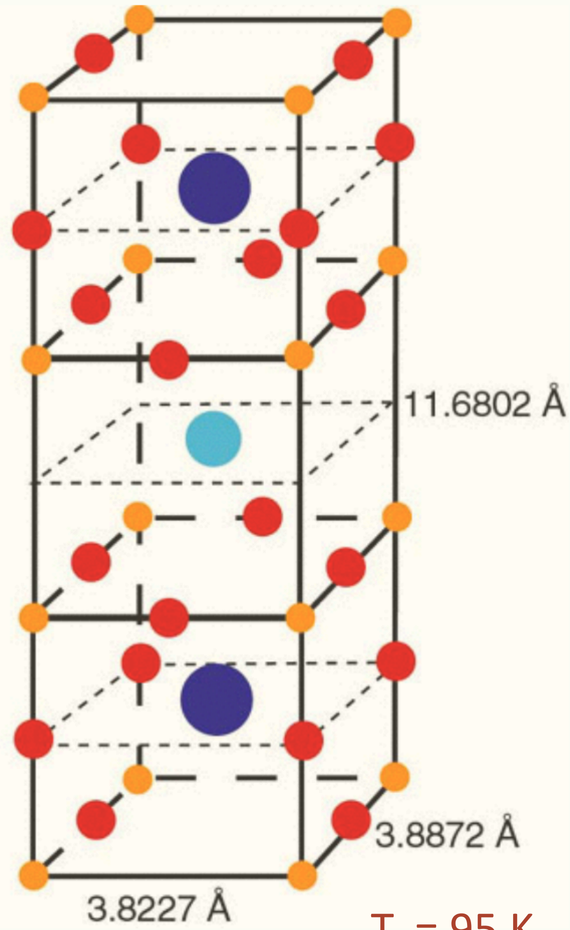
Punk *et al*, PNAS **112**, 9552 (2015)

Cuprate superconductors

High temperature superconductors

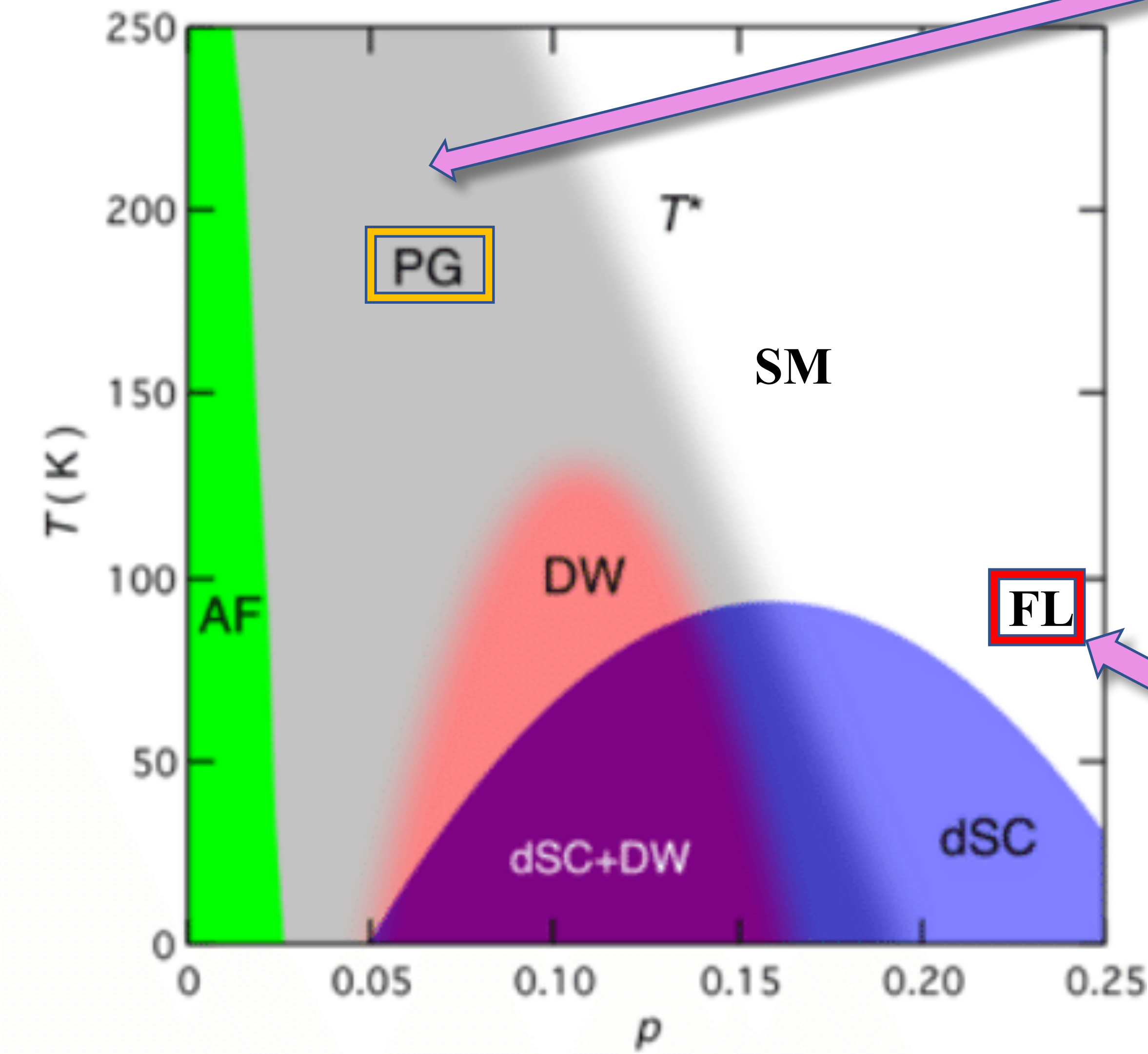


CuO_2 plane

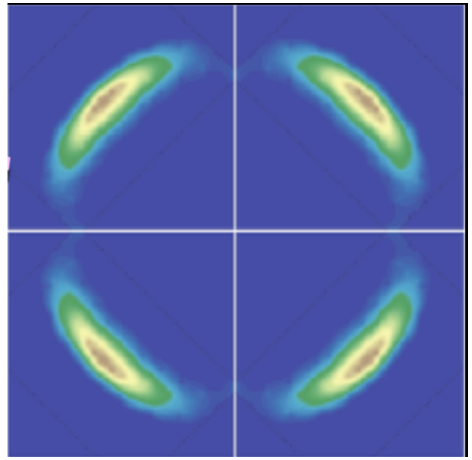


T_c = 95 K

$YBa_2Cu_3O_{6+x}$



Pseudogap metal:
Fermi arcs



Conventional Fermi liquid:
Large hole Fermi surface of size 1 + p.

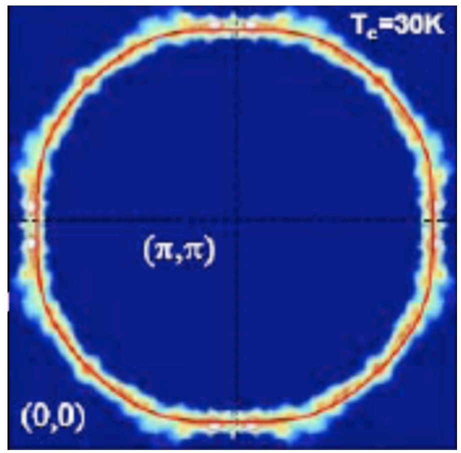
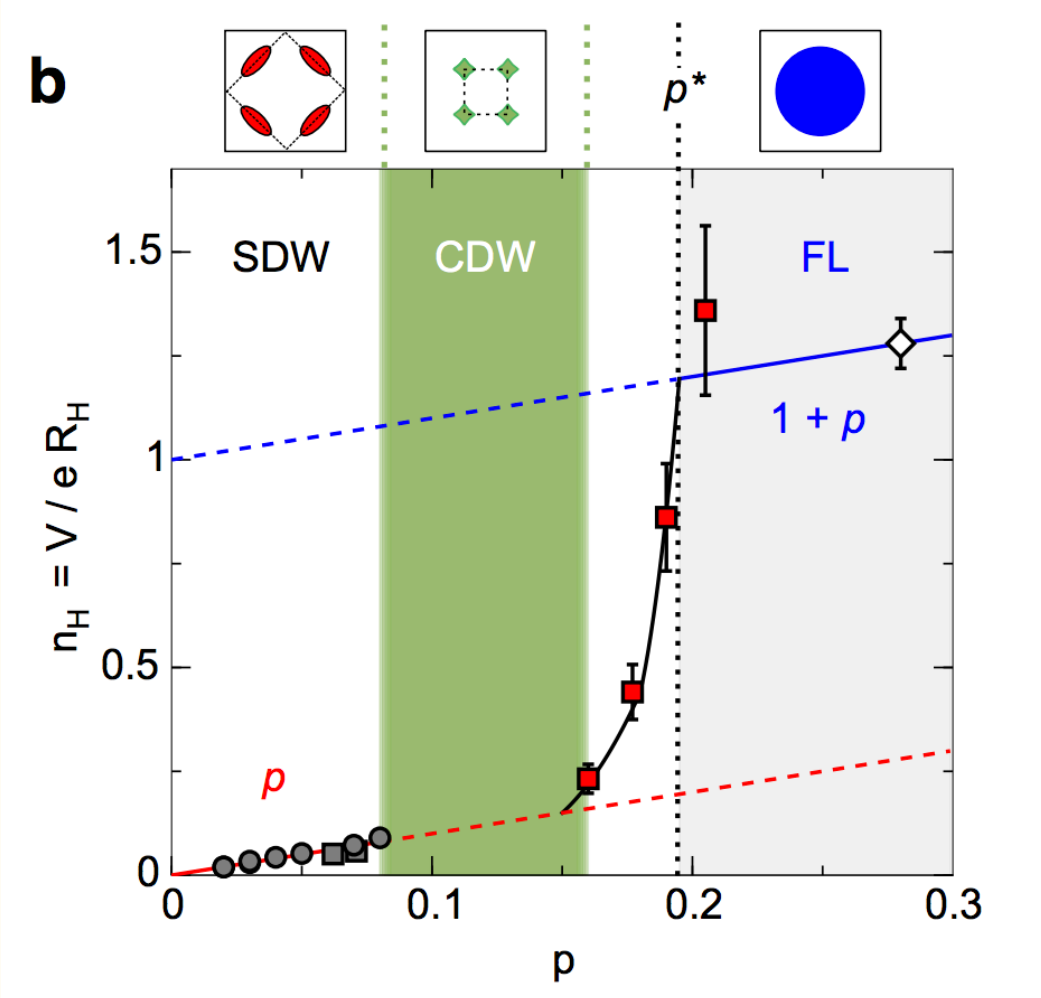


Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)
M. Plate *et al*, PRL. 95, 077001 (2005)

Hall effect in cuprates at high fields



Badoux, Proust, Taillefer *et al*, Nature **531**, 210 (2016)

How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order

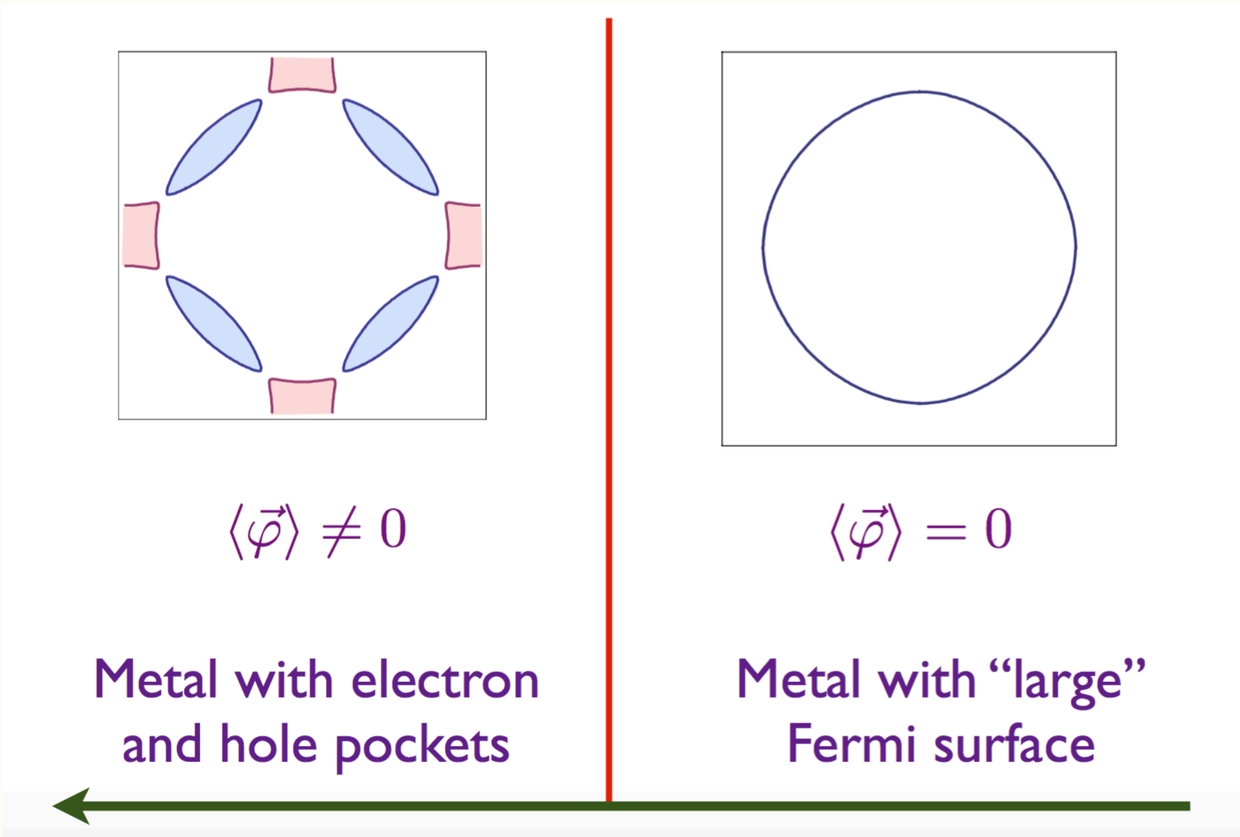
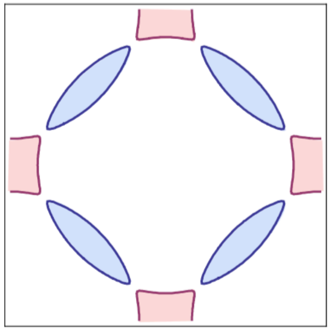


Image credits: S. Sachdev, Harvard

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)



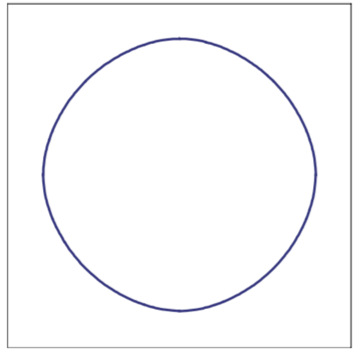
$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

Algebraic Charge liquid (ACL) or Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and pocket Fermi surfaces



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

S. Sachdev, E Berg. *S. Chatterjee*, Y. Schattner , PRB **94**, 115147 (2016)

Additional discrete broken symmetries

- Nematic order: Broken C_4 symmetry

Daou *et al*, Nature **463**, 519 (2010)

- Broken time-reversal symmetry $\boldsymbol{\theta}$ (?)

Mangin-Thro *et al*, Nat. Comms **6**, 7705 (2015), Simon & Varma, PRL **89**, 247003, 2002

- Broken inversion symmetry C_2. However, $\boldsymbol{\theta}$ C_2, the product of inversion and time-reversal seems to be preserved.

Zhao, Belvin, Hsieh *et al*, Nature Physics **13**, 250 (2017)

- No evidence of translation symmetry breaking in large parts of the phase diagram: Even with discrete broken symmetries, Small FS violates Luttinger's Theorem and requires *topological order*.

T. Senthil *et al*, PRL **90**, 216403 (2003)
Paramekanti *et al*, PRB **70**, 245118 (2004)

What is going on under the dome?

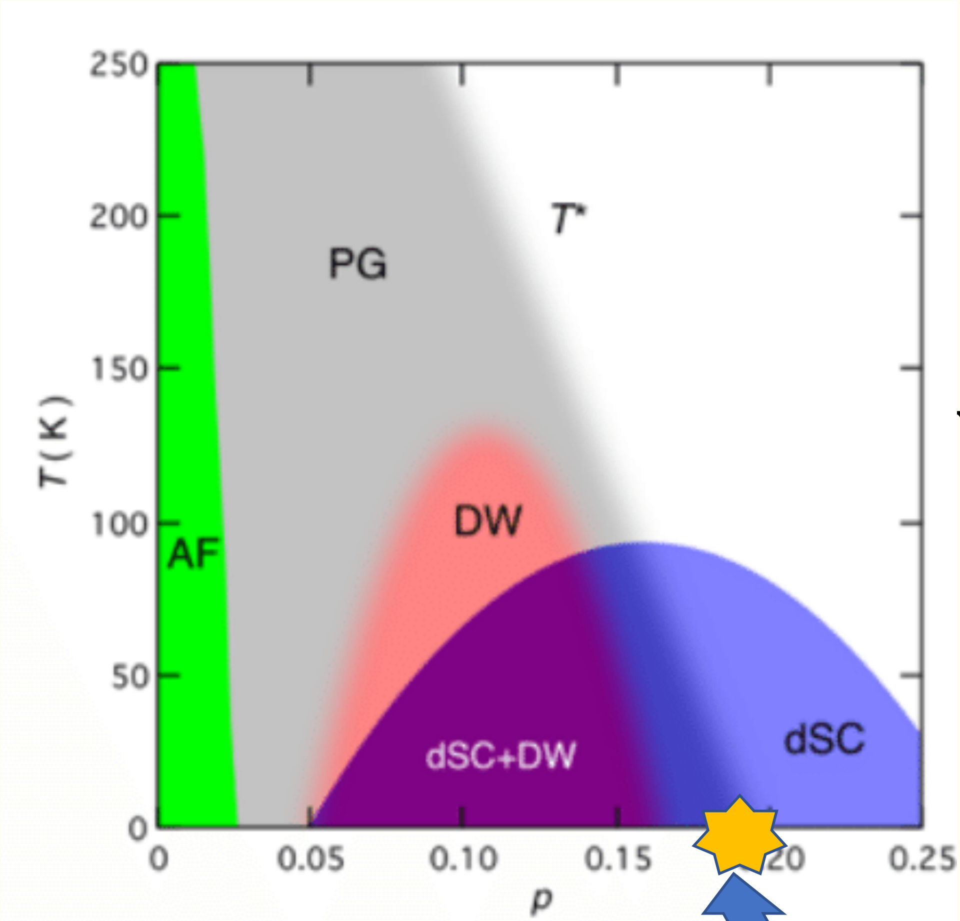


Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)

'L

Is there a quantum critical point (QCP) at optimal doping under the superconducting dome?

What is the nature of the associated phase transition? Symmetry-breaking or topological?

Figure credits: Wikipedia

Figure credits: Disney Clip Art

What is going on under the dome?

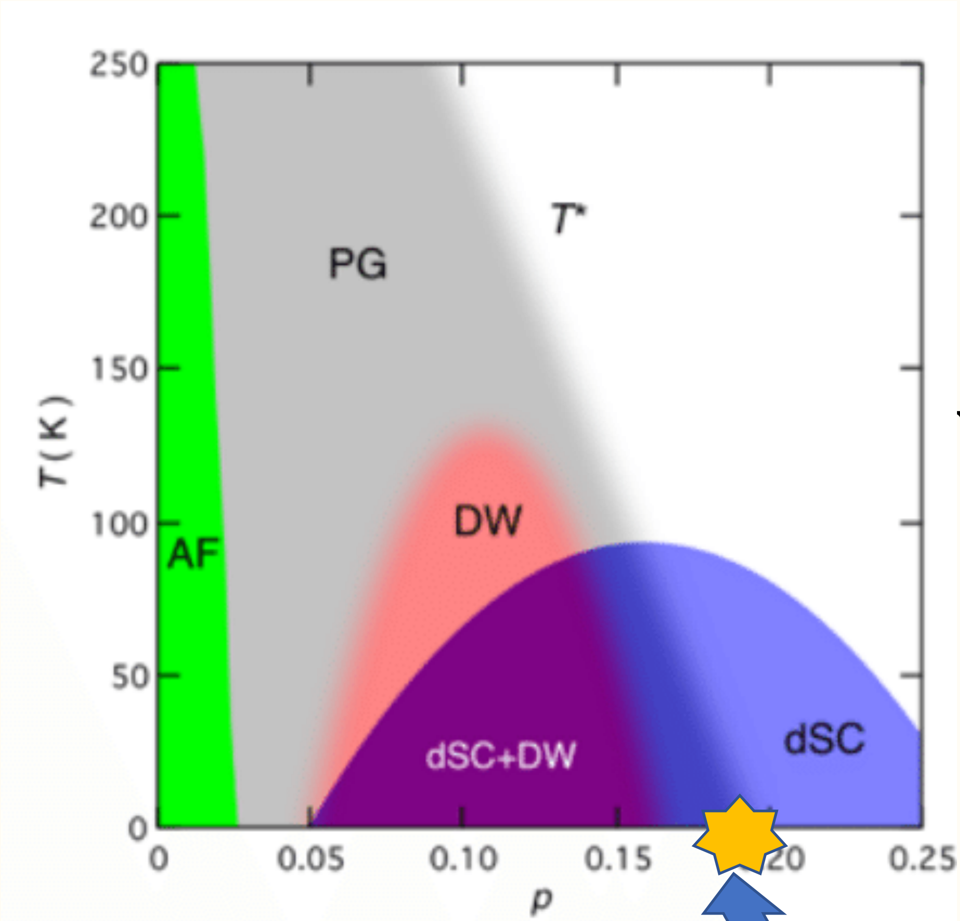


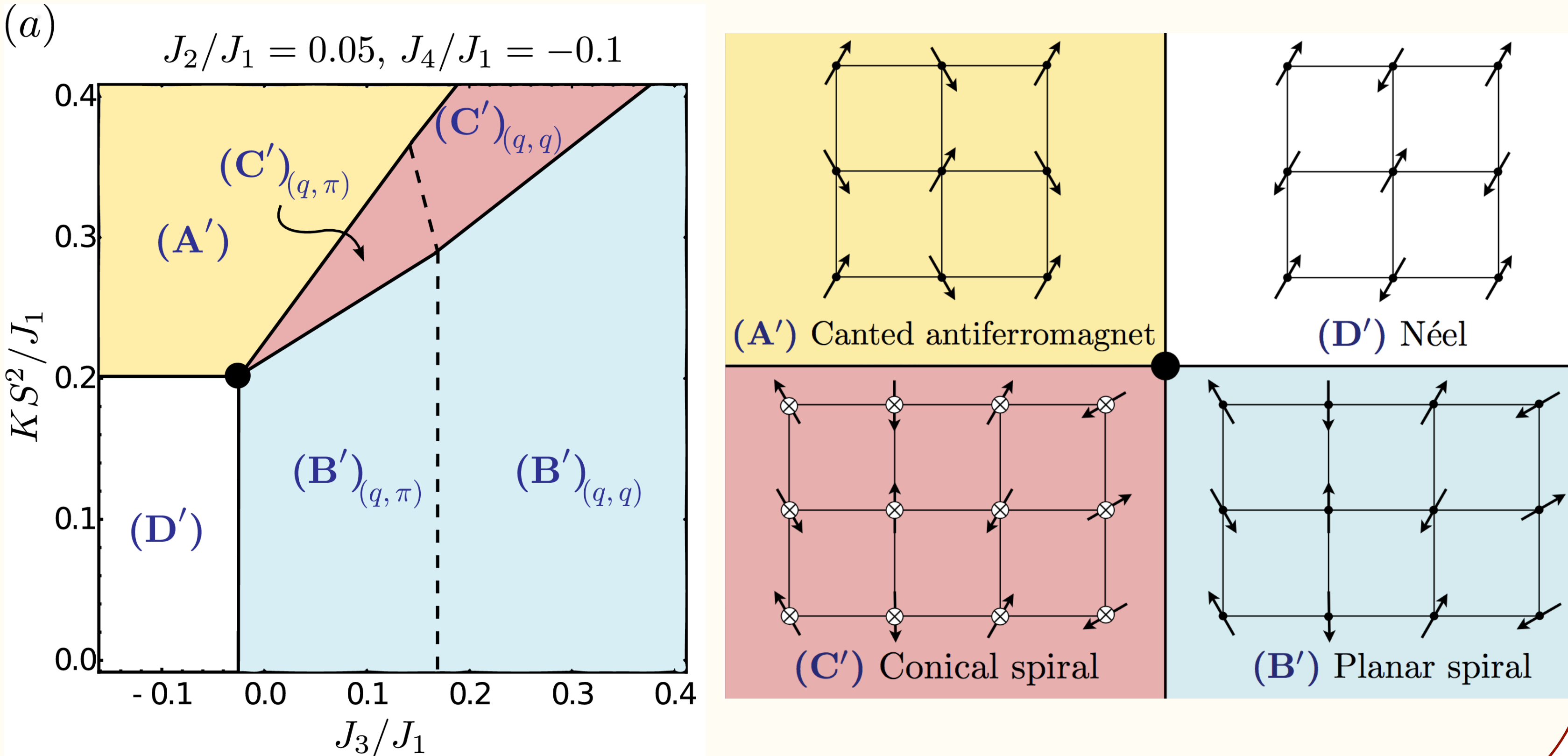
Figure credits: K. Fujita *et al*,
Nature Physics **12**, 150–156 (2016)

Why not both?

Topological QCP with associated discrete symmetry breaking!

Phases near the Neel state: Classical phase diagram

Square lattice AF with Heisenberg exchanges J_1, J_2, J_3 and J_4 and ring exchange K



Topological order in the CP¹ theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

$$\mathbf{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

$$\alpha, \beta = \uparrow, \downarrow, \quad \sum_\alpha |z_\alpha|^2 = 1$$

$$S = \frac{1}{2g} \int d^2 r dt \ (\partial_\mu \mathbf{n})^2$$
$$\rightarrow \frac{1}{2g} \int d^2 r dt \, |(\partial_\mu - i a_\mu) z_\alpha|^2$$

The CP¹ theory an has emergent U(1) gauge field a_μ: Unstable to confinement

Simplest Higgs fields: Spin rotation invariant long-wavelength spinon pairs:

$$P \sim \varepsilon_{\alpha\beta} z_\alpha \partial_t z_\beta \quad , \quad Q_a \sim \varepsilon_{\alpha\beta} z_\alpha \partial_a z_\beta$$

Read & Sachdev, PRL **66**, 1773 (1991)

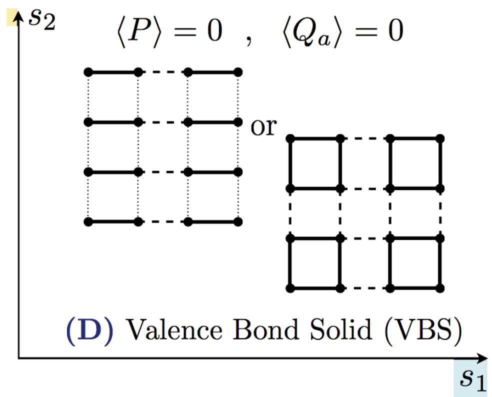
Gauge invariance + symmetry:

$$\mathcal{L} = \frac{1}{g} |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_1 |P|^2 + s_2 |Q_a|^2$$

Topological order in the CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large g with $\langle z_\alpha \rangle = 0$



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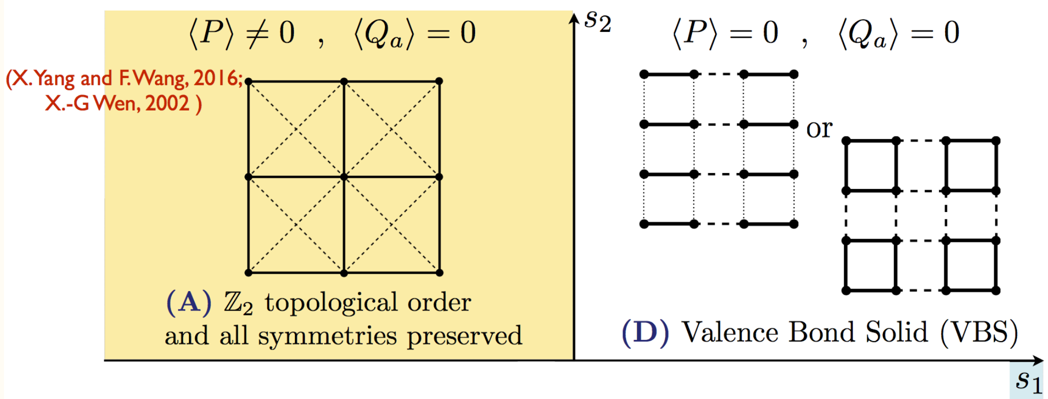

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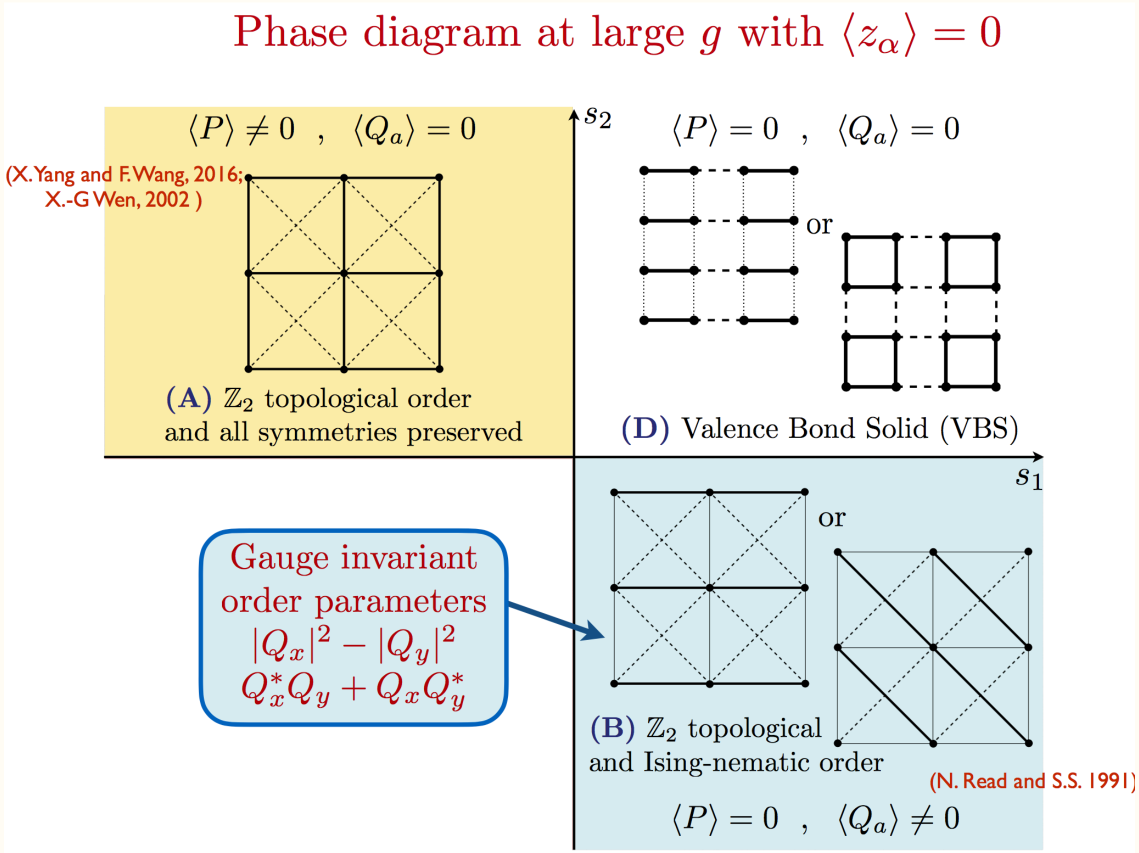

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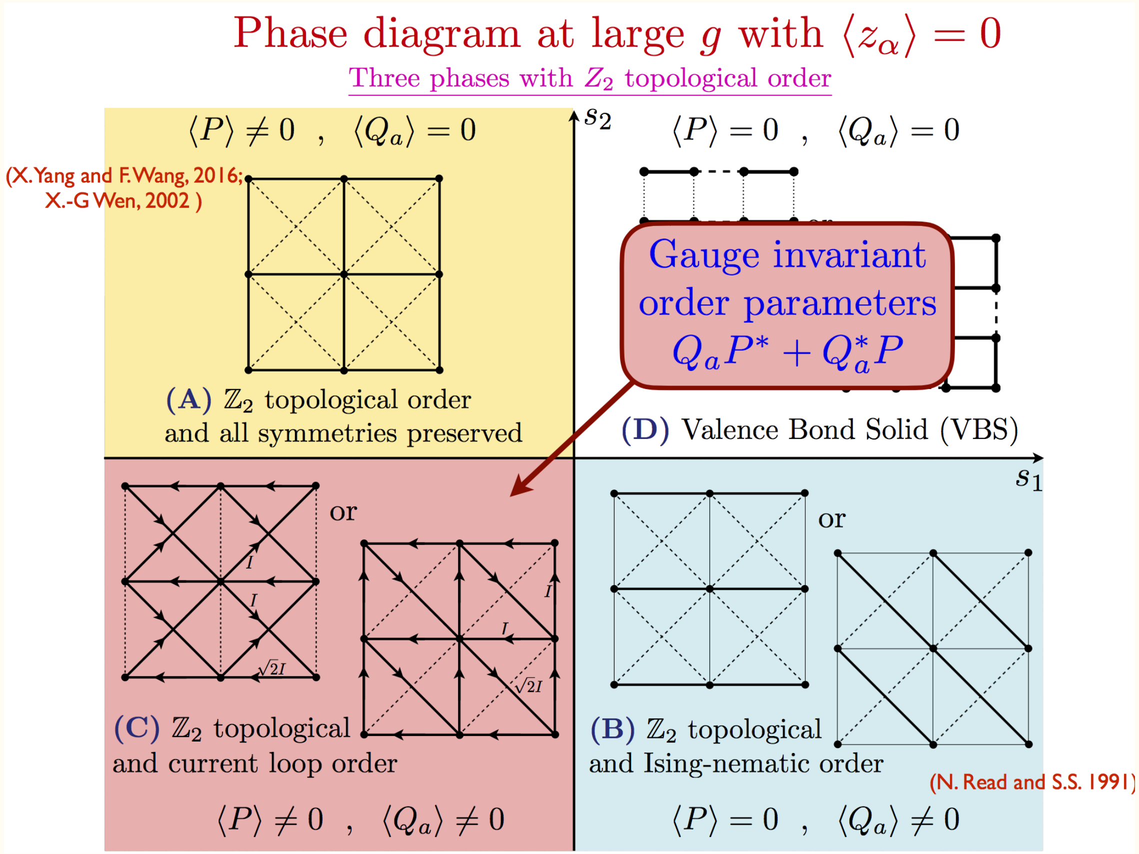
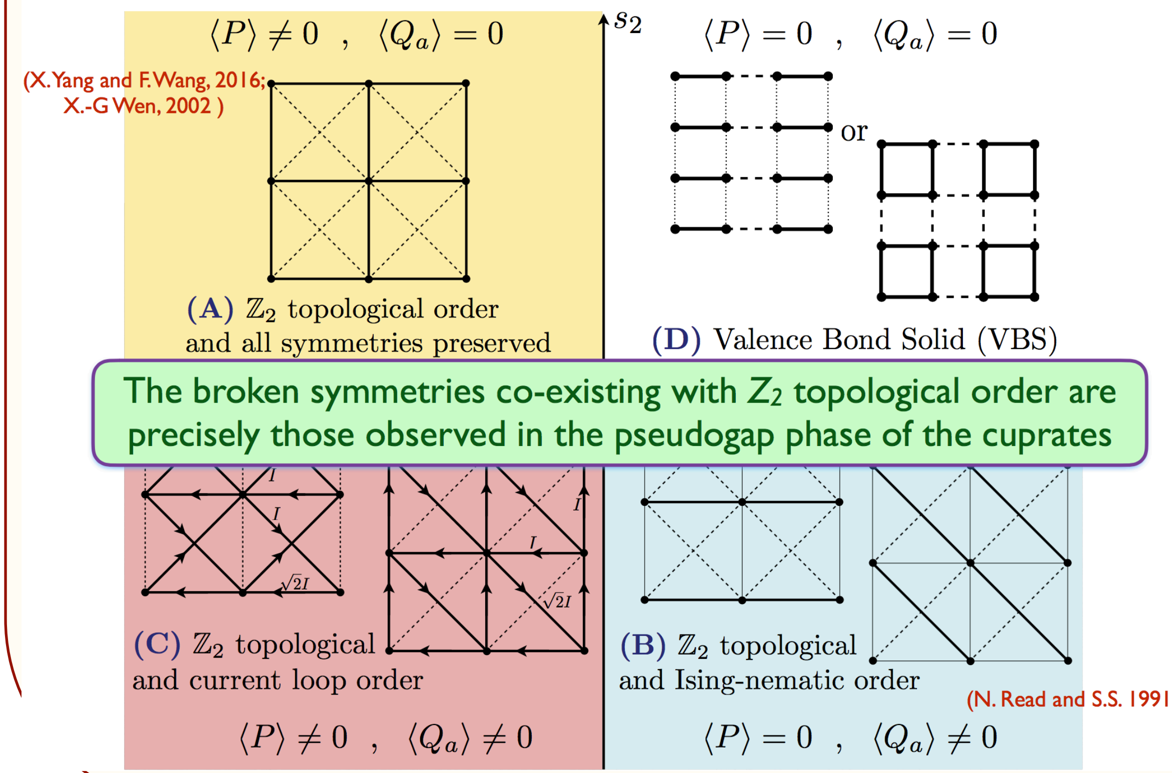


Topological order in the CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order



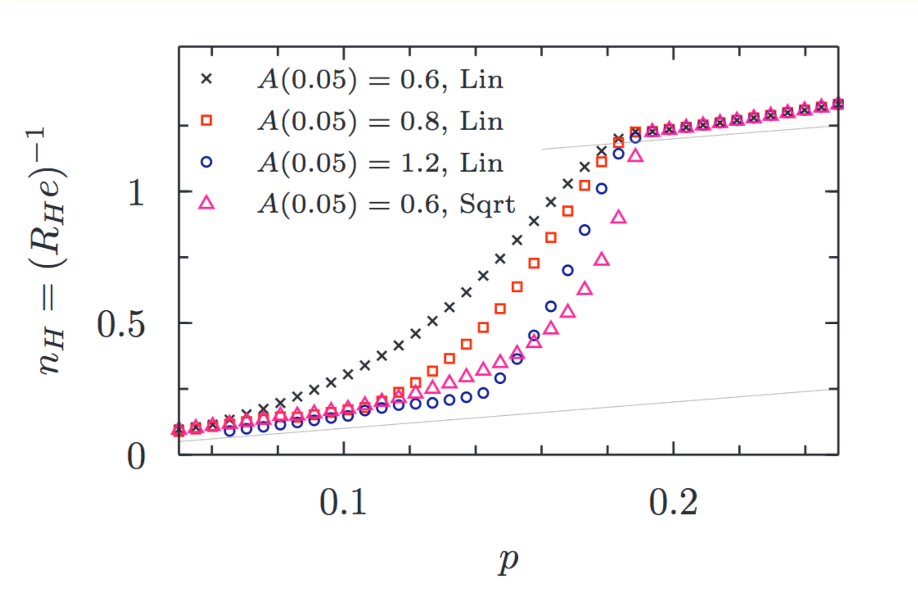
The broken symmetries co-existing with Z_2 topological order are precisely those observed in the pseudogap phase of the cuprates

Dope the insulator: SU(2) gauge theory of the electrons with small Fermi pockets + discrete broken symmetries

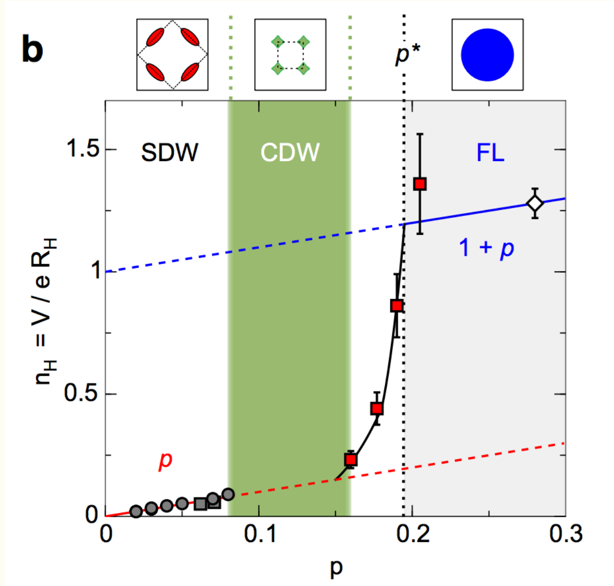
S. Chatterjee and S. Sachdev, PRB **95**, 2015133, 2017;
S. Chatterjee, S. Sachdev and Mathias S. Scheurer, PRL **119**, 227007, 2017

Comparisons with experiments

Hall data shows good qualitative agreement, as do data on longitudinal thermal and electric transport



A. Eberlein *et al*, PRL, **117,** 187001 (2016)

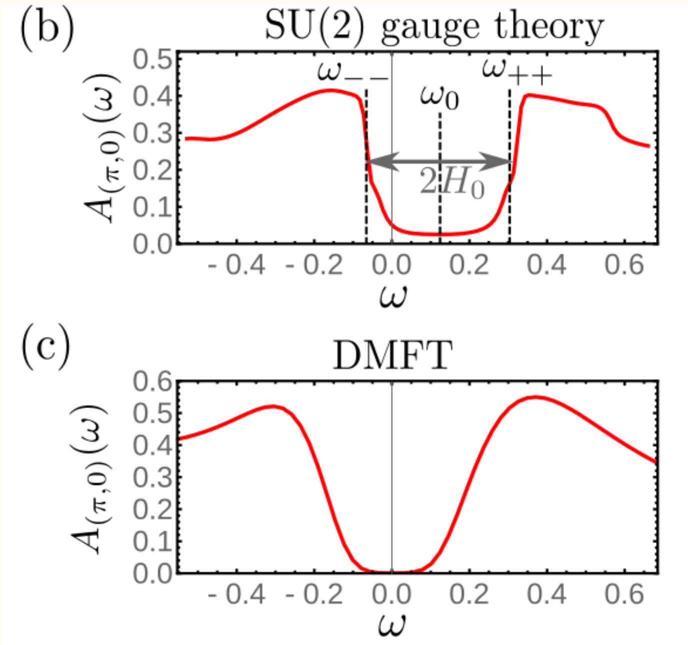


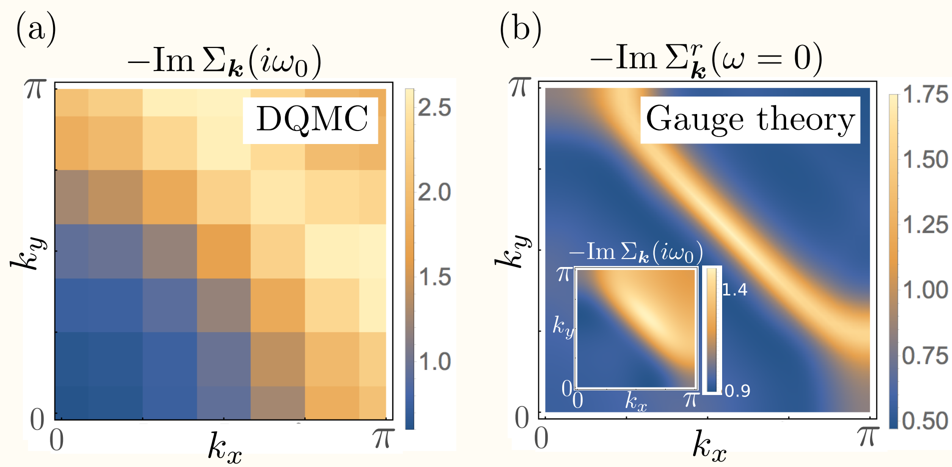
Badoux, Proust, Taillefer *et al*, Nature **531**, 210 (2016)

S. Chatterjee, S. Sachdev and A. Eberlein, PRB, **96,** 075103 (2017)

Comparisons with numerics

Electron spectral functions / self-energies from the SU(2) gauge theory closely resemble those from DMFT/QMC on 2d Hubbard model



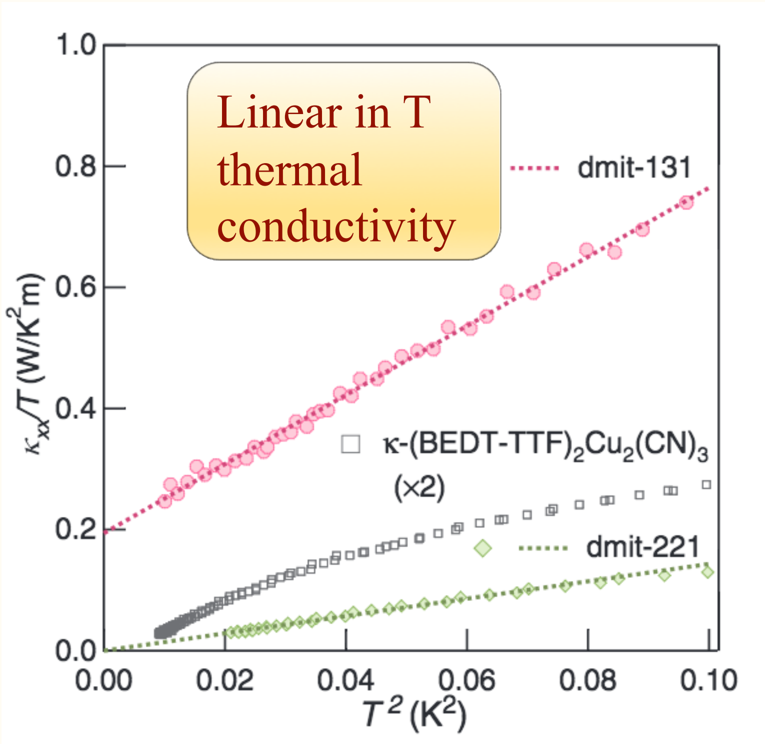


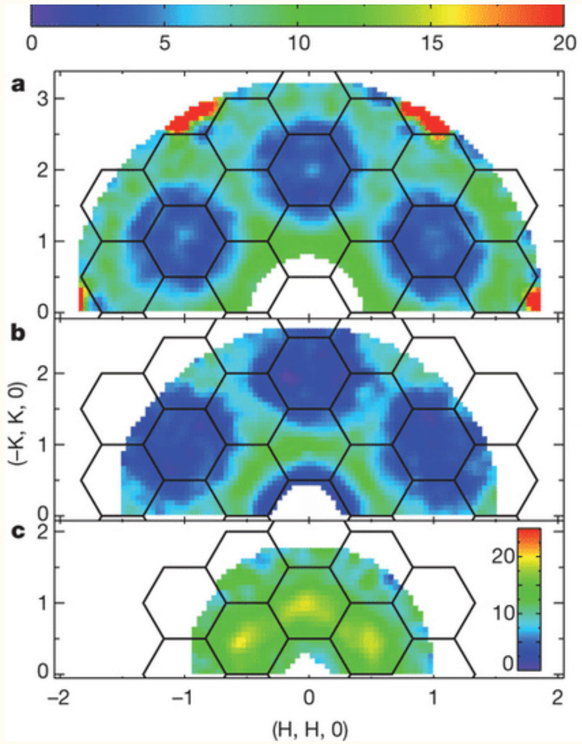
M. Scheurer, *S. Chatterjee*, W. Wu, M. Ferrero, A. Georges and S. Sachdev, arXiv:1711.09925
W. Wu, M. Scheurer, *S. Chatterjee,* S. Sachdev, A. Georges and M. Ferrero, arXiv:1707.06602

Topological order in insulators: Spin liquids

Several experimental candidates (organics, Herbertsmithite, etc)

Identified by anomalous signatures in transport/structure factor in insulators





Broad continuum of two-particle like spectrum in neutron scattering

M. Yamashita *et al*, Science, **328,** 5983 (2010) Han *et al*, Science, **328,** 7429 (2012)

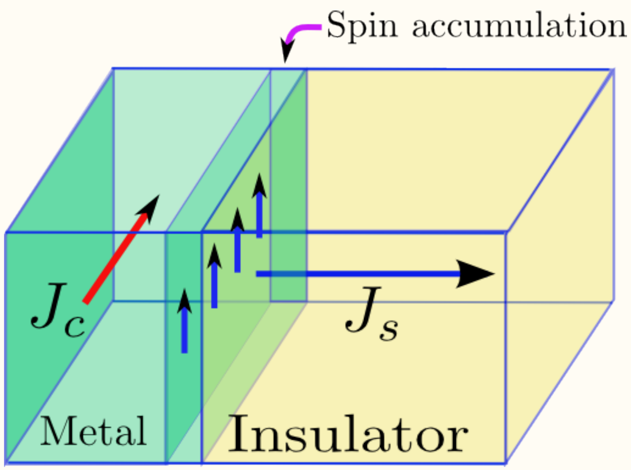
Topological order in insulators: Spin liquids

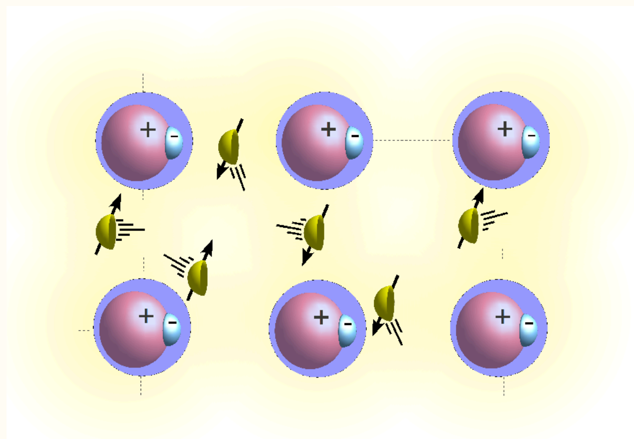
Several experimental candidates (organics, Herbertsmithite, etc)

Cool material down to low temperatures: Look for signatures of *nothing*

What are some experimental probes (other than neutrons) that can confirm a spin liquid phase in an insulator?

Use the spin sector: Local operators create two spinons but one magnon





Study $J_s(V_s)$

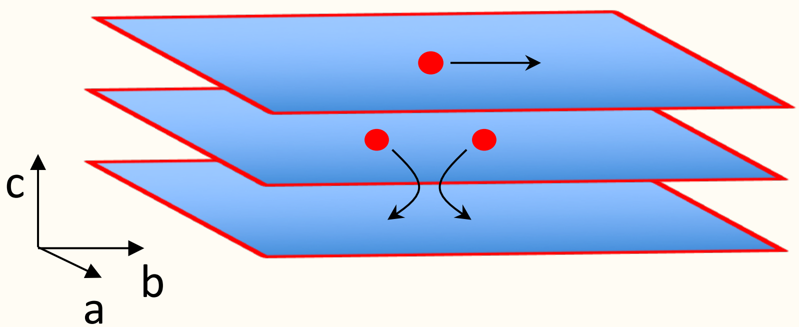
S. Chatterjee, S. Sachdev, PRB **92**, 165113 (2015)

Topological order in insulators: Spin liquids

What are some experimental probes (other than neutrons) that can confirm a spin liquid phase in an insulator?

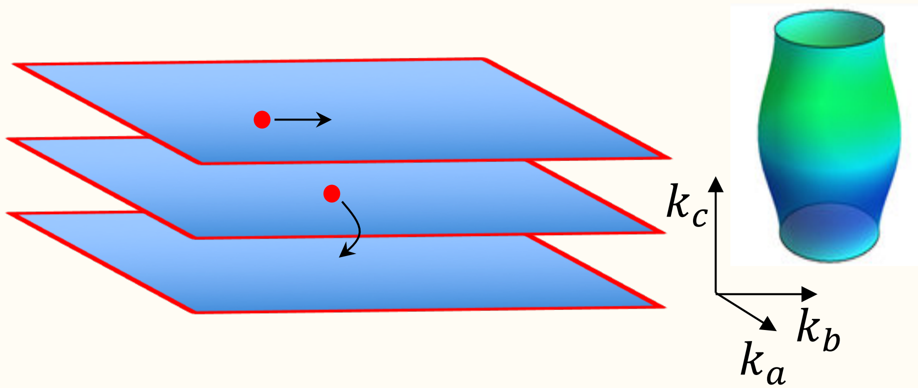
Use the energy sector: κ_c and κ_{ab} show parametrically different behavior as a function of temperature, in some gapless spin liquids κ_c dominates phonons

Fractionalized



Parametrically anisotropic thermal transport

Non-fractionalized



at low T, $\frac{\kappa_{ab}}{\kappa_c} \to const \gg 1$

Y. Werman, *S. Chatterjee*, S. C. Morampudi, E. Berg, arXiv:1708:02584

Topological order in insulators: Spin liquids

What are some experimental probes (other than neutrons) that can confirm a spin liquid phase in an insulator?

Use the energy sector: κ_c and κ_{ab} show parametrically different behavior as a function of temperature, in some gapless spin liquids κ_c dominates phonons

	In-plane		c-axis	
	Clean	Disordered	Clean	Disordered
$\mathbb{Z}_2$ Dirac	T	T	T^5	$T^{5-\alpha}$
$\mathbb{Z}_2$ FS	$1/T$	T	T^3	$\boxed{T^2}$
U(1)	$T^{1/3}$	T	$\boxed{T^{5/3}}$	$\boxed{T^2}$

Upto logarithmic corrections

Durst and Lee, PRB **62**, 1270 (2000)

Nave and Lee, PRB **76**, 235124 (2007)

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Julia Steinberg,
Harvard University