

Density matrix renormalization group evidence of superconductivity via skyrmion-pairing

Shubhayu Chatterjee

University of California, Berkeley

Correlated Systems with Multicomponent Local Hilbert Spaces

KITP, UC Santa Barbara

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In collaboration with:



Matteo Ippoliti
Stanford University

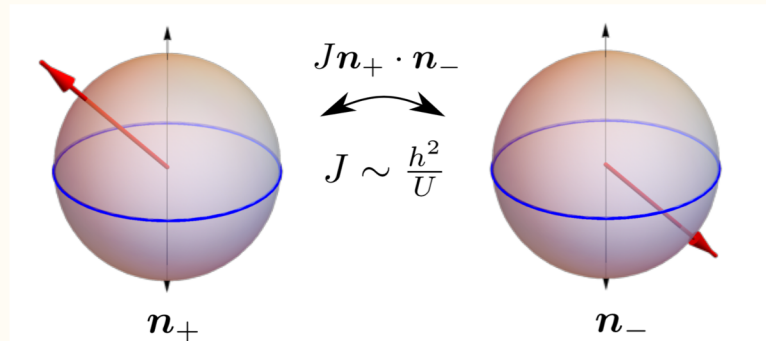


Mike Zaletel
UC Berkeley

SC, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Skyrmion-pairing mechanism

- Setting: Interacting electrons in tunnel-coupled (nearly flat) spin-ful Chern bands with opposite Chern numbers
- Within each Chern sector/*layer*: Interaction driven quantum-Hall ferromagnet (Stoner criteria)
- Tunnel-coupling leads to an antiferromagnetic (super-)exchange
- Ground state at half-filling (2 out of 4 bands) is a correlated antiferromagnetic insulator



Skyrmion-pairing mechanism

- In addition to particle-hole excitations, have topological textures: skyrmions in each *layer* carry charge

Sondhi *et al*, PRL (1993)

Moon *et al*, PRB (1994)

$$Q_{\text{physical}} = C Q_{\text{topological}}$$

- Can charge-e skyrmions pair and condense to give rise to superconductivity?
- Superconductivity from 2e skyrmion condensation has been proposed in doped QSH insulators – does this physics survive Coulomb repulsion?

Abanov and Weigeman, PRL (2001)

Grover and Senthil, PRL (2008)

Christos *et al*, PNAS (2020)

Wang *et al*, arXiv: 2006.13239

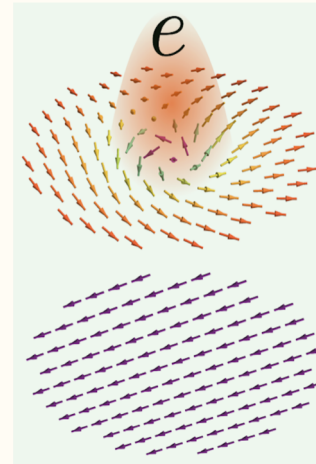
Skyrmion-pairing mechanism

- Consider a skyrmion in one QH layer and an anti-skyrmion in the opposite layer

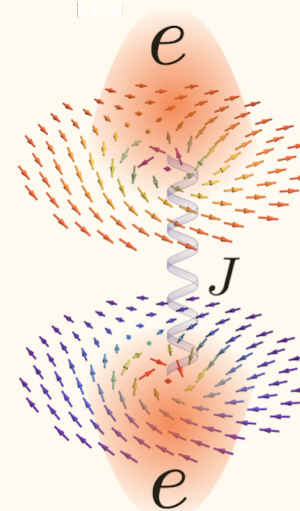
$$Q_{physical} = CQ_{topological}$$

- Both carry same charge: Repelled by Coulomb but attracted by local antiferromagnetism J
- All electronic pairing mechanism without phonons/retardation/bosonic fluctuations*

SC, N. Bultinck, M. Zaletel, PRB 2020
E. Khalaf, SC *et al*, arXiv:2004.00638



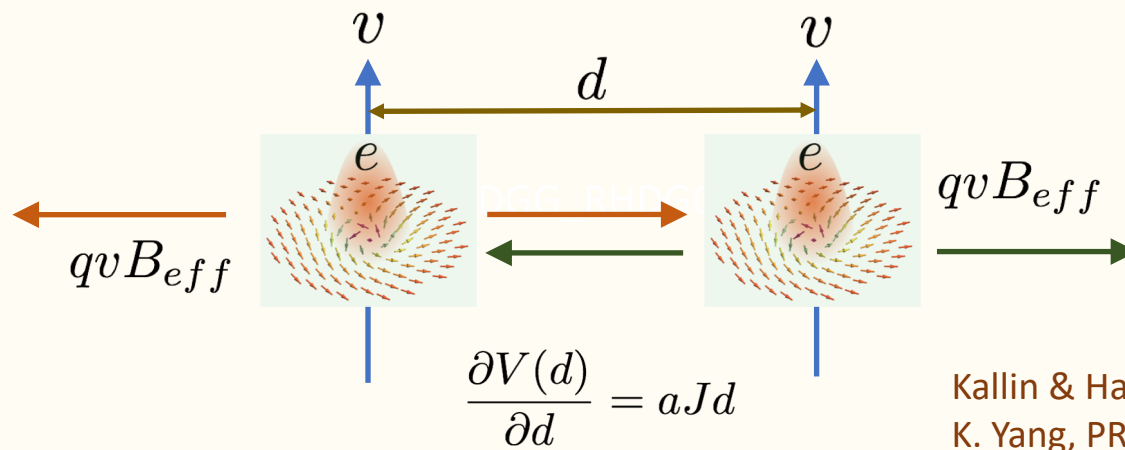
Single skyrmion pays a large exchange penalty



Sk-Ask pair can spread out to minimize Coulomb without losing exchange

Skyrmion-pairing mechanism

- For charge e textures, kinetic energy quenched by magnetic field
- Charge $2e$ skyrmion with charge e in each layer sees *no net magnetic field*, can therefore be mobile

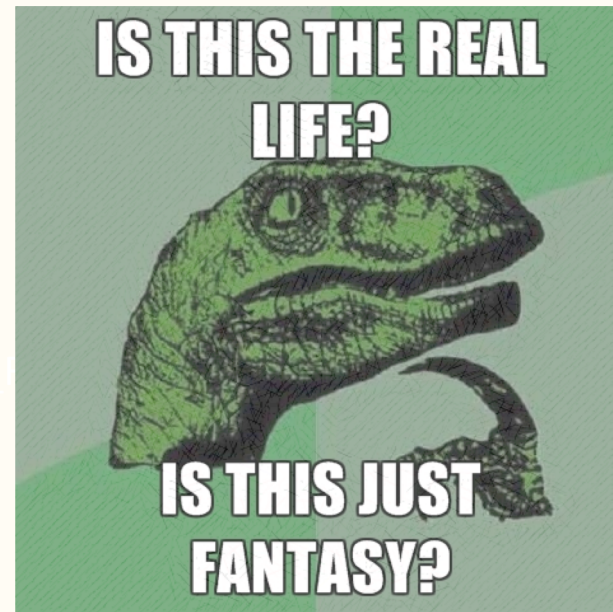
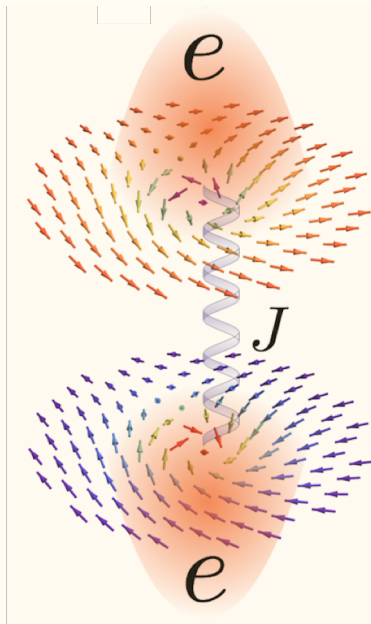


$$E = V_0 + aJd^2/2 = V_0 + \frac{(qB_{eff})^2}{2aJ}v^2$$

$$T_c \sim 1/M_{pair} \sim J \sim 1 \text{ K in MAG}$$

Skyrmion-pairing mechanism

Skyrmion-pairing
superconductivity

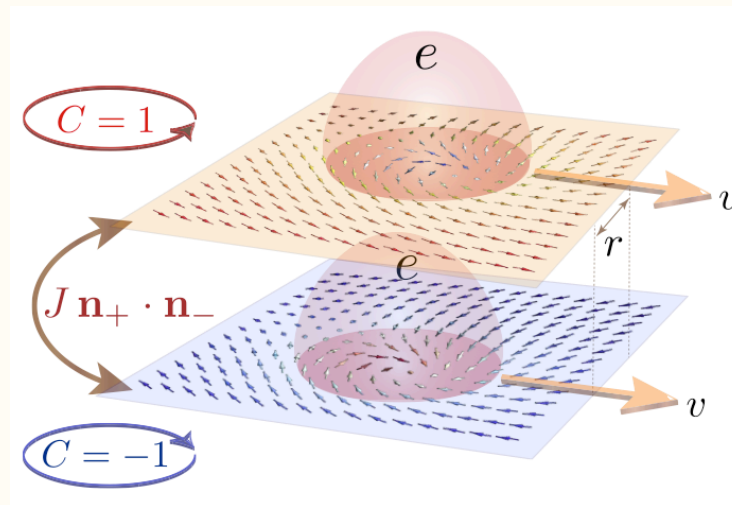


Quote: Queen, Figure credits:
<http://creatememe.chucklesnetwork.com/memes/16712>

DMRG: Model and phase diagram

- Essential ingredients:

1. Spinful (nearly) flat bands with opposite Chern number ± 1
2. AF interaction between the Chern sectors, in addition to Coulomb repulsion



For connections to
moiré graphene:
Talks by:
Eslam (Nov 30)
Mathias (today)
Ashvin (tomorrow)

- Test: AF couple spinful lowest Landau levels, amenable to DMRG

Zaletel *et al*, PRL (2013)

DMRG: Model and phase diagram

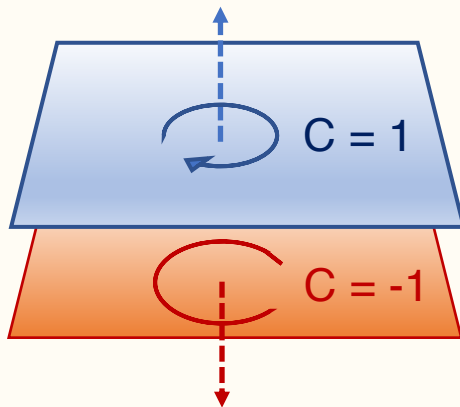
- iDMRG for coupled Landau level model on a cylinder ($L_y = 8-12 \ell_B$)

Ippoliti *et al*, PRB (2018)

$$H = \psi^\dagger \frac{(\mathbf{p} + e\gamma^z \mathbf{A})^2}{2m} \psi + \frac{1}{2} \int : n(r) V_C(r - r') n(r') : - E_C \ell_B^2 \sum_{i=x,y,z} J_i : (\psi^\dagger \gamma^z \eta^i \psi(r))^2 :$$

Kinetic term

$\gamma = \text{layer}, \eta = \text{spin}$



Coulomb repulsion

AF super-exchange

$$J_x = J_y = J + \lambda, J_z = J - \lambda \rightarrow$$

Easy plane/easy axis anisotropy

Isotropic super-exchange

Purely repulsive model for $J < 3.24$ ($d_s = 3\ell_B$)

Related work: Kang and Vafeek, PRB (2020)

Soejima, Parker *et al*, PRB (2020)

Eugenio and Dag, arXiv: 2004.10363

DMRG: Model and phase diagram

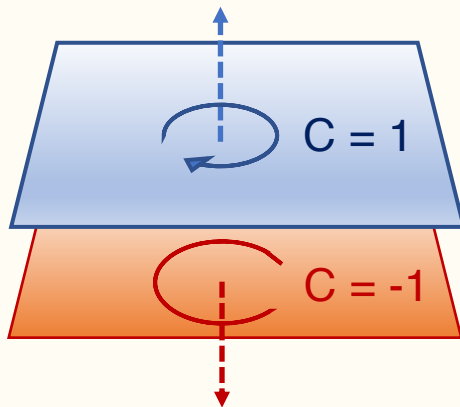
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2/4 filling: AF insulator, preserves $T' = i \gamma^x \eta^y K$

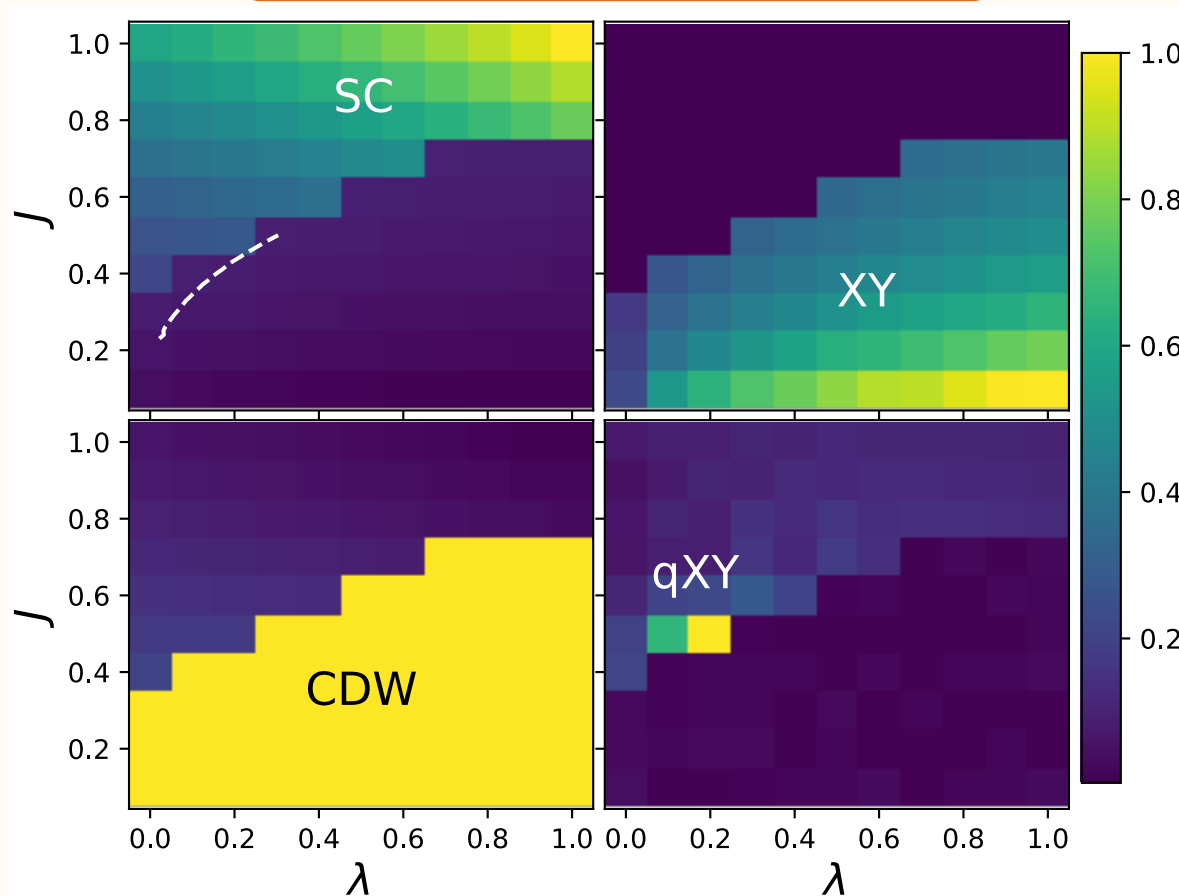
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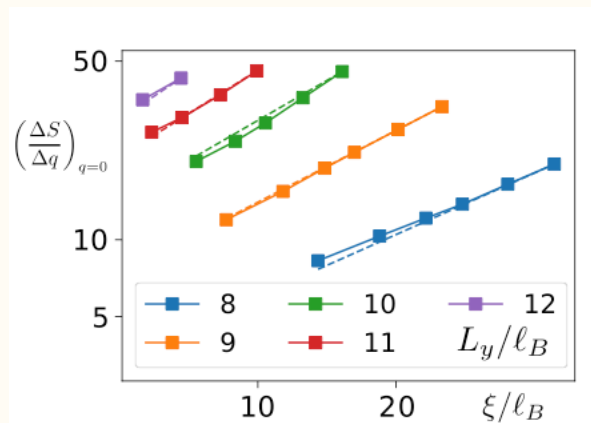
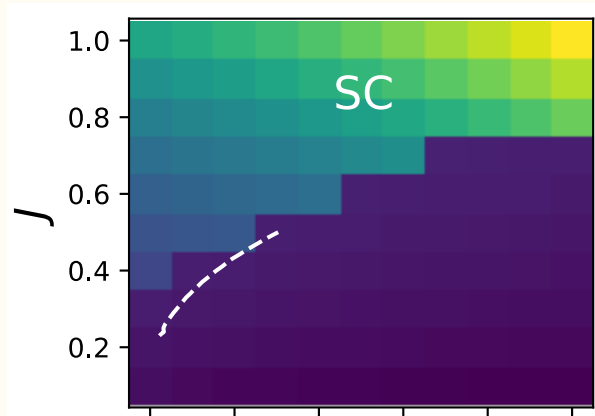
DMRG: Model and phase diagram

Phase diagram at doping $2 + 1/4$



DMRG: Model and phase diagram

Phase diagram at doping $2 + 1/4$



- Superconductivity at large J (layer-unpolarized)

- Single particle excitations have gap $\sim E_C$

- Algebraic decay of Kramers-pair correlations

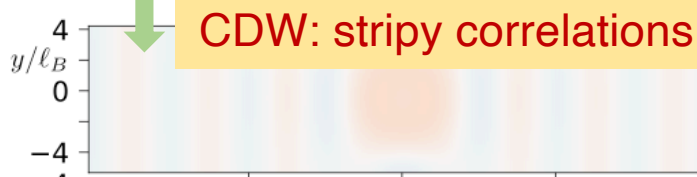
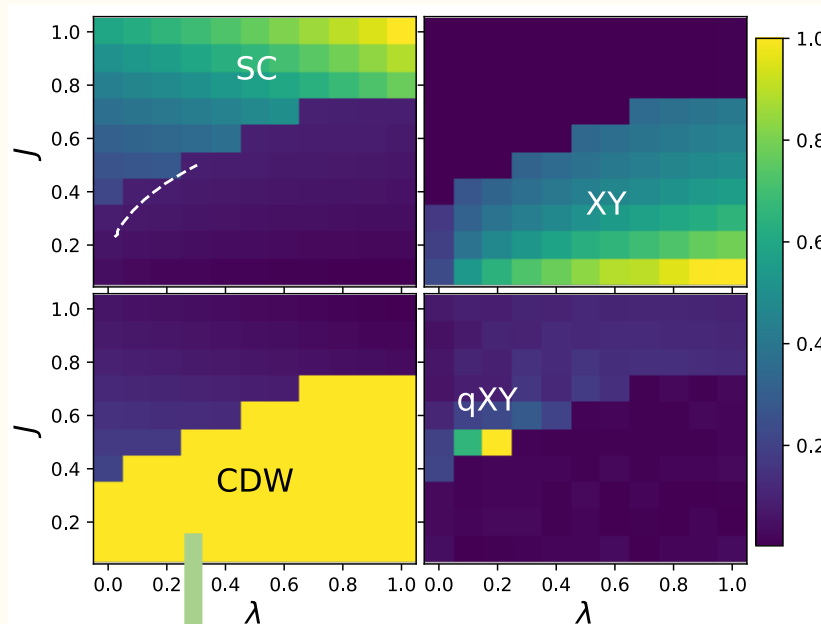
$$\langle \Delta^\dagger(x, 0) \Delta(0, 0) \rangle \propto x^{-\eta_{SC}}$$

$$\eta_{SC} \propto L_y^{-1}$$

- Scaling analysis shows true long range SC order in 2d limit ($L_y \rightarrow \infty$)

DMRG: Model and phase diagram

Phase diagram at doping $2 + 1/4$

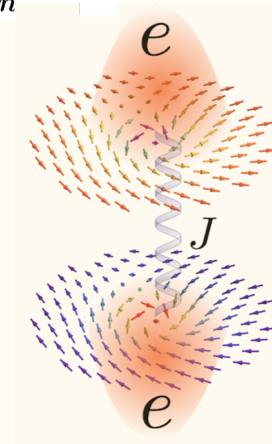
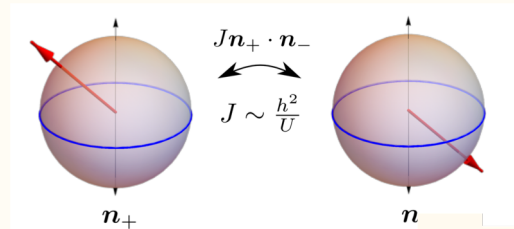


- Coexisting XY-AF and CDW at small J (layer-polarized)
- Transition (first order) between CDW and SC as J is increased
- Small region of coexistence of SC and XY-AF order at finite q_* (tied to the doping)
- The competing state is layer-polarized, but depends on the filling (CDW at $2+1/4$, CFL at $2+1/2$, IQHE at $2+1$)

Evidence for skyrmion-pairing

What is the mechanism of SC? Are skyrmions relevant?

Intuition from NLSM:
Yes, for small anisotropy



$$\mathcal{L} = \int_r \sum_{\gamma} \left[\frac{1}{2A_M} \mathcal{A}_{\gamma} \cdot \partial_{\tau} \mathbf{n}_{\gamma} + \frac{g}{2} (\nabla \mathbf{n}_{\gamma})^2 + \mathbf{A}_{\mu} \cdot \mathbf{j}_{\gamma}^{\mu} \right]$$

$$+ \frac{1}{2} \int_{r,r'} \sum_{\gamma,\gamma'} \rho_{\gamma}(r) V_C(r-r') \rho_{\gamma'}(r') - \bar{J}^i \int_r (\mathbf{n}_{+}^i - \mathbf{n}_{-}^i)^2$$

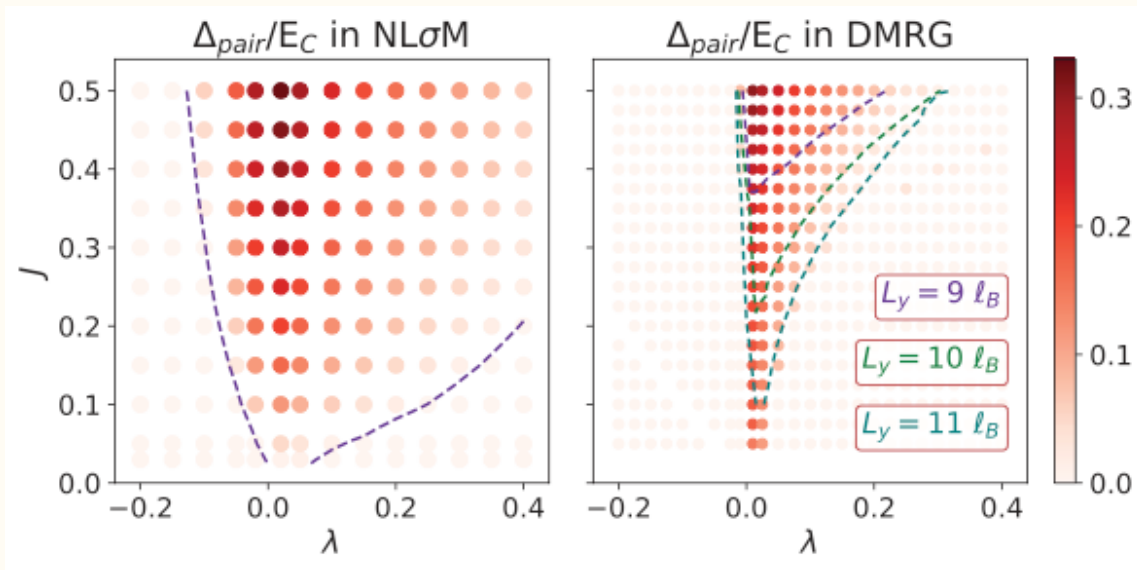
$$\mathbf{j}_{\pm}^{\mu} = \pm \frac{e}{8\pi} \epsilon^{\mu\nu\rho} \mathbf{n}_{\pm} \cdot (\partial_{\nu} \mathbf{n}_{\pm} \times \partial_{\rho} \mathbf{n}_{\pm})$$

Both NLSM and DMRG give energy of charged excitations above insulator

Evidence for skyrmion-pairing

- NLSM + Segment DMRG to determine energy of charged excitations

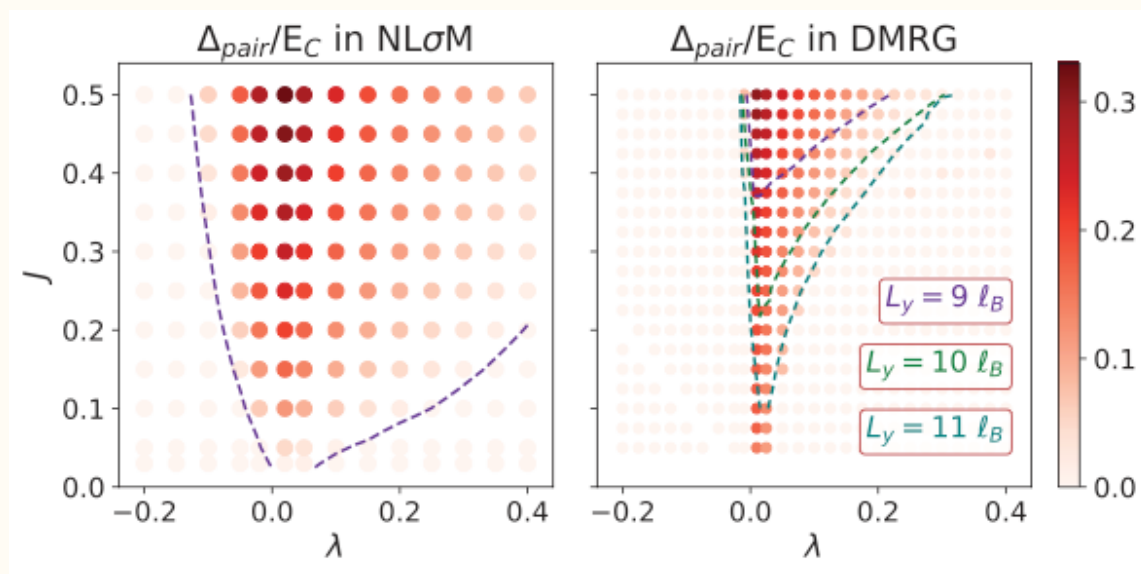
$$\Delta_{\text{pair}} = 2 E_{1e} - E_{2e}$$



- Numerics for quantum system confirm classical expectations!

Evidence for skyrmion-pairing

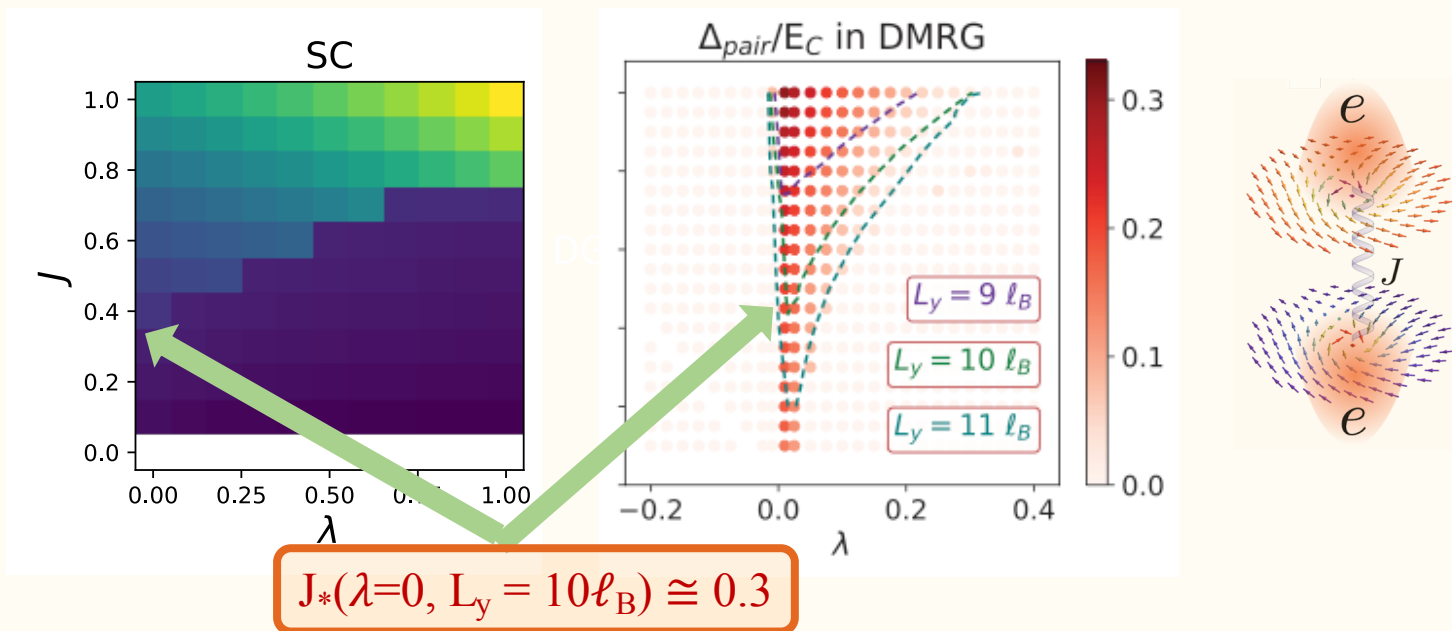
- Critical $J_*(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$, indicative of collective pairing mechanism
- Pairing is much more favorable in the easy plane case (good for MAG!)



- Good qualitative agreement between quantum and classical numerics

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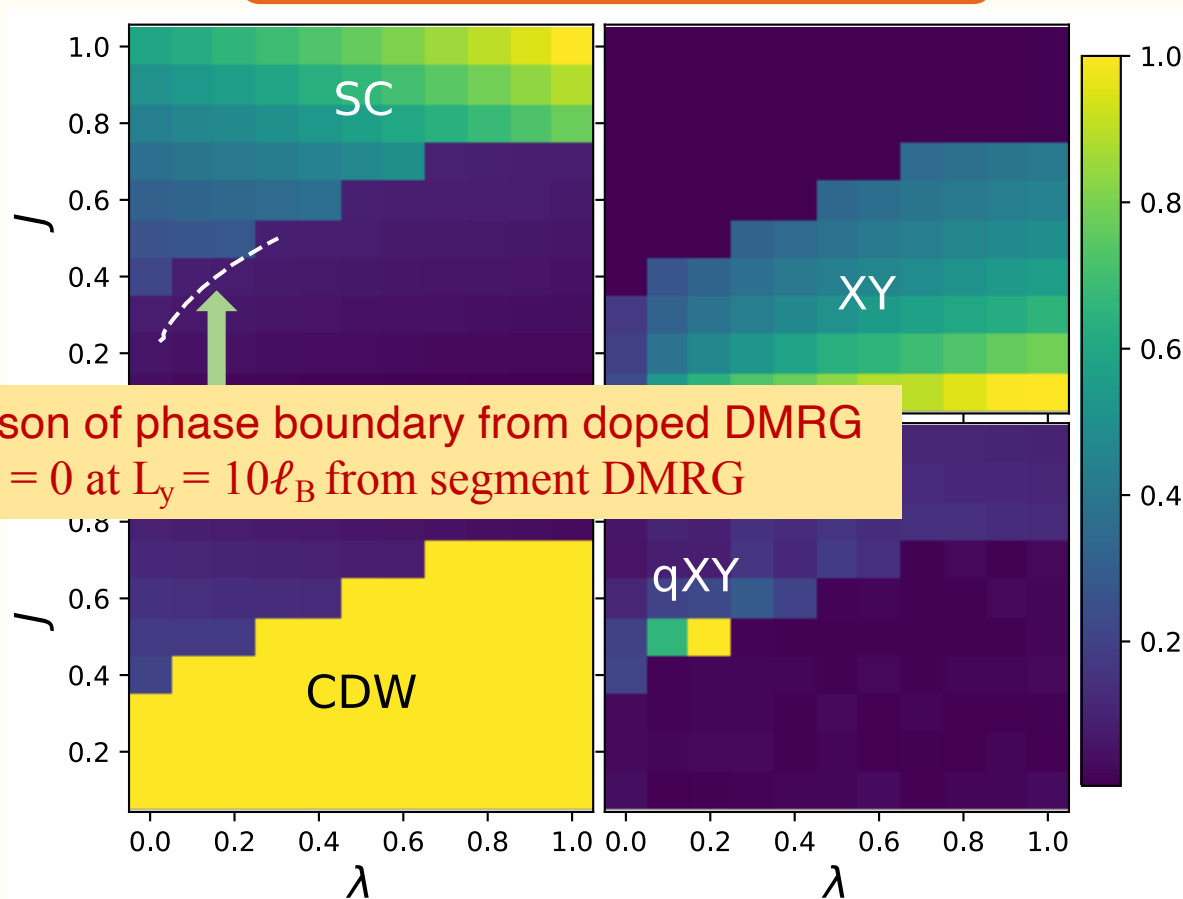
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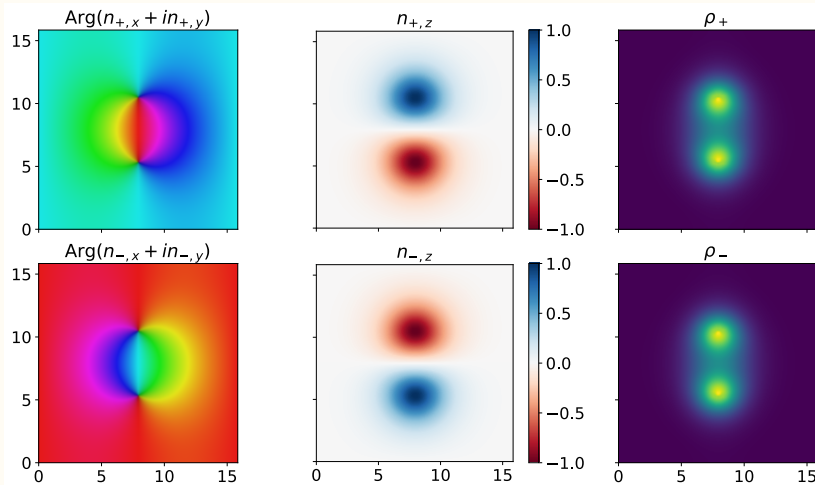
Phase diagram at doping $2 + 1/4$



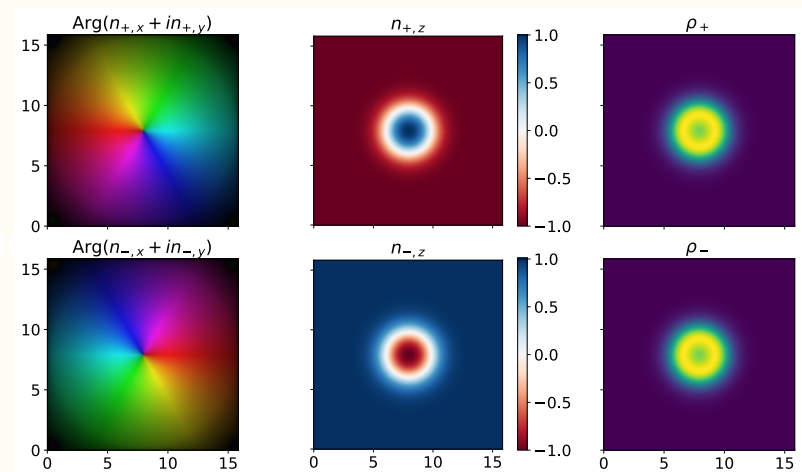
Comparison of phase boundary from doped DMRG with $\Delta_{\text{pair}} = 0$ at $L_y = 10\ell_B$ from segment DMRG

Evidence for skyrmion-pairing

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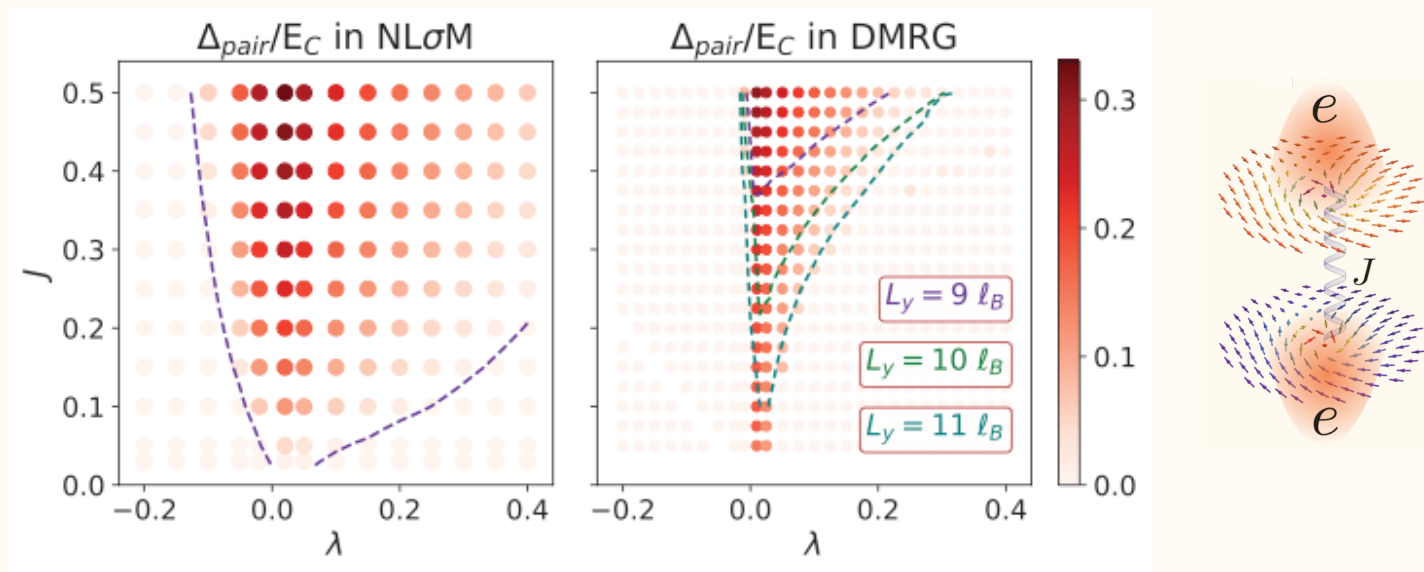
Easy plane: Charges deform into topologically equivalent meron pairs



Easy axis: Charge density remains radially symmetric, incurs larger Coulomb penalty

Evidence for skyrmion-pairing

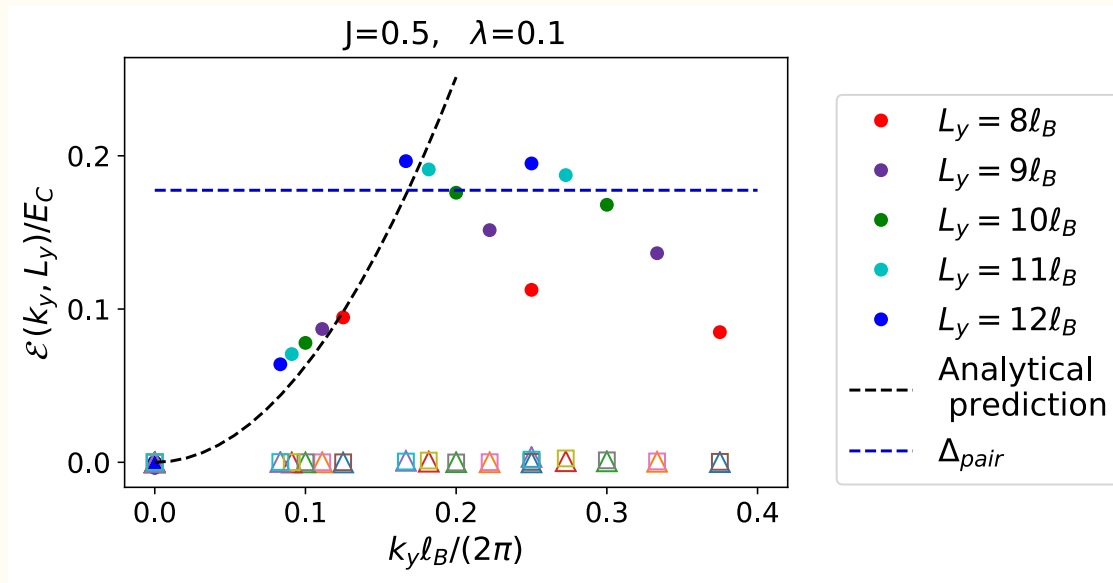
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- Quantum zero-point fluctuations $\propto J$ raise the energy of $2e$ skyrmions

Evidence for skyrmion-pairing

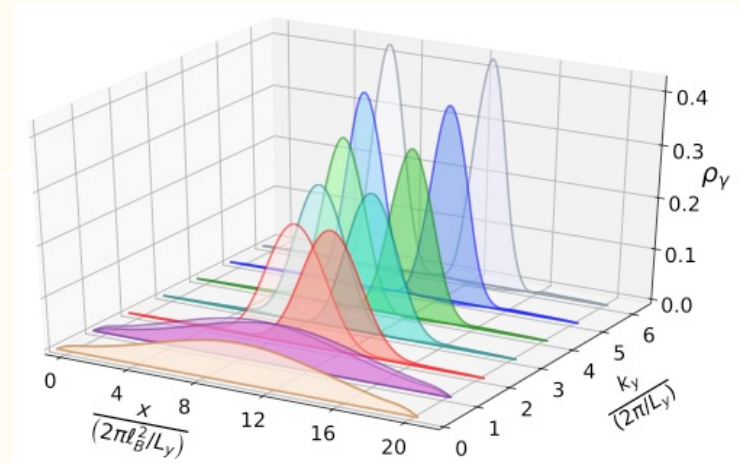
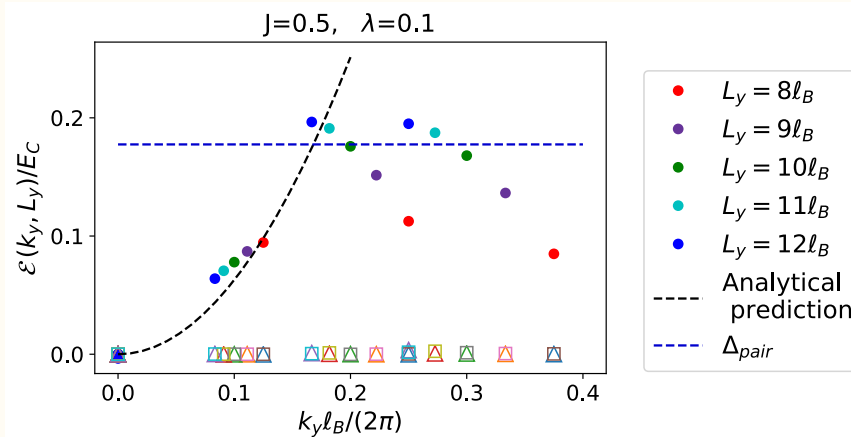
- As $\lambda \rightarrow 0$, effective mass $M_{2e} \propto J^{-1}$ as expected from semiclassical study
- At larger momenta k_y , charges from opposite layers get separated, with Chern resolved dipole moment $\propto k_y$: paying AF exchange penalty



Zaletel *et al*, PRB (2018)

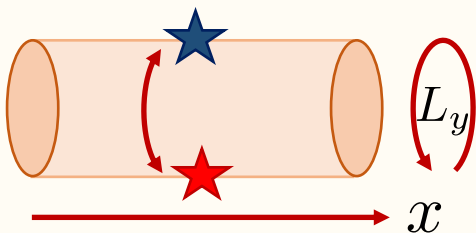
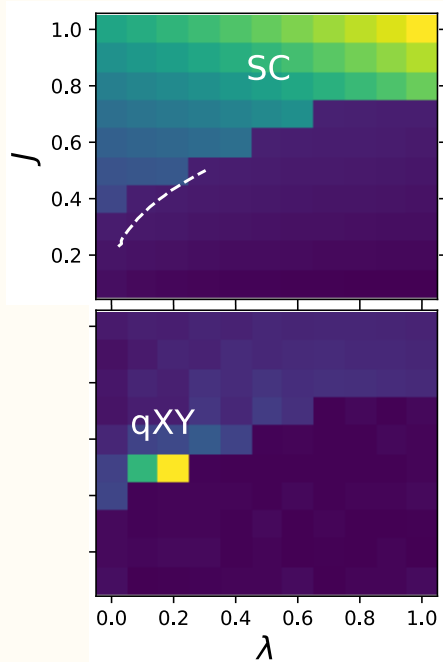
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- Again, numerics for quantum system confirm classical expectations!

Evidence for skyrmion-pairing



- When XY-AF order coexists with SC, it has finite momenta q_* tied to doping
- Phase θ of XY order jumps by π every-time it crosses $2e$ charge (on top of insulator)
- Easy-plane anisotropy: $2e$ skyrmion deforms into meron-antimeron pair
- To avoid Coulomb repulsion, meron and anti-meron lie half-way apart on the cylinder

$$\theta = \arg [\sinh(2\pi(z - z_0)/2L_y) \sinh(2\pi(\bar{z} - \bar{z}_1)/2L_y)]$$

$$\Delta\theta = \theta(x = \infty, y) - \theta(x = -\infty, y) = 2\pi(y_1 - y_0)/L_y$$

- Further evidence of skyrmion mechanism!

Conclusions and Outlook

- Numerically established skyrmion-antiskyrmion pair condensation as a viable mechanism for superconductivity
- Band topology plays a crucial role (not seen in bands with same C)
- MATBG has the right physical ingredients to realize this mechanism: required band topology and low iso-spin stiffness ~ 1 meV (perhaps mirror symmetric MATLG too?)
- Open questions --- Effects of:
 1. Non-uniform Berry curvature
 2. Disorder
 3. Spin-orbit coupling

Saito *et al*, arXiv:2008:10830

Park *et al*, arXiv:2012.01434

Hao *et al*, arXiv:2012.01434

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Thank you for your attention!

