Skyrmion superconductivity: Application to moiré graphene platforms and DMRG evidence

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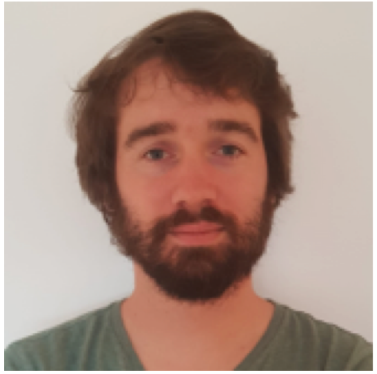
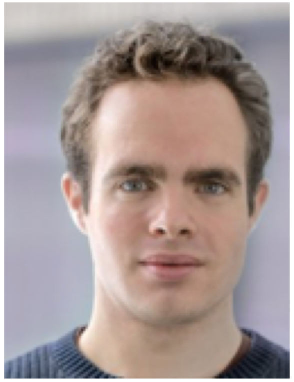


Nick Bultinck
UC Berkeley → Oxford



Mike Zaletel
UC Berkeley

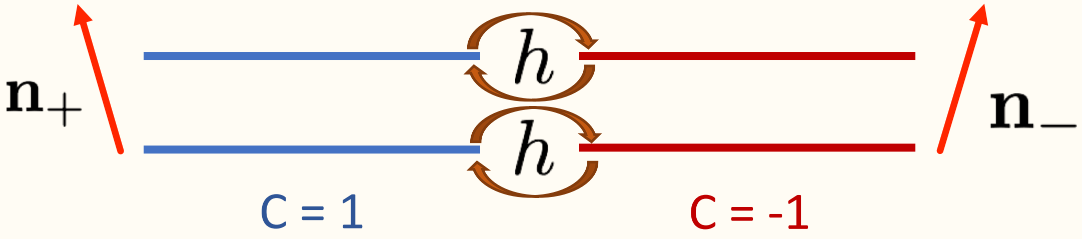
SC, N. Bultinck, M. P. Zaletel, PRB **101**, 165141 (2020)

E. Khalaf, SC, N. Bultinck, M. P. Zaletel, A. Vishwanath, Sci. Adv. **7**, eabf5299 (2021)

SC, M. Ippoliti, M. P. Zaletel, arXiv:2010.01144

Setting: Coupled spinful Chern bands

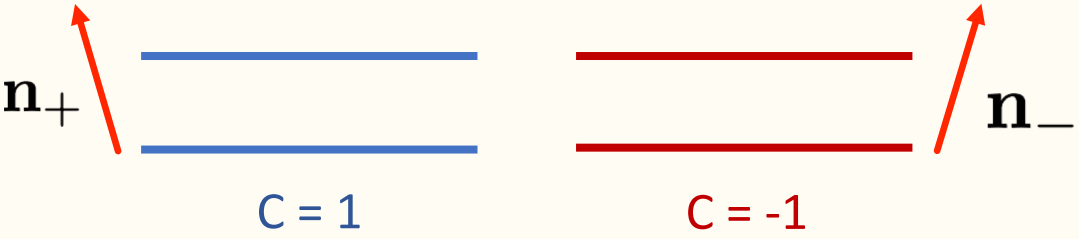
- Setting: Interacting electrons in tunnel-coupled (nearly flat) spin-ful Chern bands with opposite Chern numbers



- Consider a half-filled state (2/4 bands)

Setting: Coupled spinful Chern bands

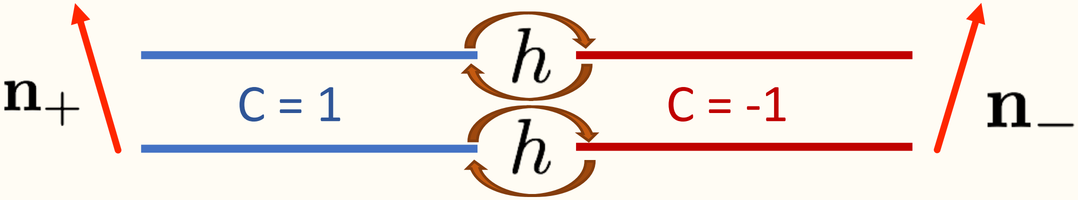
- Setting: Interacting electrons in tunnel-coupled (nearly flat) spin-ful Chern bands with opposite Chern numbers



- Consider a half-filled state (2/4 bands)
- Without tunnel coupling, Coulomb repulsion leads to a ferromagnetic state in each Chern sector.
- Expected from the Stoner criteria in the flat band limit, $\nu(\varepsilon_F) \to \infty$

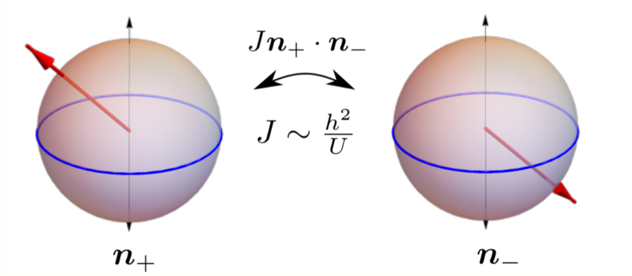
Setting: Coupled spinful Chern bands

- Setting: Interacting electrons in tunnel-coupled (nearly flat) spin-ful Chern bands with opposite Chern numbers



- In each Chern sector: Interaction driven quantum-Hall ferromagnet

- Tunnel-coupling leads to an antiferromagnetic (super-)exchange
- Ground state at half-filling (2/4 bands) is a spin-layer locked insulator



Bultinck *et al*, PRX (2020)
Repellin *et al*, PRL (2020)

Setting: Coupled spinful Chern bands

- Concrete Hamiltonian: Interacting electrons in tunnel-coupled zeroth Landau levels with opposite magnetic fields

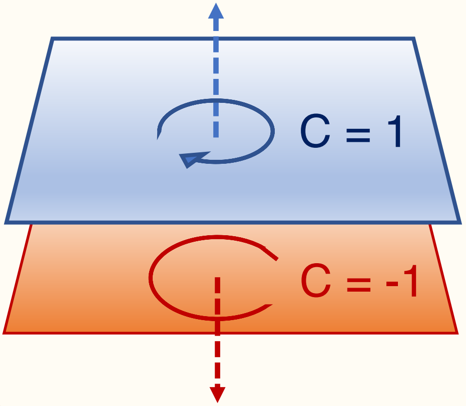
$$H = \psi^\dagger \frac{(\mathbf{p} + e\gamma^z \mathbf{A})^2}{2m}\psi + \frac{1}{2}\int :n(r)V_C(r-r')n(r'): - E_C \ell_B^2 \sum_{i=x,y,z} J_i : \left(\psi^\dagger \gamma^z \eta^i \psi(r)\right)^2 :$$

Kinetic term

Coulomb repulsion

AF super-exchange

γ = layer, η = spin



$$J_x = J_y = J + \lambda, \; J_z = J - \lambda$$

Easy plane/easy axis anisotropy

Isotropic super-exchange

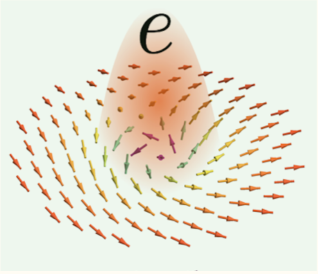
2/4 filling: AF insulator, preserves T′ = i γ^x η^y K

Expect robust pairing of fermions related by Kramers T′

Setting: Coupled spinful Chern bands

- In addition to particle-hole excitations, have topological textures: skyrmions in each Chern sector/*layer* carry charge

$$\boxed{Q_{physical} = CQ_{topological}}$$



Sondhi *et al*, PRL (1993)
Moon *et al*, PRB (1994)
Parameswaran *et al*, PRB (2012)

1. Assuming the charge e skyrmions are energetically relevant (low spin-stiffness) – can they bind together into 2e pairs?

2. Can these 2e pairs give rise to superconductivity on doping the half-filled insulator?

3. If there is superconductivity, what is T_c for the BKT transition?

Setting: Coupled spinful Chern bands

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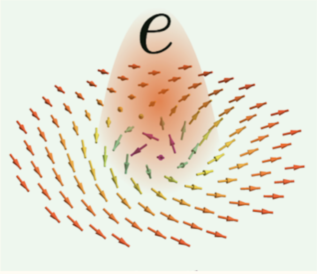

Sondhi *et al*, PRL (1993)
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3. If there is superconductivity, what is T_c for the BKT transition?

Appeal to semi-classics

SC, N. Bultinck, M. Zaletel, PRB 2020, E. Khalaf, SC *et al*, Sci. Adv. (2021)

Setting: Coupled spinful Chern bands

- Can these 2e pairs give rise to superconductivity on doping the half-filled insulator?

- Superconductivity from 2e skyrmion condensation has been proposed in doped QSH insulators, and seen in sign-problem free Quantum Monte Carlo

Abanov and Weigeman, PRL (2001)
Grover and Senthil, PRL (2008)
Christos *et al*, PNAS (2020)
Khalaf *et al*, arXiv:2012.05915
Wang *et al*, arXiv: 2006.13239

- What is the phase diagram at T = 0 in presence of Coulomb repulsion?

- Can we rule out Wigner crystals of 2e bosons?

Need alternate numerical methods: DMRG

SC, Ippoliti, Zaletel, arXiv:2010.01144

Skyrmion-pairing mechanism

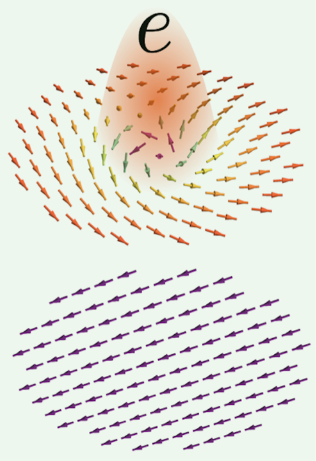
- Consider a skyrmion in one QH layer and an anti-skyrmion in the opposite layer

$$Q_{physical} = C Q_{topological}$$

- Both carry same charge: Repelled by Coulomb but attracted by local antiferromagnetism J

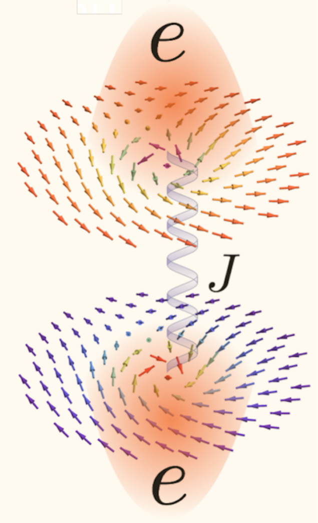
- *All electronic pairing mechanism* without phonons/retardation/bosonic fluctuations

SC, N. Bultinck, M. Zaletel, PRB 2020
E. Khalaf, SC *et al*, Sci. Adv. (2021)



Single skyrmion pays exchange penalty

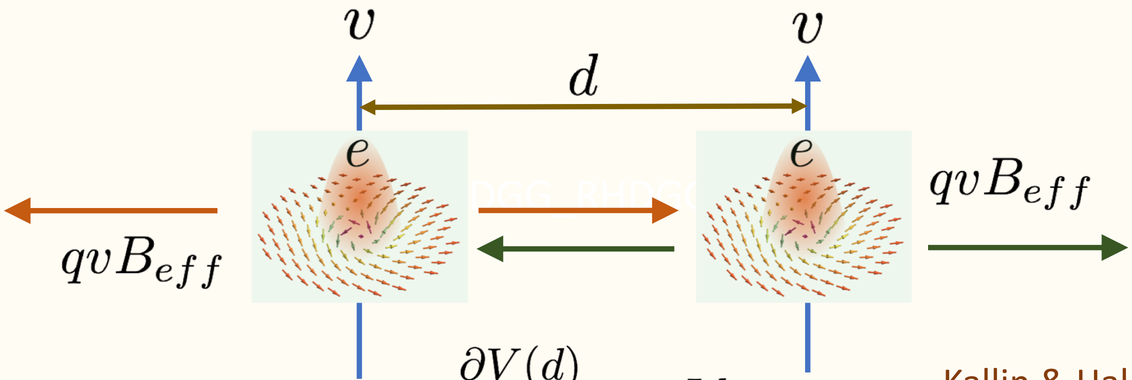
Belavin, Polyakov , JETP (1975)
Sondhi *et al*, PRL (1993)
Moon *et al*, PRB (1994)



Sk-Ask pair can spread out to minimize Coulomb without losing exchange

Skyrmion-pairing mechanism

- For charge e textures, kinetic energy quenched by magnetic field
- Charge 2e skyrmion with charge e in each layer sees *no net magnetic field*, can therefore be mobile



$$\frac{\partial V(d)}{\partial d} = aJd$$

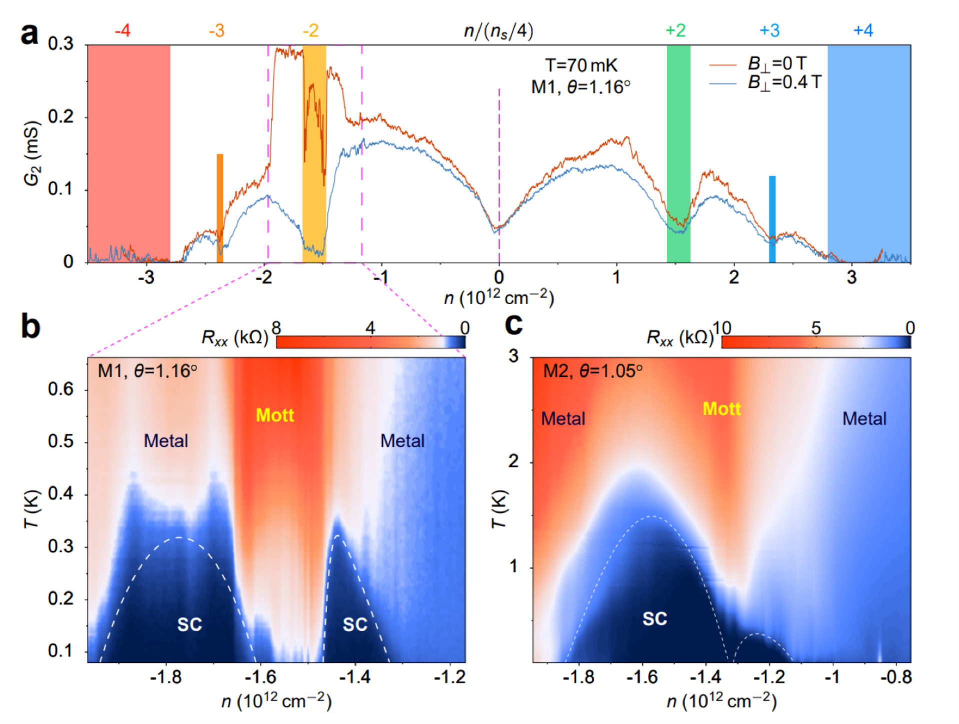
Kallin & Halperin, PRB (1984)
K. Yang, PRL (2001)

$$E = V_0 + aJd^2/2 = V_0 + \frac{(qB_{eff})^2}{2aJ}v^2$$

$T_c \sim 1/M_{pair} \sim J \sim 1$ K in MAG

E. Khalaf, SC *et al*, Sci. Adv. (2021)

Application to moire graphene: MATBG



Experiments:

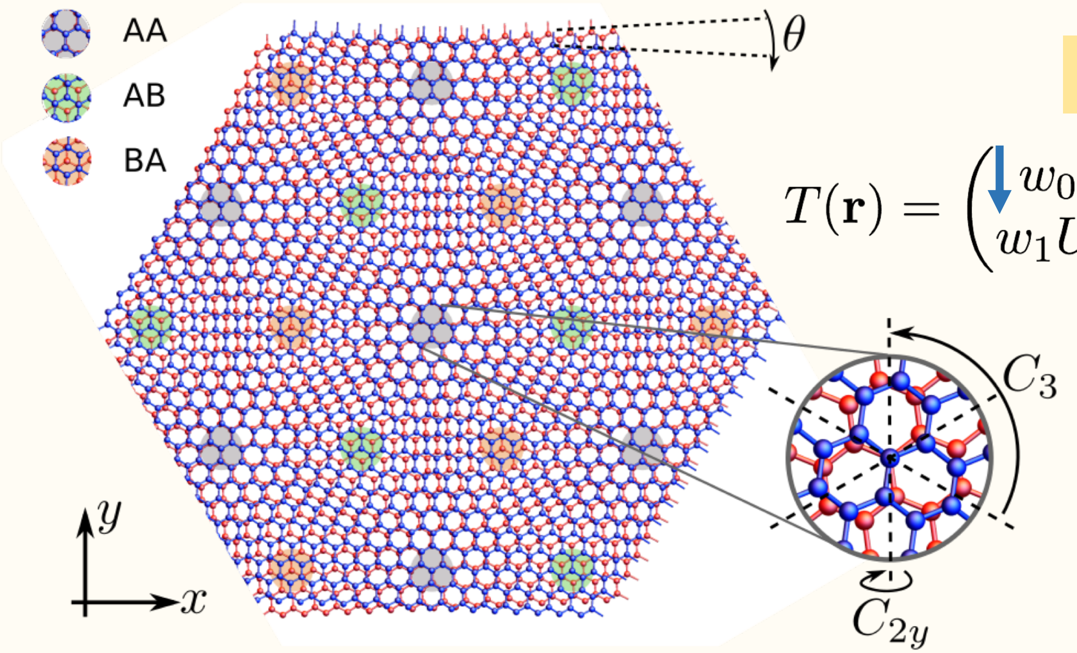
Cao *et al,* Nature (2018), Lu *et al,* Nature (2019), Yankowitz *et al*, Science (2019), Several others…

Theory:

Po *et al* , Yuan *et al*, Isobe *et al*, Zou *et al*, Kang *et al*, Lewandowski *et al,* lots of others…

Application to moire graphene: MATBG

- Approach from a quantum Hall perspective: the chiral limit



$$T(\mathbf{r}) = \begin{array}{c} \begin{matrix} \quad A_1 & \qquad\qquad B_1 \end{matrix} \\ \begin{pmatrix} \downarrow w_0U_0(\mathbf{r}) & w_1U_1(\mathbf{r}) \\ w_1U^*(-\mathbf{r}) & \downarrow w_0U_0(\mathbf{r}) \end{pmatrix} \begin{matrix} A_2 \\ B_2 \end{matrix} \end{array}$$

w_0/w_1 goes down due to lattice relaxation

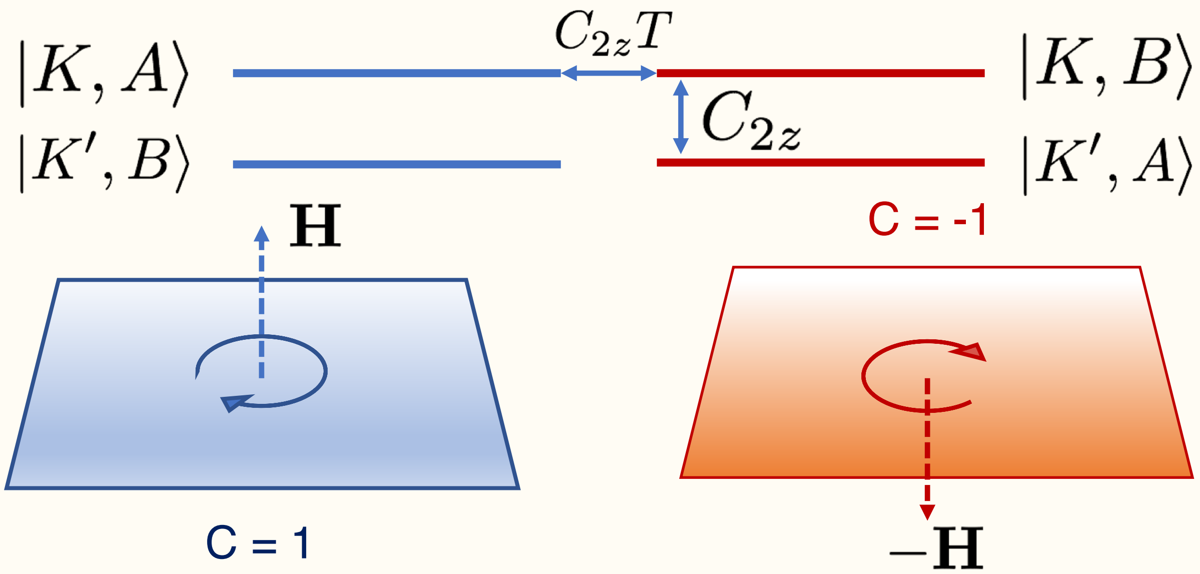
Bistritzer, MacDonald PNAS (2011),

Application to moire graphene: MATBG

- Chiral limit (turn off $w_0 =$ AA hopping between layers): Additional chiral symmetry allows for sublattice and valley polarized basis

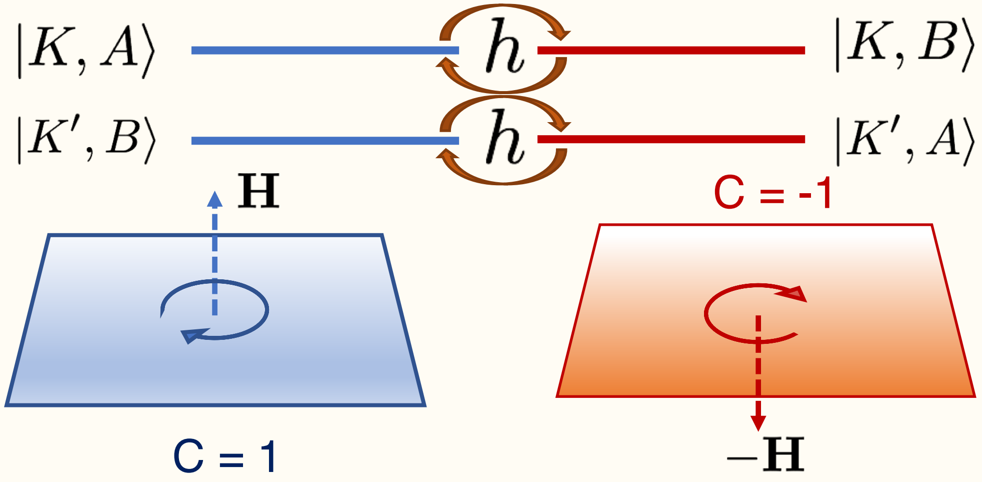
Jose et al, PRL (2012), Tarnopolsky *et al*, PRL (2019), J. Liu *et al,* PRB (2019)

- Exactly flat Chern bands: each band behaves like a lowest Landau level, but different bands see opposite effective magnetic fields



Application to moire graphene: MATBG

- Adding dispersion introduces AF super-exchange between Chern sectors (breaks U(2) × U(2) to U(2))

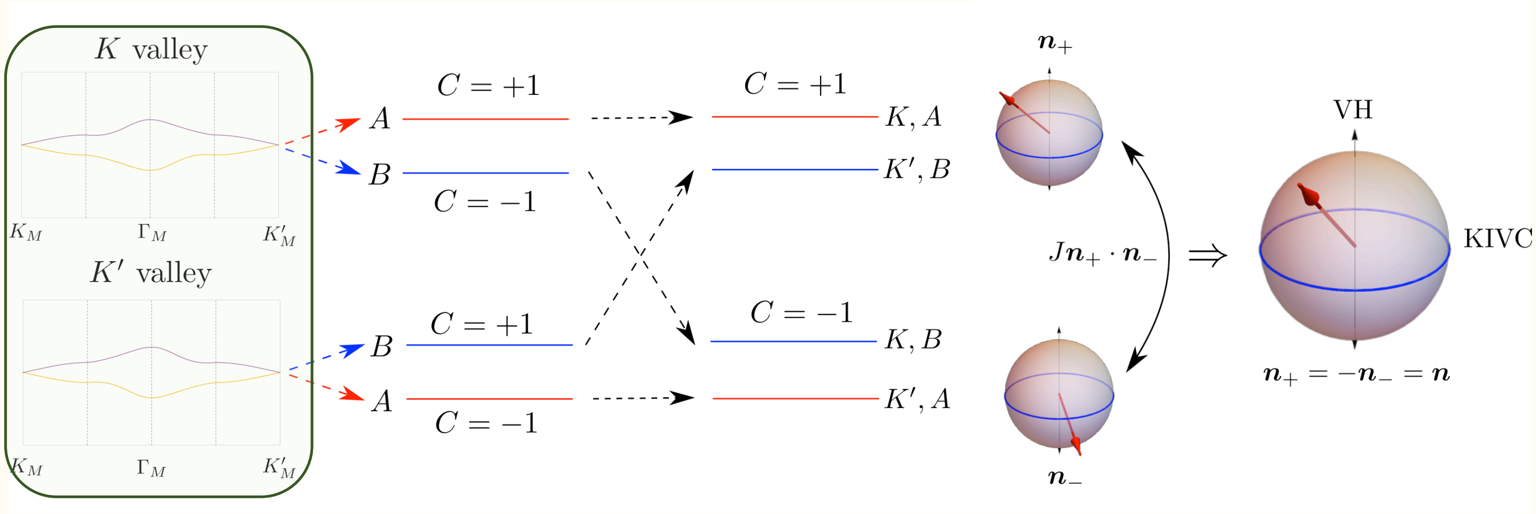


- Effective Hamiltonian resembles weakly dispersive iso-spinful Chern bands with antiferromagnetic exchange between opposite Chern sectors

Bultinck *et al*, PRX (2020), Kang & Vafek, PRL (2020)
Lian, Bernevig et al, arXiv:2009:13530

Application to moire graphene: MATBG

Schematic overview



MAG flat bands

Chiral limit

Chern bands

FM in each Chern sector

Dispersion ➛ AF superexchange

Ground state: easy-plane AF or KIVC

Bultinck *et al*, PRX (2020), Kang & Vafek, PRL (2020)
Lian, Bernevig et al, arXiv:2009:13530

Application to moire graphene: MATBG

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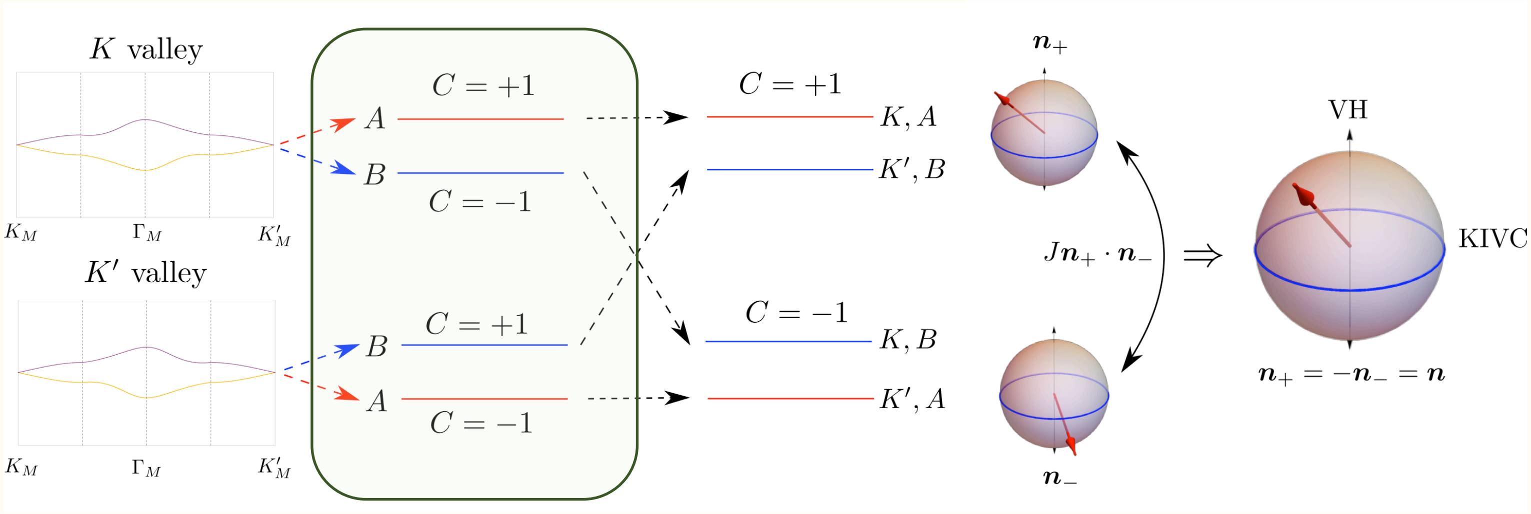

MAG flat bands → Chiral limit

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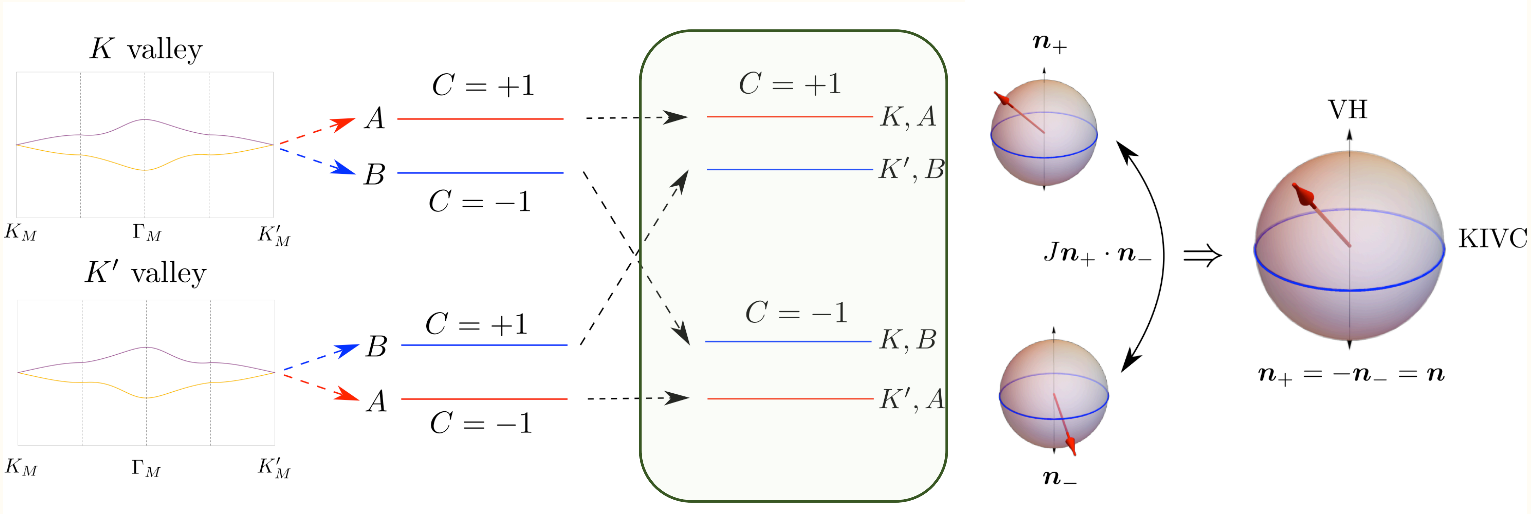

MAG flat bands

Chiral limit → Chern bands

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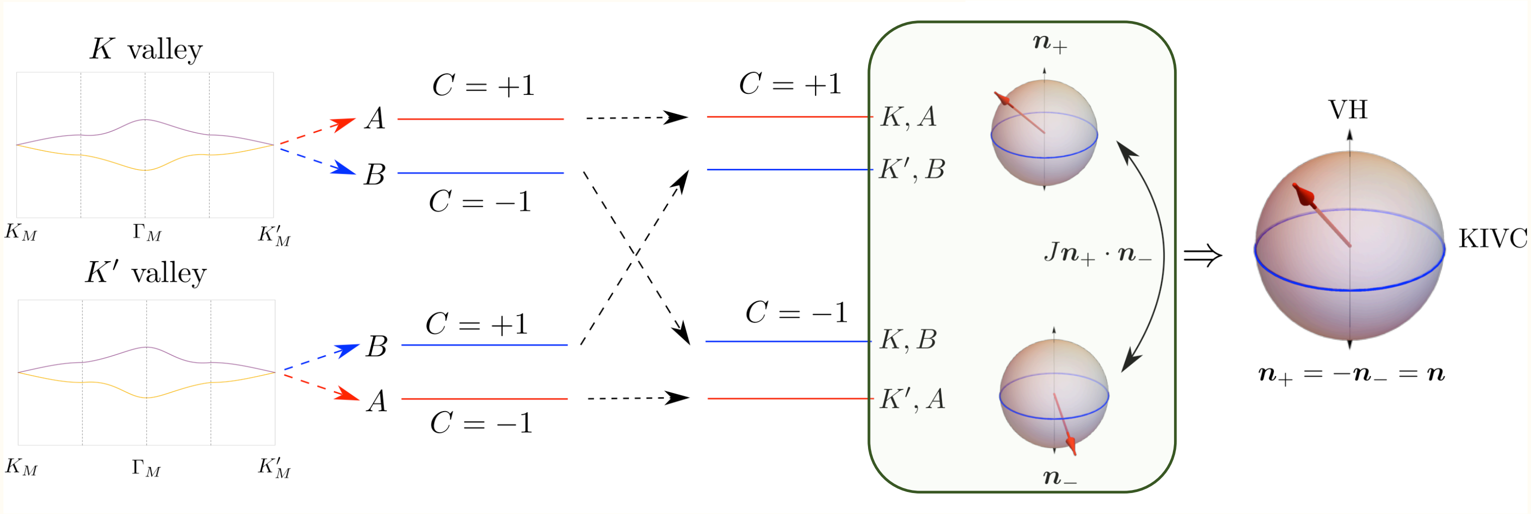

MAG flat bands

Chiral limit

Chern bands → FM in each Chern sector

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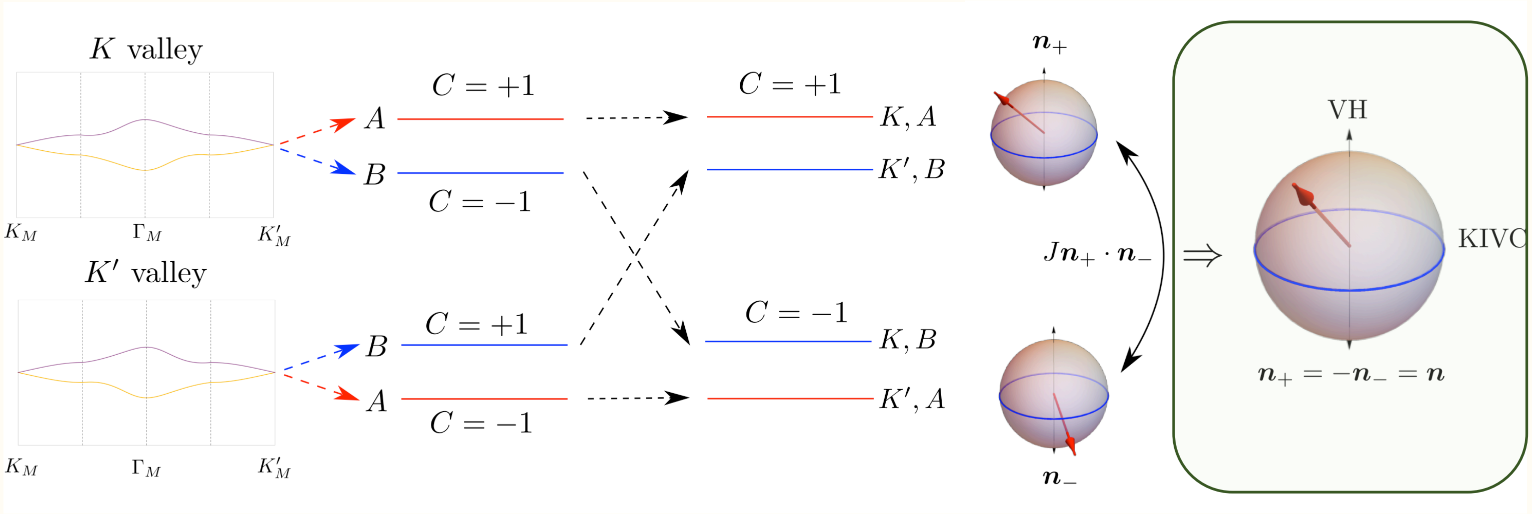


MAG flat bands

Chiral limit

Chern bands

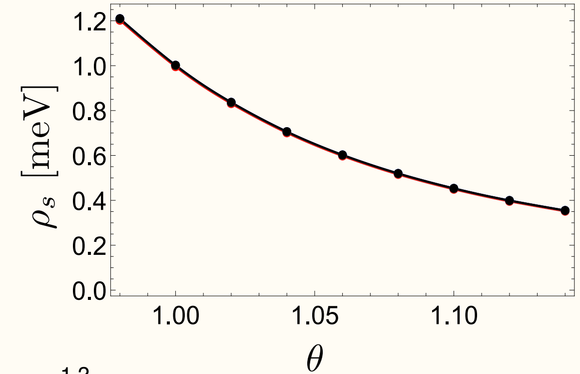
FM in each Chern sector → Dispersion → AF superexchange

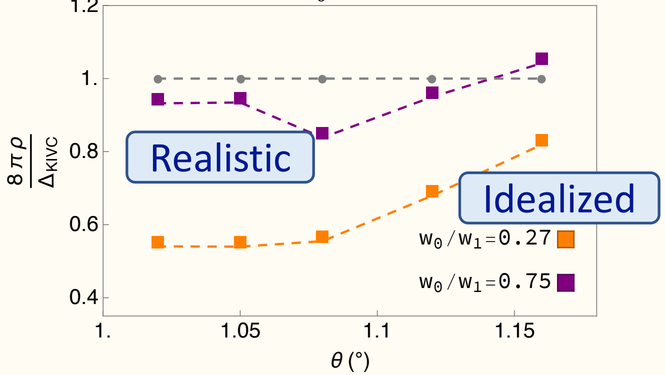
Ground state: easy-plane AF or KIVC

Bultinck *et al*, PRX (2020), Kang & Vafek, PRL (2020)
Lian, Bernevig et al, arXiv:2009:13530

Application to moire graphene: MATBG

- Established requisite band topology, but how about energetics?
- Need stiffness to be small



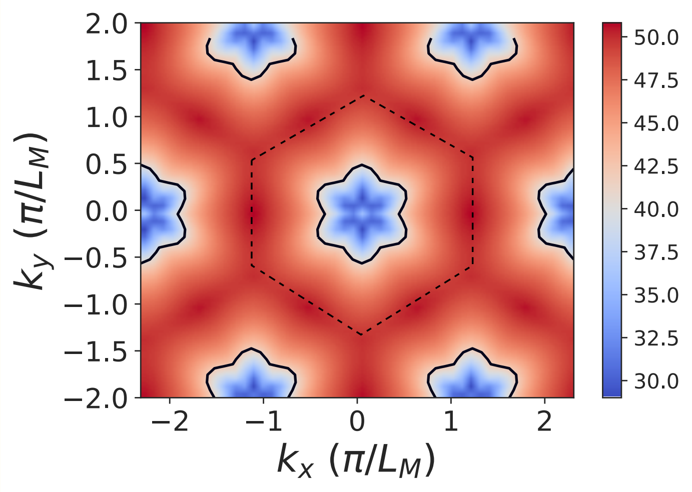


Realistic

Idealized

Δ_{KIVC} = gap from HF numerics

Small critical doping beyond which charges enter as 2e skyrmions

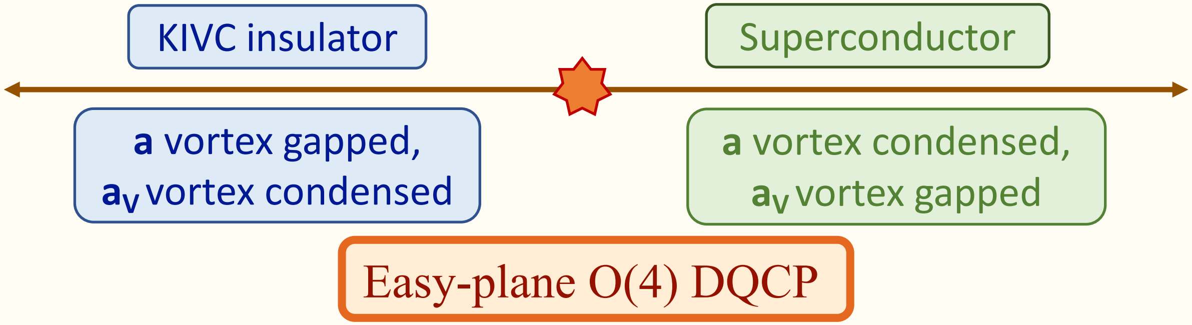


E. Khalaf, SC *et al*, Sci. Adv. (2021)

Detour: Doped phase diagram

- CP^1 formulation in the AF manifold: $\mathbf{n} = (n_1, n_2, n_3) = z^\dagger \boldsymbol{\eta}\, z$
- Introduces gauge field a_μ (phase of spinor z), with charge density $\propto$ curl($\mathbf{a}$)
- Each flux (vortex) of **a** carries $U(1)_c$ charge 2e
- We can write $\Delta_{SC} = n_4 + i\, n_5$
- Alternate CP^1 formulation: $\mathbf{m} = (n_3, n_4, n_5) = w^\dagger \boldsymbol{\eta}\, w$
- Emergent gauge field $\mathbf{a_V}$ carries valley-charge $U(1)_v$

Senthil *et al*, Science. (2004)
Senthil *et al*, PRB (2004)
Wang, Nahum *et al*, PRX (2017)



KIVC insulator

Superconductor

a vortex gapped,
a$_V$ vortex condensed

a vortex condensed,
a$_V$ vortex gapped

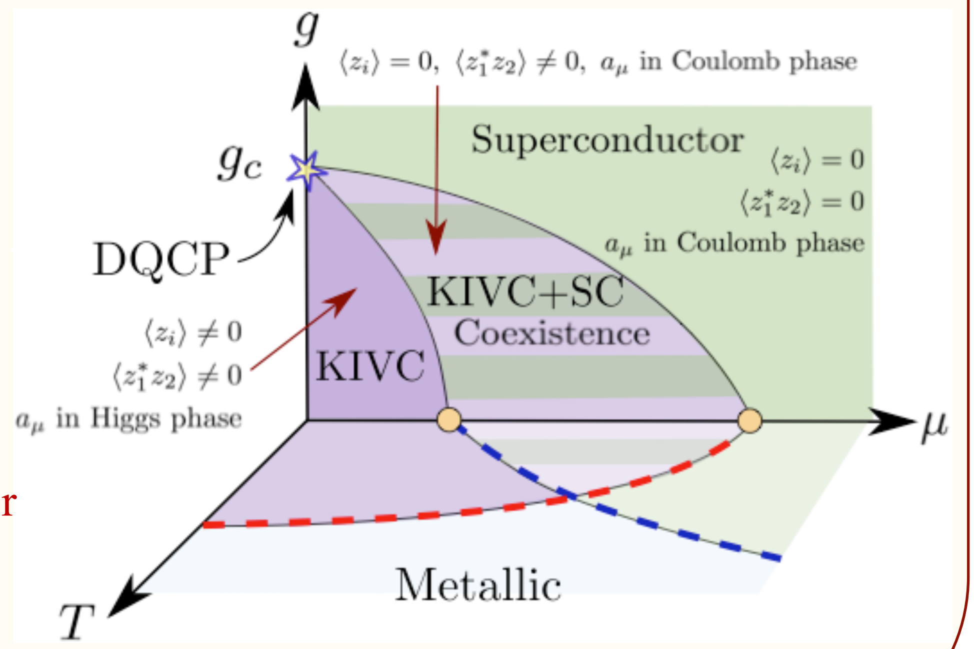
Easy-plane O(4) DQCP

Detour: Doped phase diagram

- KIVC insulator: z_i condensed, $z_1^* z_2$ condensed, vortices in a_μ (charges) disallowed
- Chemical potential acts act magnetic field, nucleates vortices in **n** (2e skyrmions): coexistence: a_μ in Coulomb phase, but $z_1^* z_2$ condensed (relative phase well-defined)
- Further raising μ: expect disappearance of KIVC
- Large N theory gives first order transition with

$$T_c = \frac{3JA_M\nu}{\pi N}$$

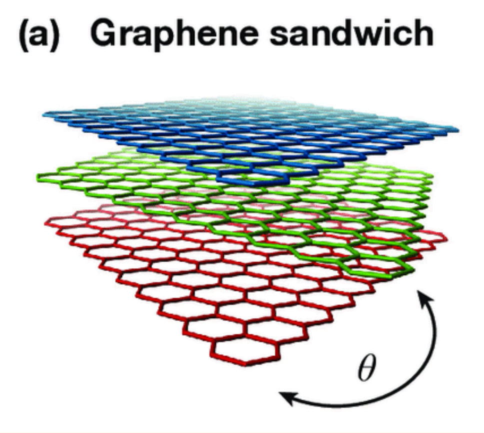
Doped phase diagram

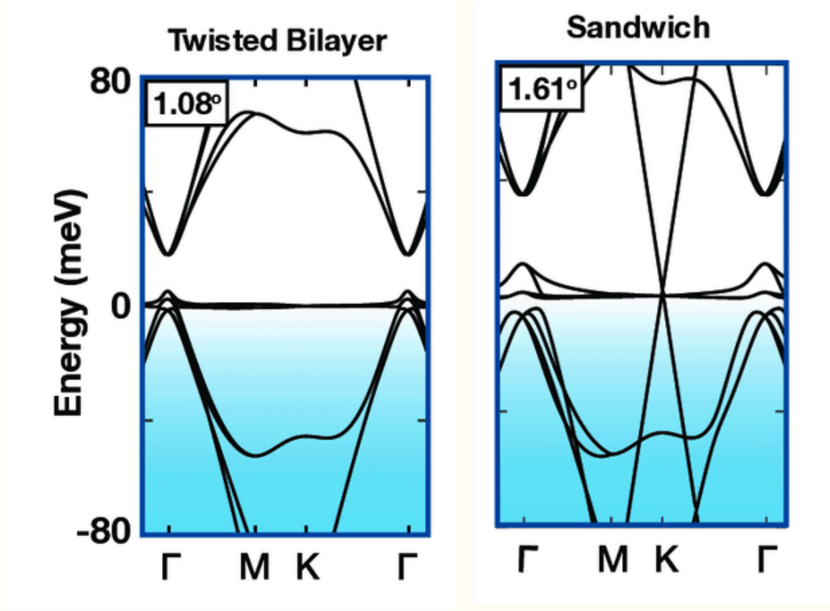


E. Khalaf, SC *et al*, Sci. Adv. (2021)

Application to moire graphene: MATLG

- Alternating angle twisted trilayer graphene = TBG flat bands + highly dispersive Dirac cone (like monolayer graphene)



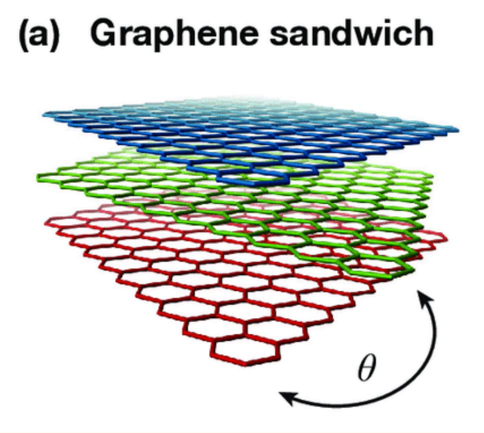


Khalaf *et al*, PRB (2019).
Carr et al, Nano Letters (2020)

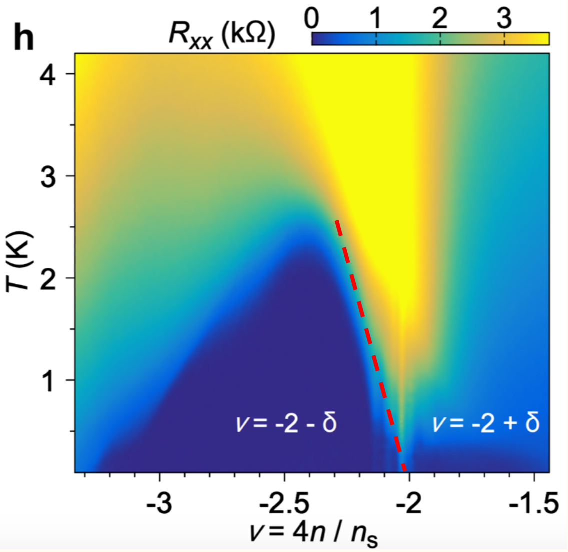
$$\boxed{\theta^{\mathrm{M}}_{\mathrm{TLG}} = \sqrt{2}\,\theta^{\mathrm{M}}_{\mathrm{TBG}}}$$

Application to moire graphene: MATLG

- Recently, robust superconductivity has been observed in MATLG, with multiple evidences in favor of a strong-coupling origin



Khalaf *et al*, PRB (2019).
Carr et al, Nano Letters (2020)



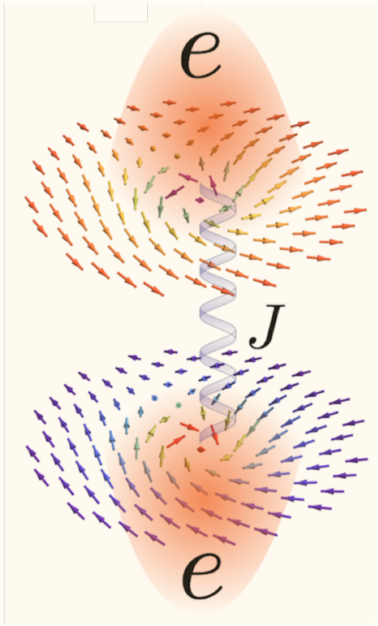
Park *et al*, Nature (2021)
Hao *et al*, Science (2021)

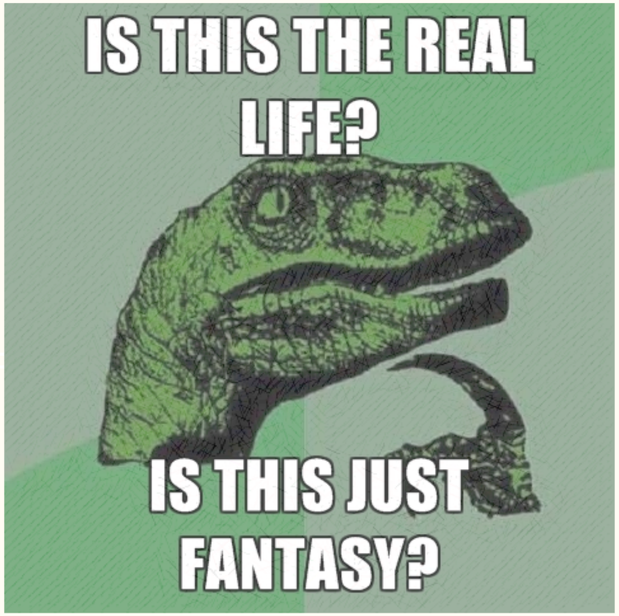
- T_c appears to be proportional to doping ν

Khalaf, SC *et al*, arXiv:2004.00638

Skyrmion-pairing mechanism

Skyrmion-pairing superconductivity



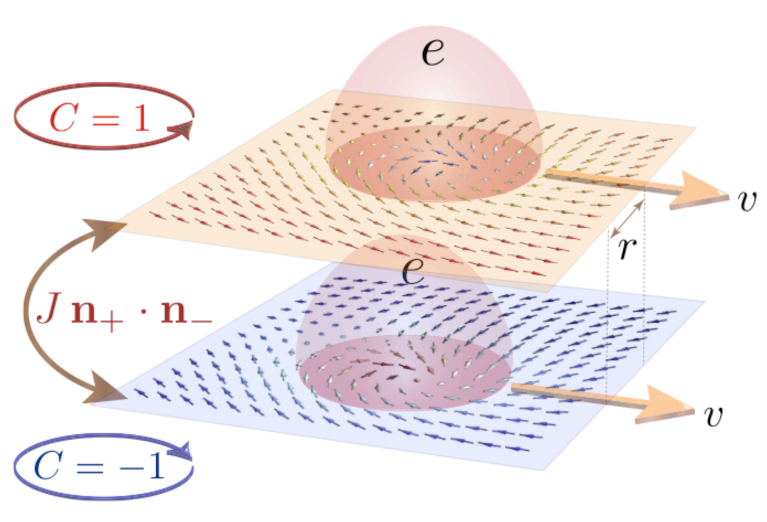


Quote: Queen, Figure credits:
http://creatememe.chucklesnetwork.com/memes/16712

DMRG: Model and phase diagram

- Essential ingredients:

1. Spinful (nearly) flat bands with opposite Chern number ±1
2. AF interaction between the Chern sectors, in addition to Coulomb repulsion



Related work:
Kang, Vafek, PRB (2020)
Soejima, Parker *et al*, PRB (2020)
Eugenio, Dag, arXiv: 2004.10363

- Test: AF couple spinful lowest Landau levels, amenable to DMRG

Zaletel *et al*, PRL (2013)

DMRG: Model and phase diagram

- iDMRG for coupled Landau level model on a cylinder ($L_y = 8\text{-}12\ \ell_B$)

Ippoliti *et al*, PRB (2018)

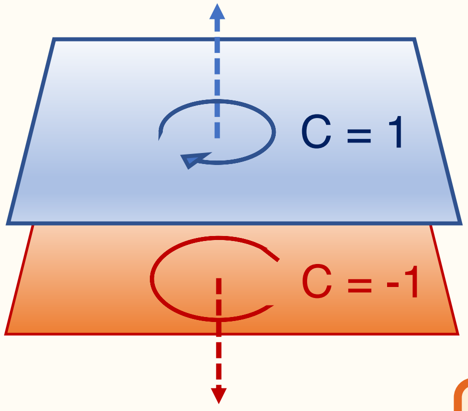
$$H = \psi^\dagger \frac{(\mathbf{p} + e\gamma^z \mathbf{A})^2}{2m} \psi + \frac{1}{2} \int : n(r) V_C(r - r') n(r') : - E_C \ell_B^2 \sum_{i=x,y,z} J_i : \left(\psi^\dagger \gamma^z \eta^i \psi(r)\right)^2 :$$

Kinetic term

Coulomb repulsion

AF super-exchange

γ = layer, η = spin



$$J_x = J_y = J + \lambda,\ J_z = J - \lambda$$

Easy plane/easy axis anisotropy

Isotropic super-exchange

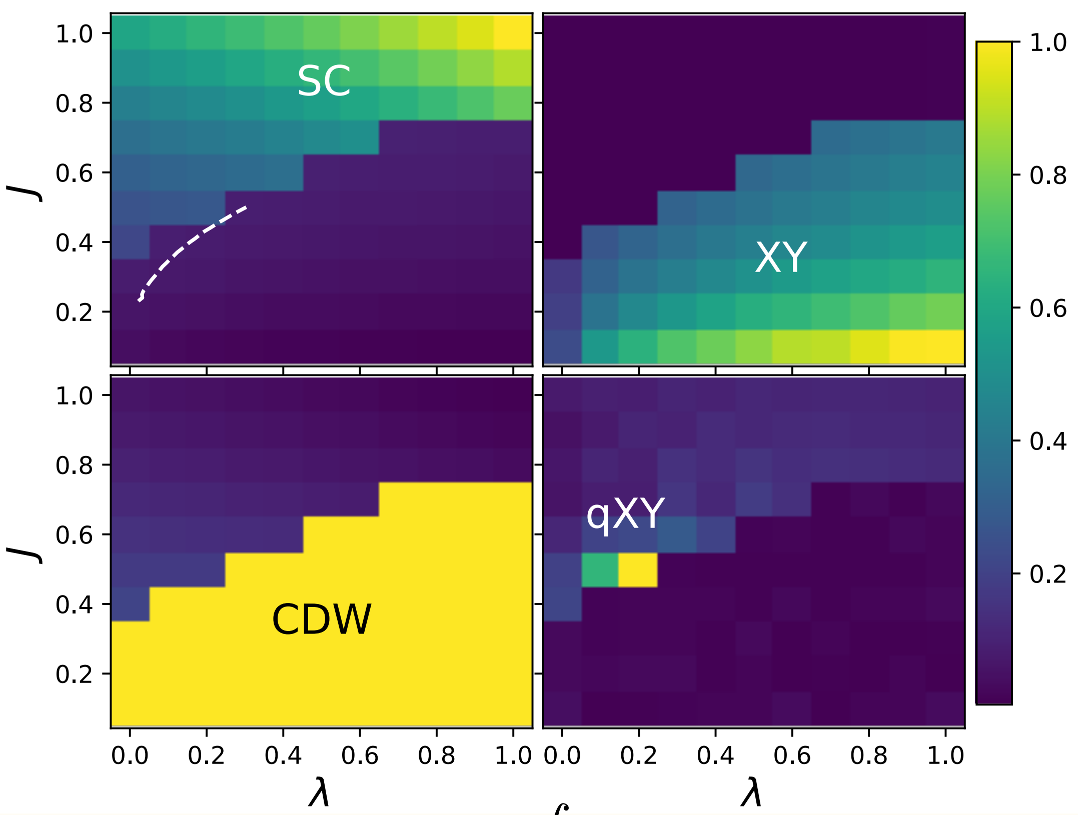
$$V_{+\uparrow,-\downarrow}(r) = V_C(r) - 2JE_C\ell_B^2\ \delta(\mathbf{r})$$

Smeared by LLL projection

Purely repulsive model for $J < 3.24$ ($d_s = 3\ell_B$)

DMRG: Model and phase diagram

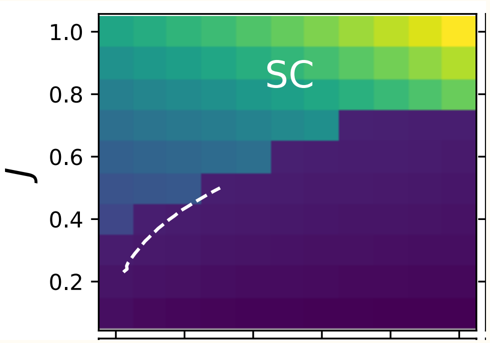
Phase diagram at doping 2 + 1/4



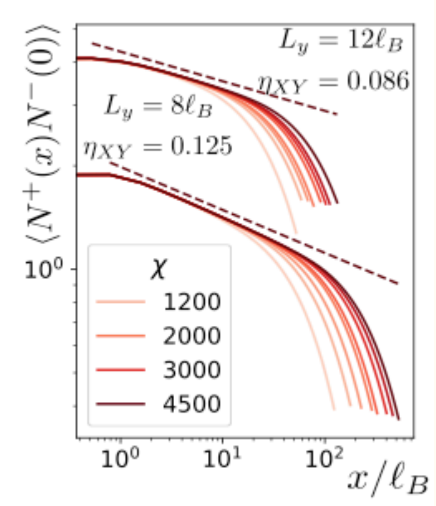
$$S_{XY/SC}(\mathbf{q}=0) = \int d^2r \langle O^\dagger(\mathbf{r}) O(\mathbf{r}) \rangle$$

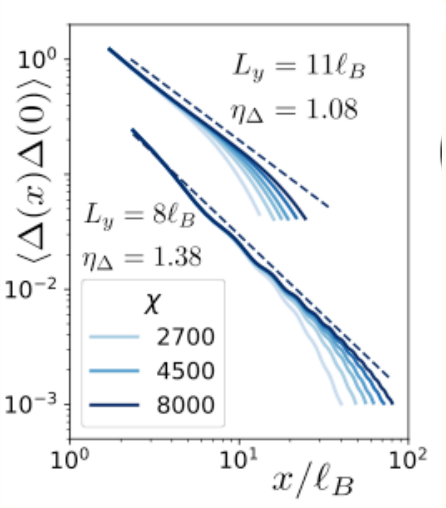
DMRG: Model and phase diagram

Phase diagram at doping 2 + 1/4



- Superconductor at large J (layer-unpolarized) – Kramers-pairing ($T' = i\,\gamma^x \eta^y K$)
- Single particle excitations have gap ~ E_C





- Algebraic decay of Kramers-pair correlations

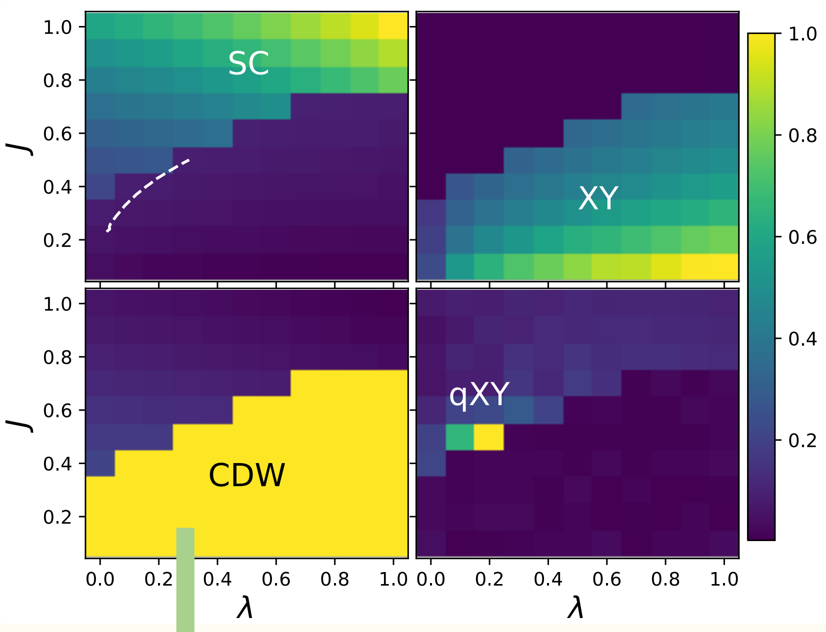
$$\langle \Delta^\dagger(x,0)\Delta(0,0)\rangle \propto x^{-\eta_{SC}}$$

$$\eta_{SC} \propto L_y^{-1}$$

- Scaling analysis shows true long range SC order in 2d limit ($L_y \to \infty$)

DMRG: Model and phase diagram

Phase diagram at doping 2 + 1/4



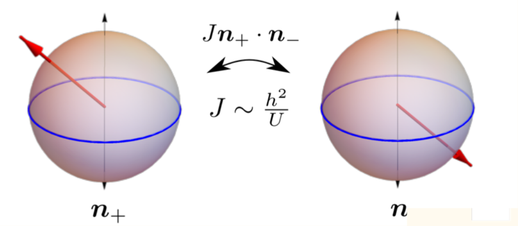
CDW: stripy correlations

- Coexisting XY-AF and CDW at small J (layer-polarized)
- Transition (first order) between CDW and SC as J is increased
- Small region of coexistence of SC and XY-AF order at finite q_* (tied to the doping)
- The competing state is layer-polarized, but depends on the filling (CDW at 2+1/4, CFL at 2+1/2, IQHE at 2+1)

Evidence for skyrmion-pairing

What is the mechanism of SC? Are skyrmions relevant?

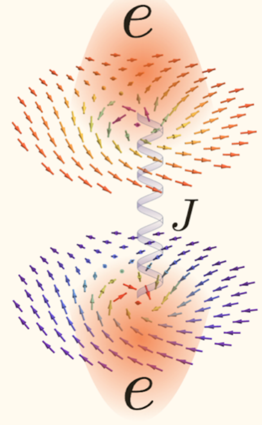
Intuition from NLSM:
Yes, for small anisotropy



$$\mathcal{L} = \int_r \sum_\gamma \left[\frac{1}{2A_M}\mathcal{A}_\gamma\cdot\partial_\tau \mathbf{n}_\gamma + \frac{g}{2}(\nabla\mathbf{n}_\gamma)^2 + \mathbf{A}_\mu\cdot\mathbf{j}^\mu_\gamma\right]$$

$$+\frac{1}{2}\int_{r,r'}\sum_{\gamma,\gamma'}\rho_\gamma(r)V_C(r-r')\rho_{\gamma'}(r') - \bar{J}^i\int_r(\mathbf{n}^i_+ - \mathbf{n}^i_-)^2$$

$$\mathbf{j}^\mu_\pm = \pm\frac{e}{8\pi}\epsilon^{\mu\nu\rho}\mathbf{n}_\pm\cdot(\partial_\nu\mathbf{n}_\pm\times\partial_\rho\mathbf{n}_\pm)$$

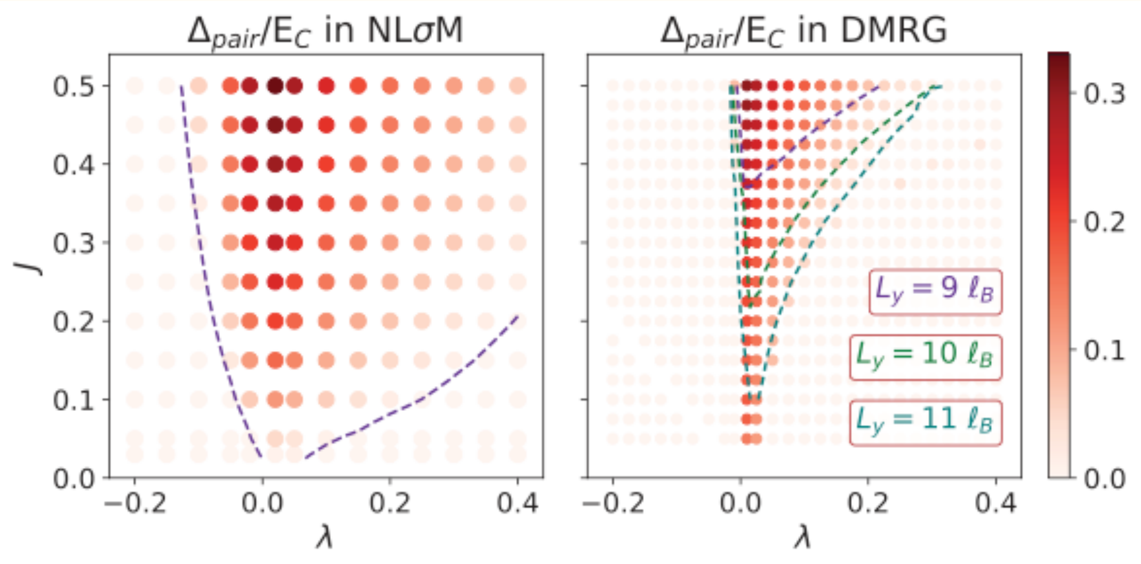


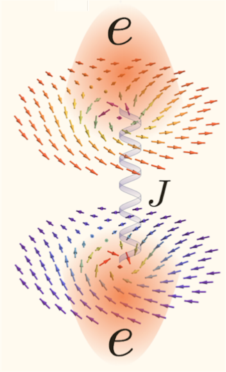
Both NLSM and DMRG give energy of charged excitations above insulator

Evidence for skyrmion-pairing

- NLSM + Segment DMRG to determine energy of charged excitations

$$\boxed{\Delta_{pair} = 2\ E_{1e} - E_{2e}}$$

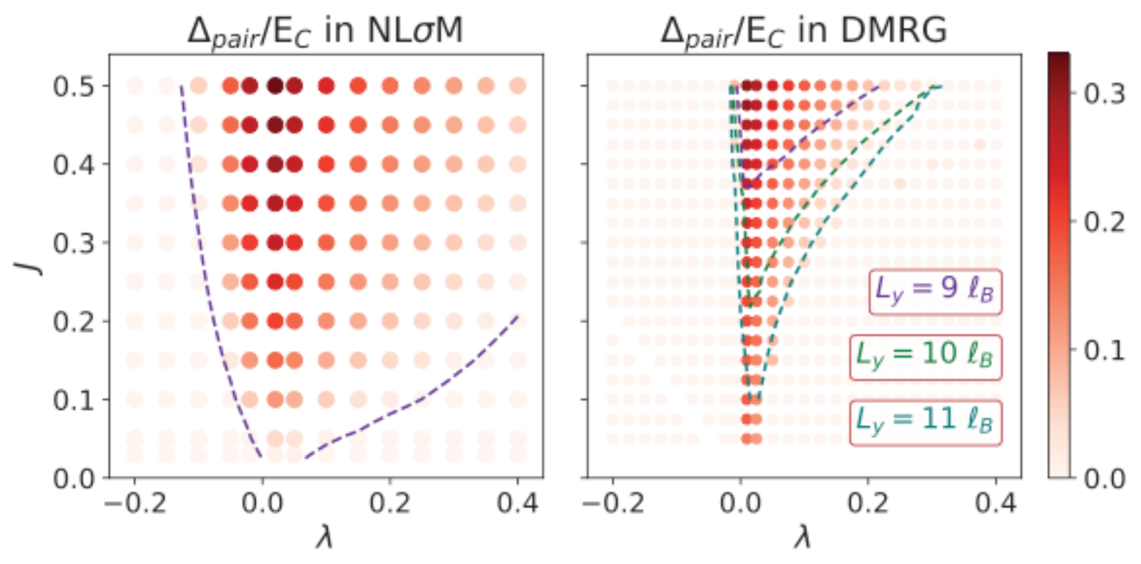


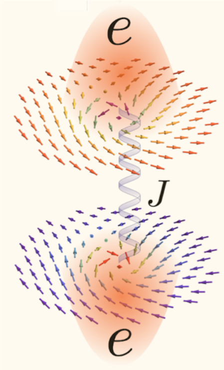


- Numerics for quantum system confirm classical expectations!

Evidence for skyrmion-pairing

- Critical $J_*(\lambda) \to 0$ as $\lambda \to 0$, indicative of collective pairing mechanism
- Pairing is much more favorable in the easy plane case (good for MAG!)





Typical RMS radius: 3 ℓ_B

- Good qualitative agreement between quantum and classical numerics

Evidence for skyrmion-pairing

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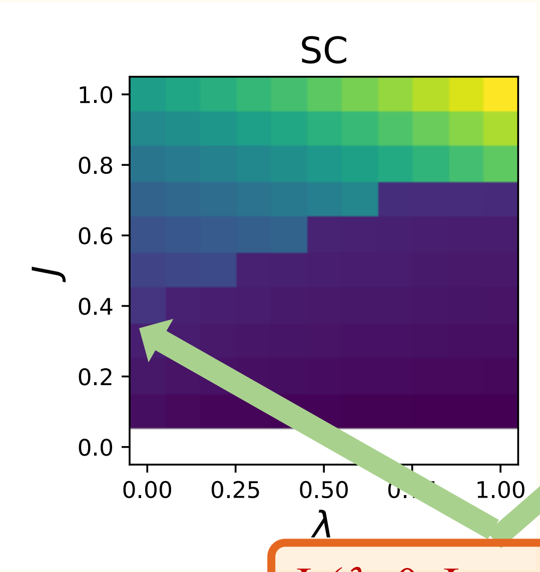
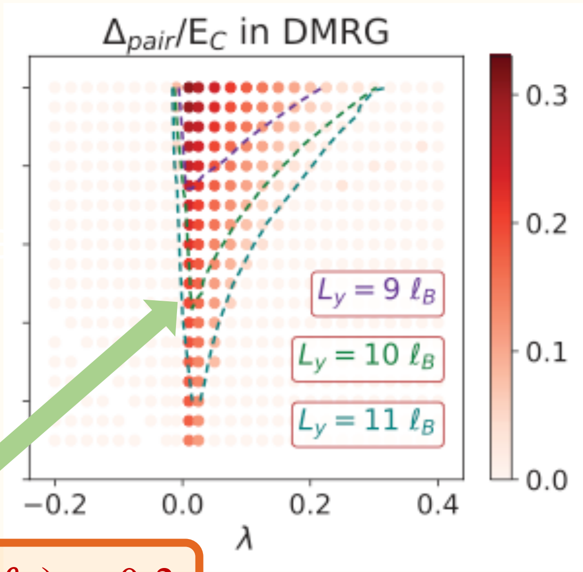
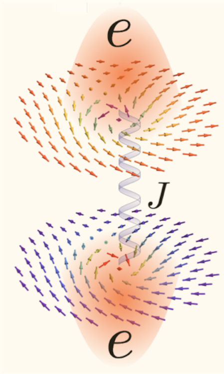




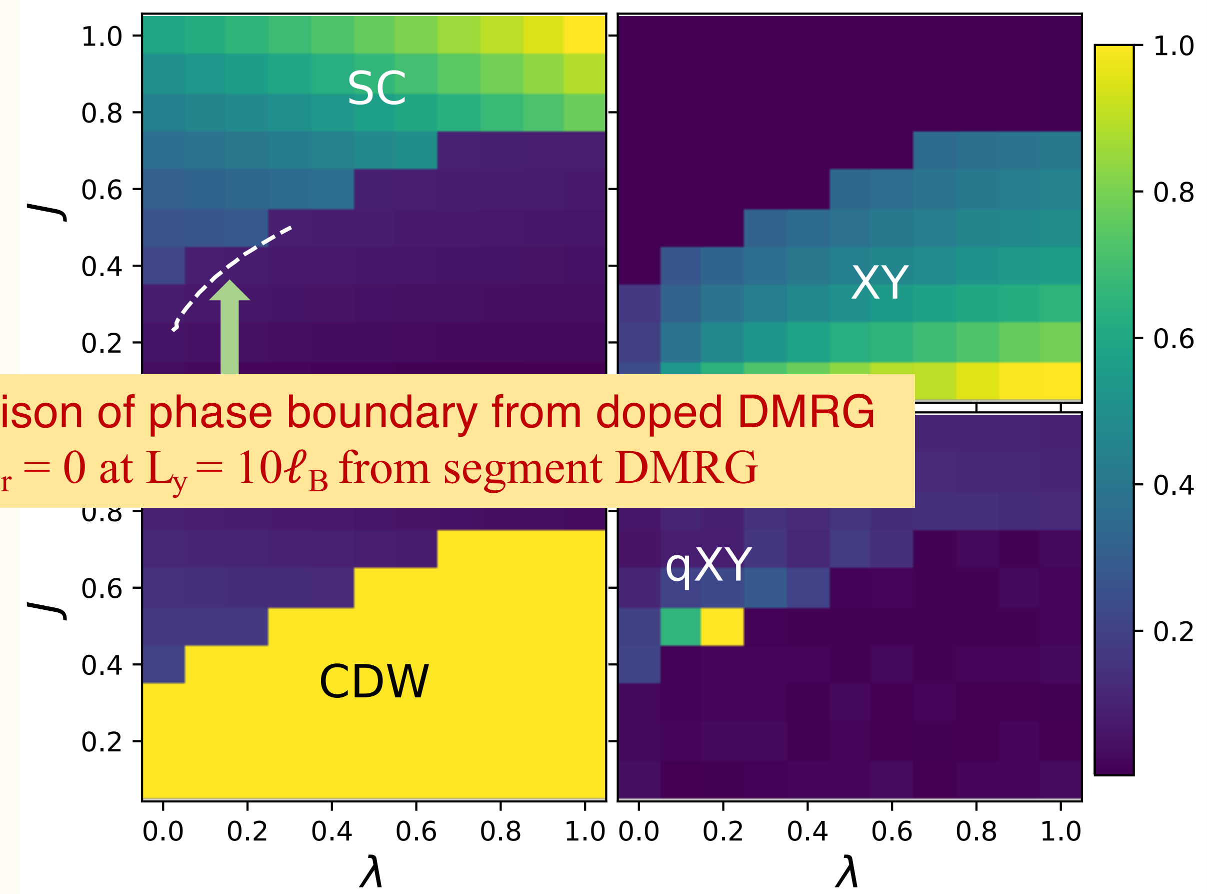


$J_*(\lambda=0,\ L_y = 10\ell_B) \cong 0.3$

- Good qualitative agreement between quantum and classical numerics

Evidence for skyrmion-pairing

Phase diagram at doping 2 + 1/4



Comparison of phase boundary from doped DMRG with $\Delta_{\text{pair}} = 0$ at $L_y = 10\ell_B$ from segment DMRG

Conclusions and Outlook

- Numerically established skyrmion-antiskyrmion pair condensation as a viable mechanism for superconductivity

- Band topology plays a crucial role (not seen in bands with same C in a *control* experiment)

- MATBG has the right physical ingredients to realize this mechanism: required band topology and low iso-spin stiffness ~ 1 meV, perhaps mirror symmetric MATLG too

Saito *et al*, Nature (2021)
Park *et al*, Nature (2021)
Hao *et al*, Science (2021)

- Open questions --- Effects of:

1. Non-uniform Berry curvature
2. Disorder
3. Spin-orbit coupling

Arora *et al*, Nature (2020)

Thank you for your attention!