DMRG evidence for superconductivity via skyrmion-condensation: Application to magic angle graphene

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In collaboration with:

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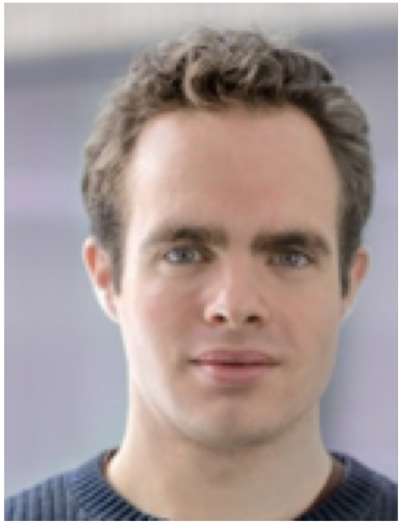


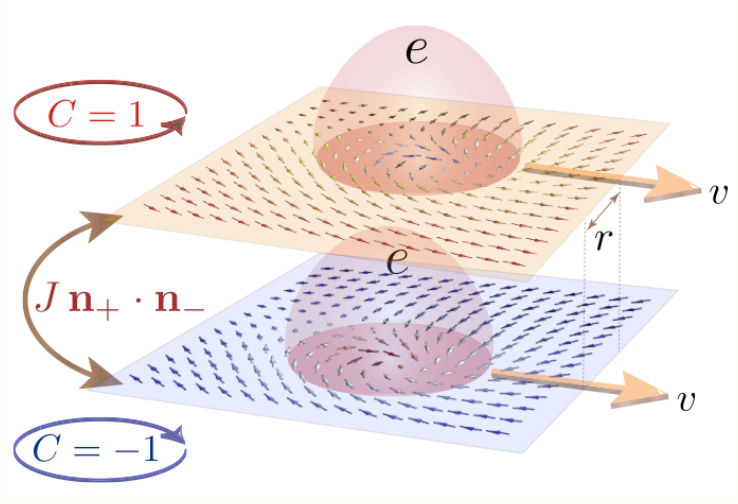
Mike Zaletel
UC Berkeley

SC, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Skyrmion superconductivity: Essential ingredients

- Essential ingredients:

1. Spinful (nearly) flat bands with opposite Chern number ±1
2. AF interaction between the Chern sectors, in addition to Coulomb repulsion



SC, N. Bultinck, M. Zaletel, PRB 2020

E. Khalaf, SC *et al*, arXiv:2004.00638 (to appear in Sci. Adv.)

- Test: AF couple spinful lowest Landau levels, amenable to DMRG

Zaletel *et al*, PRL (2013)

DMRG: Model and phase diagram

- iDMRG for coupled Landau level model on a cylinder ($L_y = 8\text{-}12\ \ell_B$)

Ippoliti *et al*, PRB (2018)

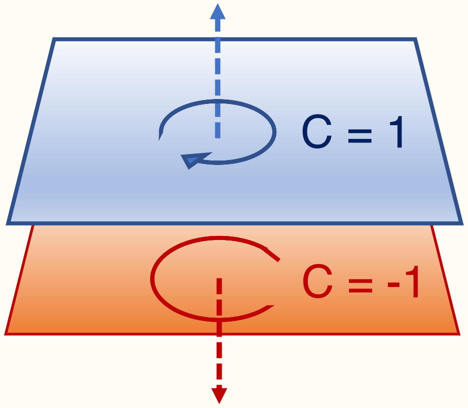
$$H = \psi^\dagger \frac{(\mathbf{p} + e\gamma^z \mathbf{A})^2}{2m} \psi + \frac{1}{2} \int : n(r) V_C(r - r') n(r') : - E_C \ell_B^2 \sum_{i=x,y,z} J_i : \left(\psi^\dagger \gamma^z \eta^i \psi(r)\right)^2 :$$

Kinetic term

Coulomb repulsion

AF super-exchange

γ = layer, η = spin



$$J_x = J_y = J + \lambda, J_z = J - \lambda$$

Easy plane/easy axis anisotropy

Isotropic super-exchange

Purely repulsive model for J < 3.24 ($d_s = 3\ell_B$)

Related work: Kang and Vafek, PRB (2020)
Soejima, Parker *et al*, PRB (2020)
Eugenio and Dag, arXiv: 2004.10363

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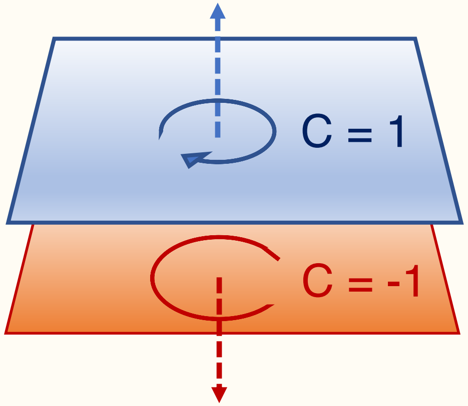

$J_x = J_y = J + \lambda,\ J_z = J - \lambda$ → Easy plane/easy axis anisotropy

↓ Isotropic super-exchange

2/4 filling: AF insulator, preserves $T' = i\,\gamma^x\,\eta^y\,K$

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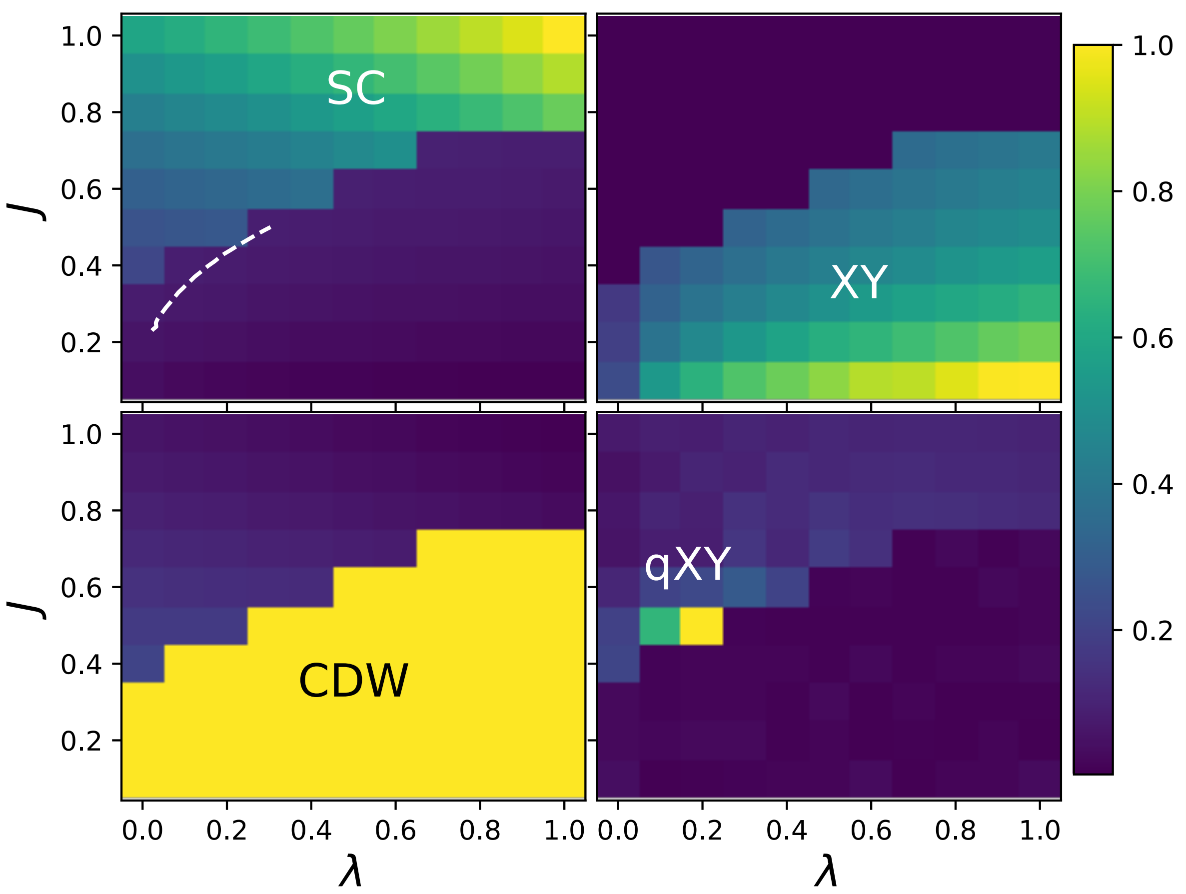
DMRG: Model and phase diagram

Phase diagram at doping 2 + 1/4



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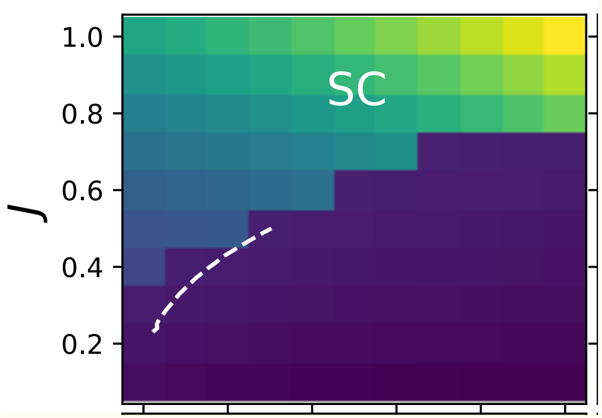
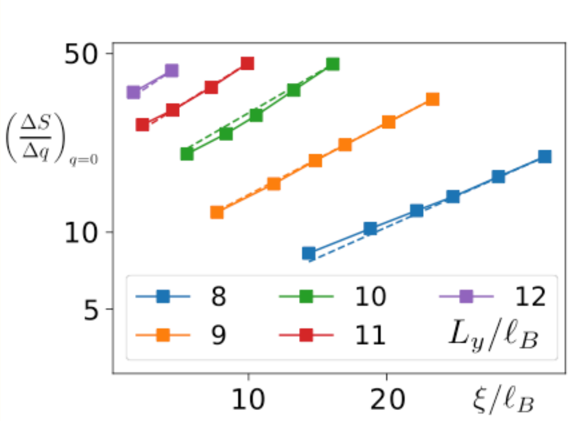




- Superconductivity at large J (layer-unpolarized)
- Single particle excitations have gap ~ E_C
- Algebraic decay of Kramers-pair correlations

$$\langle \Delta^\dagger(x,0)\Delta(0,0)\rangle \propto x^{-\eta_{SC}}$$

$$\eta_{SC} \propto L_y^{-1}$$

- Scaling analysis shows true long range SC order in 2d limit ($L_y \rightarrow \infty$)

Evidence for skyrmion-pairing

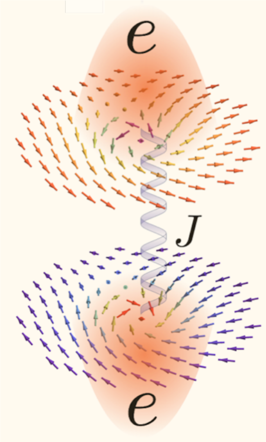
What is the mechanism of SC? Are skyrmions relevant?

- Provide 3 key pieces of evidence

1. Studying energetics of charged excitations above the insulating state, and comparing with classical NLSM of skyrmion-pairing

2. Looking at anisotropy effects on the energetics of 2e excitations, which matches with semiclassical picture of skyrmion-pairing

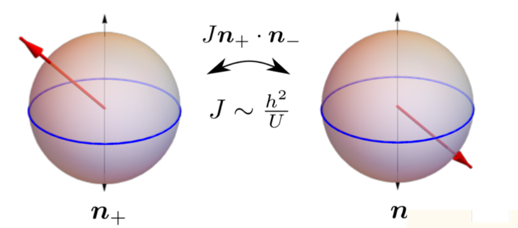
3. Estimating effective mass of 2e pairs, and comparing with analytical estimates



Evidence for skyrmion-pairing

Energetics of charged excitations

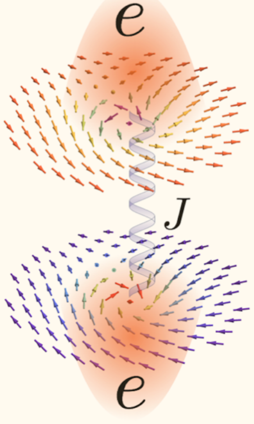
Can write an effective NLSM
in analogy with QHFM



$$\mathcal{L} = \int_r \sum_\gamma \left[\frac{1}{2A_M} \mathcal{A}_\gamma \cdot \partial_\tau \mathbf{n}_\gamma + \frac{g}{2} (\nabla \mathbf{n}_\gamma)^2 + \mathbf{A}_\mu \cdot \mathbf{j}^\mu_\gamma \right]$$

$$+ \frac{1}{2} \int_{r,r'} \sum_{\gamma,\gamma'} \rho_\gamma(r) V_C(r - r') \rho_{\gamma'}(r') - \bar{J}^i \int_r (\mathbf{n}^i_+ - \mathbf{n}^i_-)^2$$

$$\mathbf{j}^\mu_\pm = \pm \frac{e}{8\pi} \epsilon^{\mu\nu\rho} \mathbf{n}_\pm \cdot (\partial_\nu \mathbf{n}_\pm \times \partial_\rho \mathbf{n}_\pm)$$

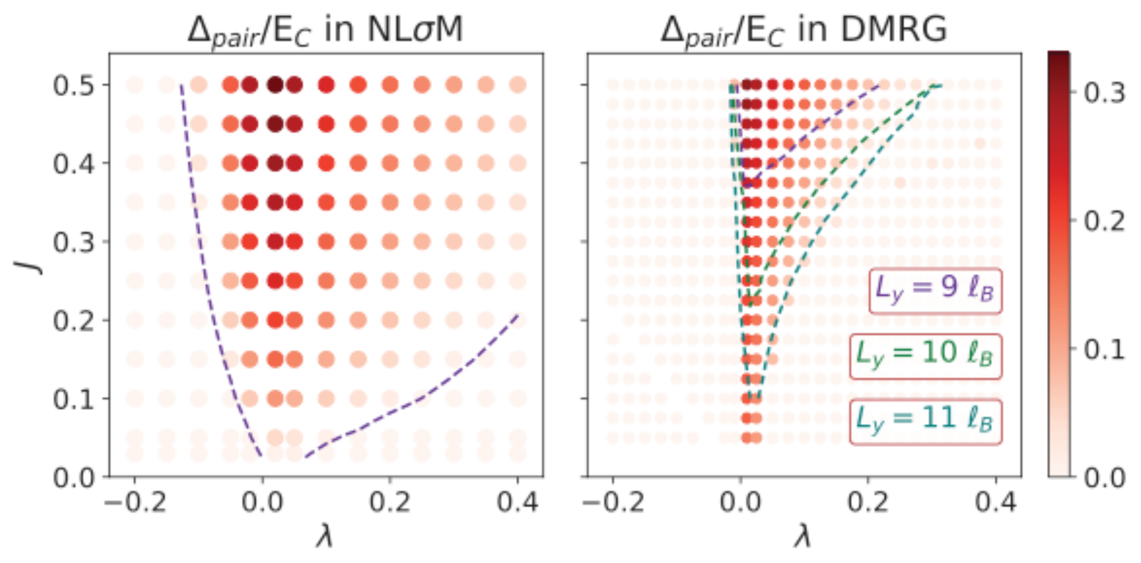


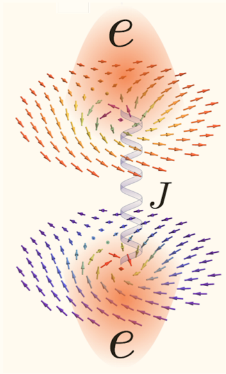
Both NLSM and DMRG give energy of charged excitations above insulator

Evidence for skyrmion-pairing

- NLSM + Segment DMRG to determine energy of charged excitations

$$\boxed{\Delta_{\text{pair}} = 2\,E_{1e} - E_{2e}}$$

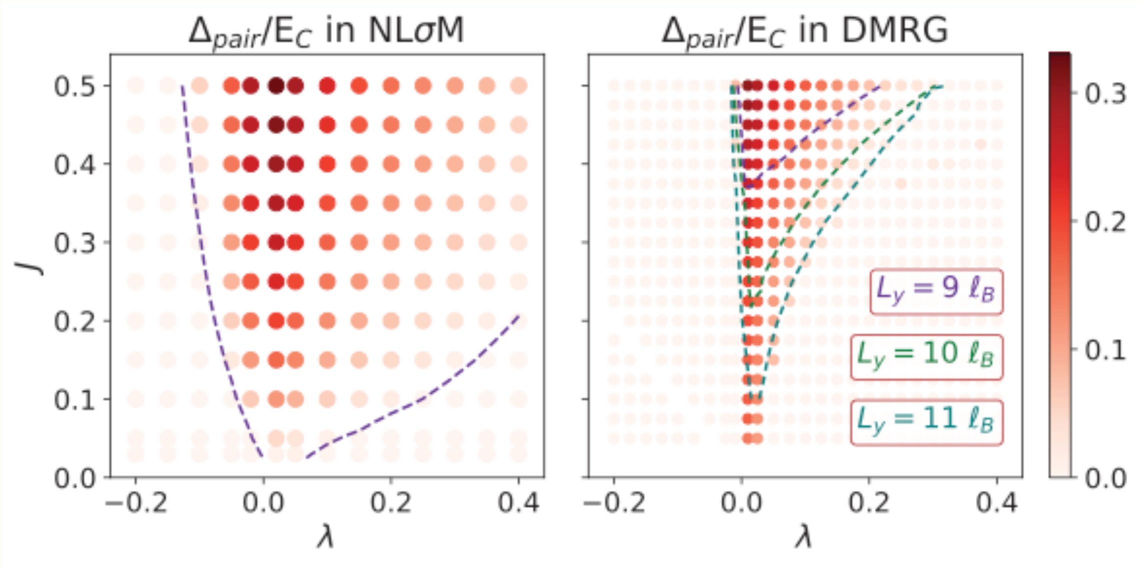


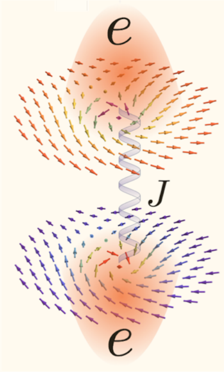


- Numerics for quantum system confirm classical expectations!

Evidence for skyrmion-pairing

- Critical $J_*(\lambda) \rightarrow 0$ as $\lambda \rightarrow 0$, indicative of collective pairing mechanism
- Pairing is much more favorable in the easy plane case (good for MAG!)





- Good qualitative agreement between quantum and classical numerics

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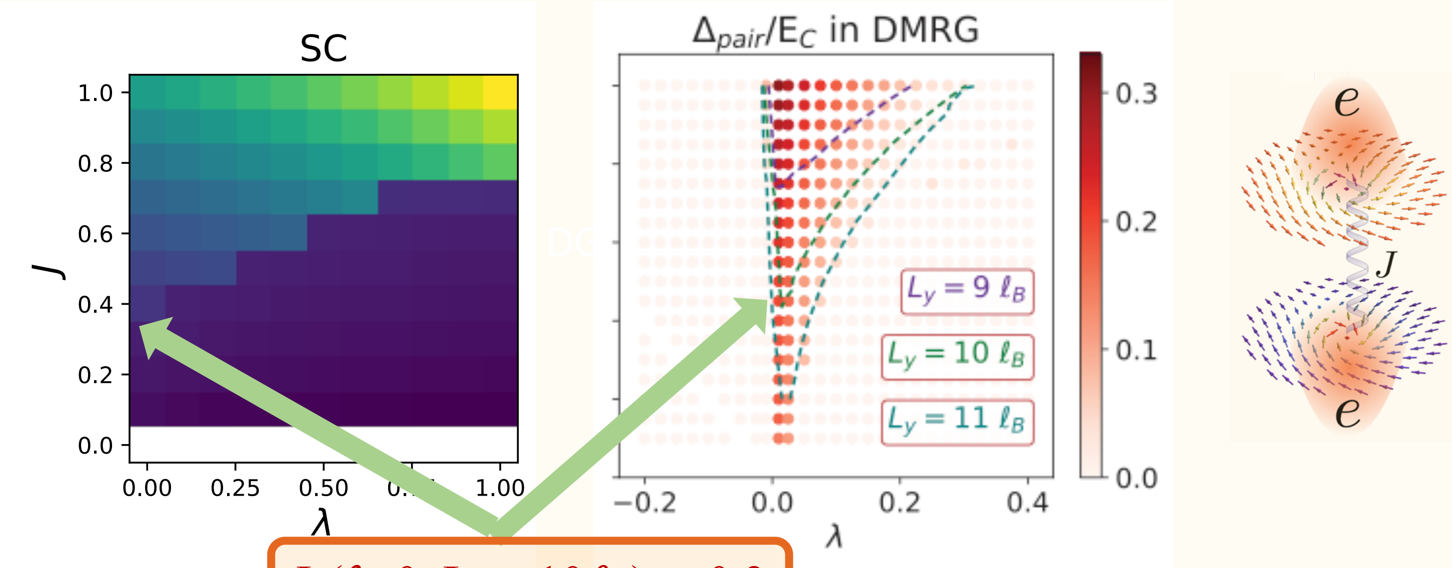
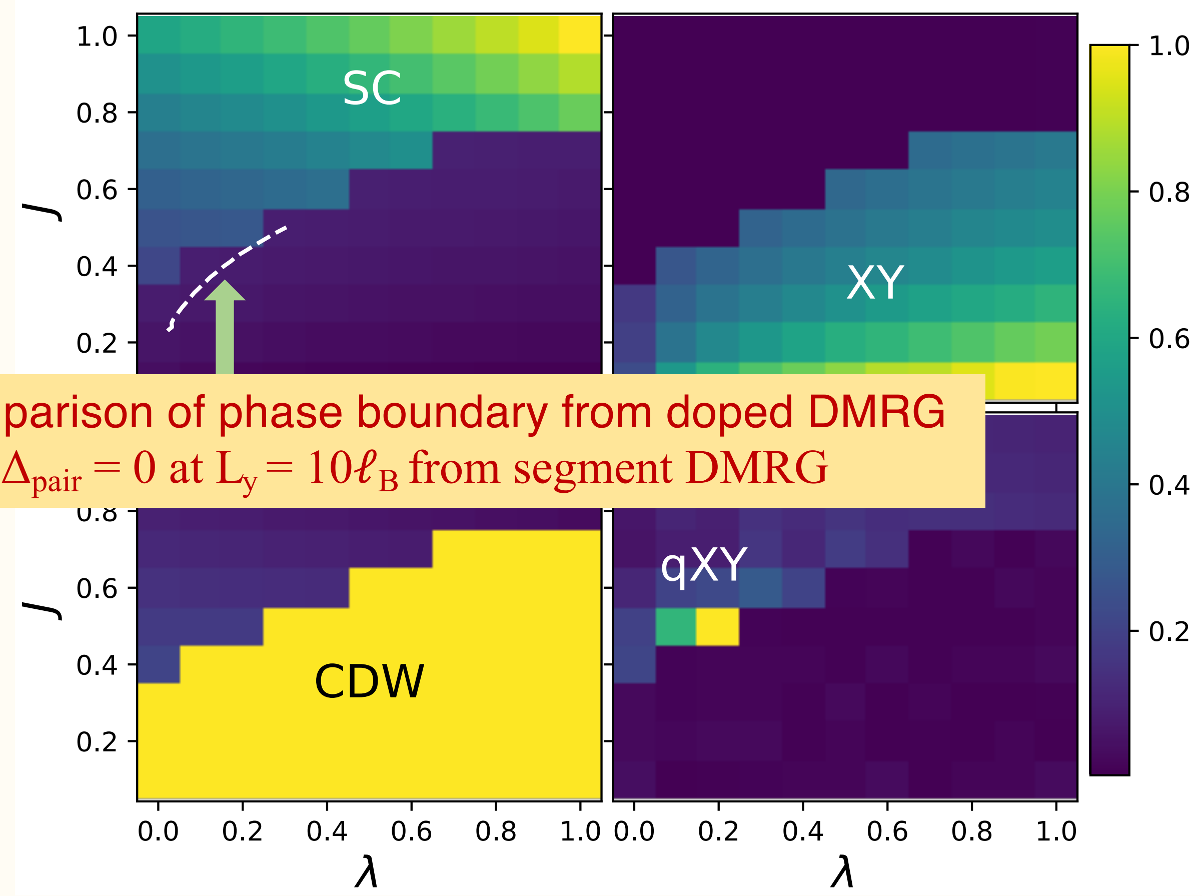


$J_*(\lambda=0,\ L_y = 10\ell_B) \cong 0.3$

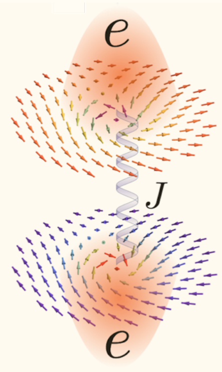
- Good qualitative agreement between quantum and classical numerics

Evidence for skyrmion-pairing

Phase diagram at doping 2 + 1/4



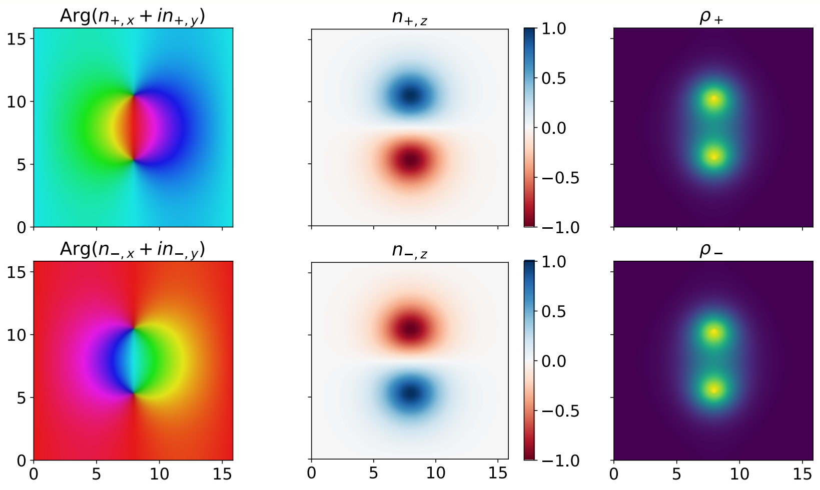
Comparison of phase boundary from doped DMRG with $\Delta_{\text{pair}} = 0$ at $L_y = 10\ell_B$ from segment DMRG

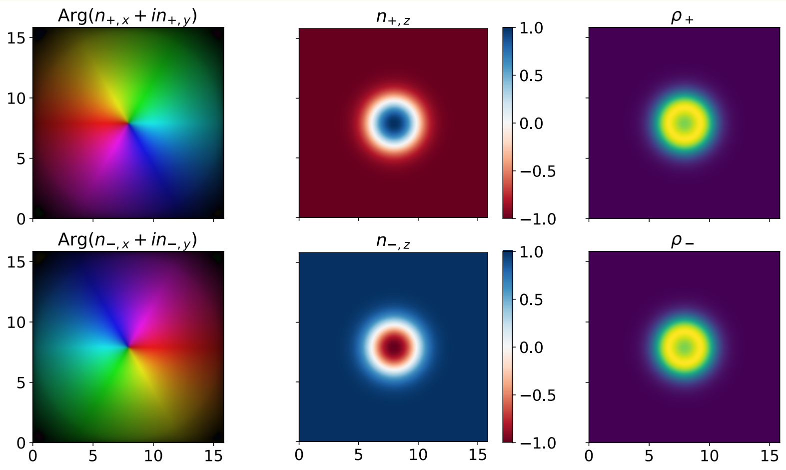


Evidence for skyrmion-pairing

Effects of anisotropy

- Pairing is much more favorable in the easy plane case (good for MAG!)





Easy plane: Charges deform into topologically equivalent meron pairs

Easy axis: Charge density remains radially symmetric, incurs larger Coulomb penalty

Evidence for skyrmion-pairing

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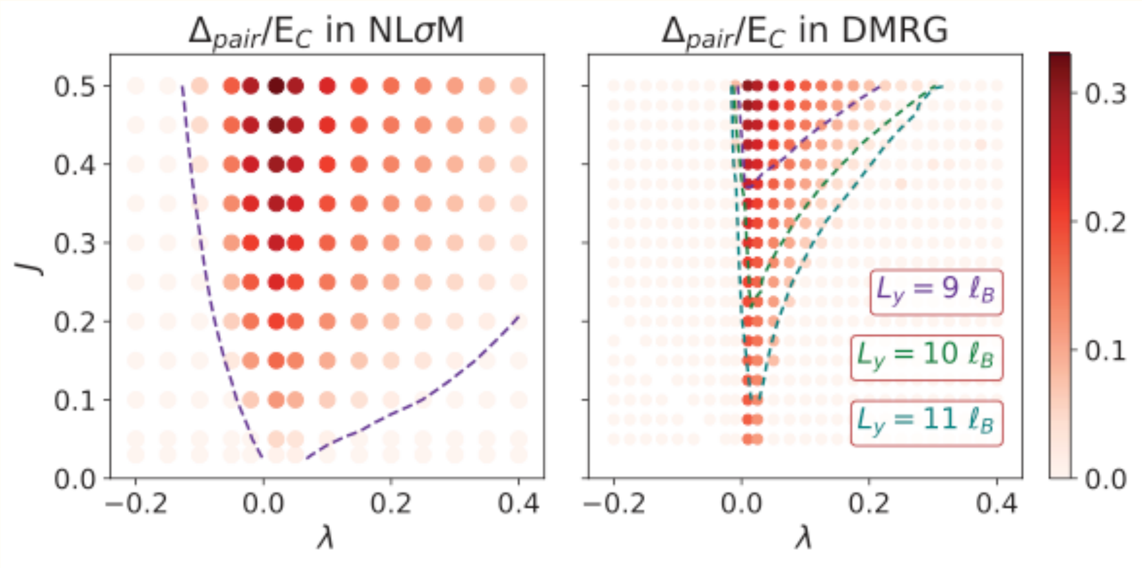
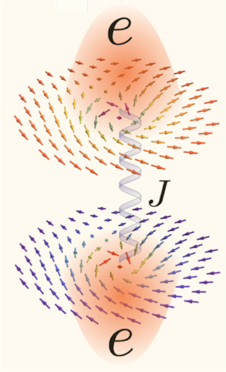
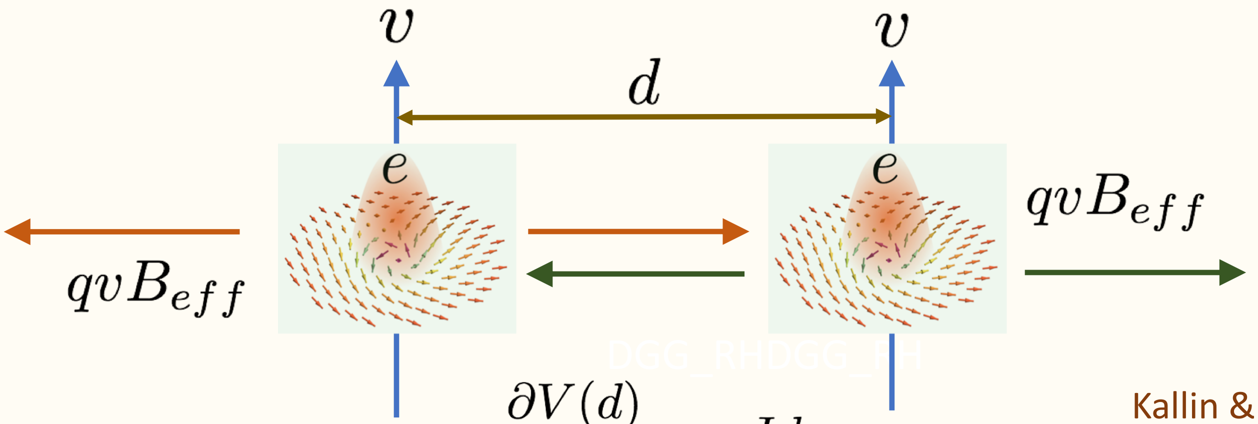




- Semi-classical expectations confirmed by numerics

Evidence for skyrmion-pairing

Effective mass



$$\frac{\partial V(d)}{\partial d} = aJd$$

Kallin & Halperin, PRB (1984)
K. Yang, PRL (2001)

$$\boxed{E = V_0 + aJd^2/2 = V_0 + \frac{(qB_{eff})^2}{2aJ} v^2}$$

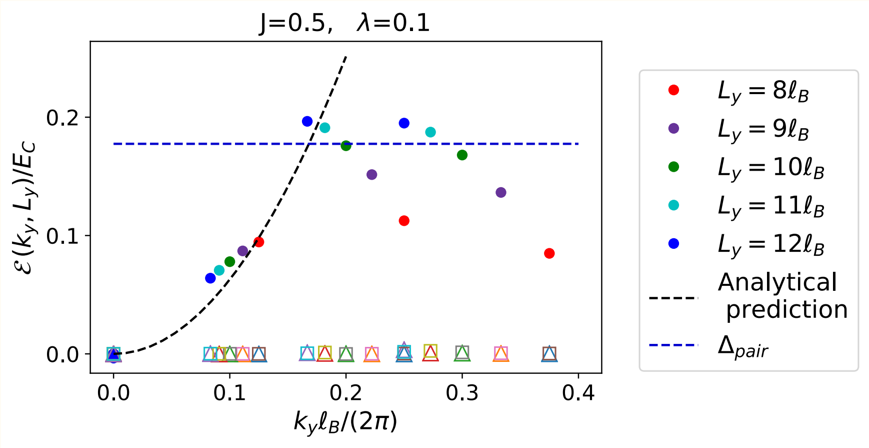
$$\boxed{M_{\text{pair}} \sim 1/J}$$

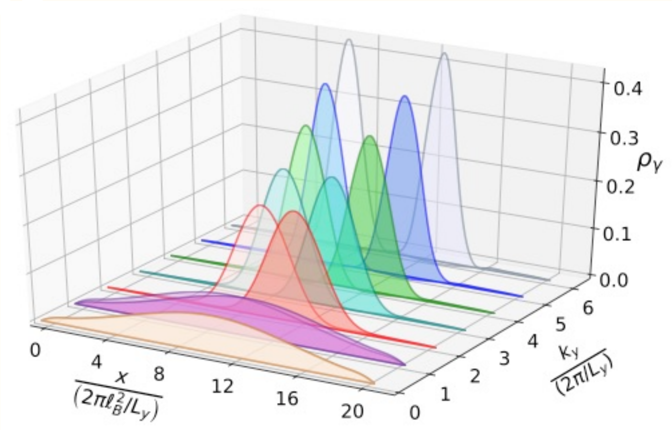
E. Khalaf, SC *et al*,
arXiv:2004.00638
(to appear in Sci. Adv.)

Evidence for skyrmion-pairing

Effective mass

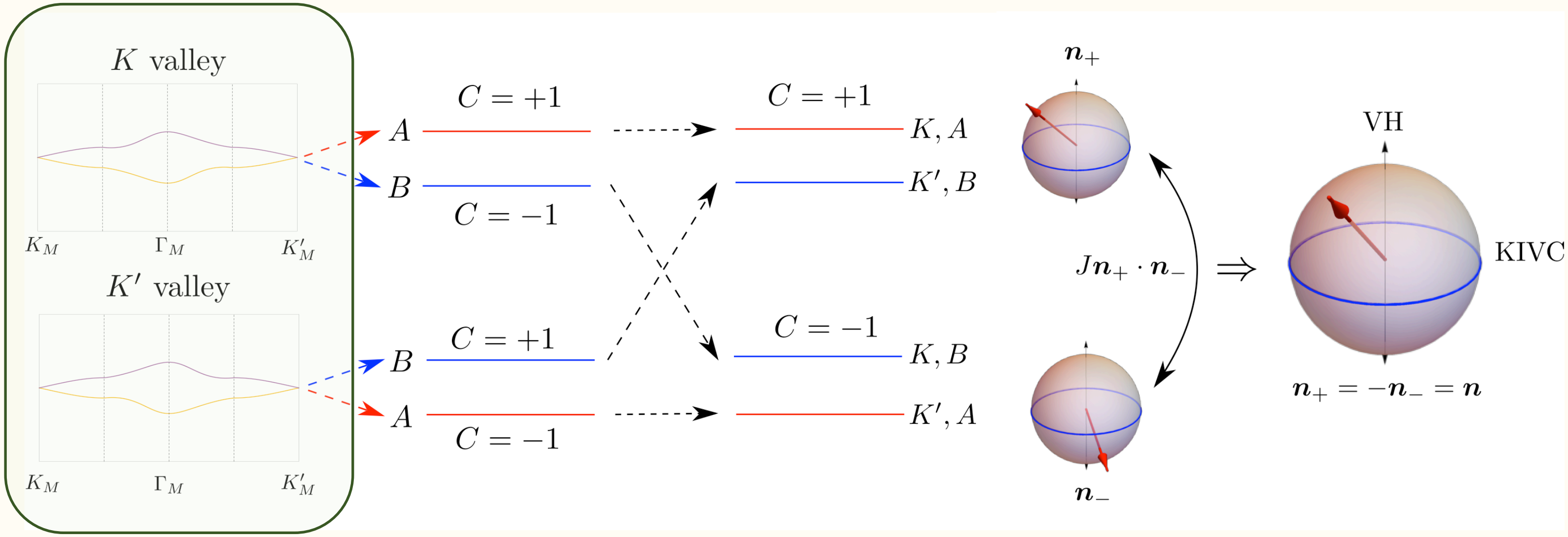
- As $\lambda \to 0$, effective mass $M_{2e} \propto J^{-1}$ as expected from semiclassical study
- At larger momenta k_y, charges from opposite layers get separated, with Chern resolved dipole moment $\propto k_y$: paying AF exchange penalty





Connection to MAG: Landau level picture

Schematic overview



MAG flat bands → Chiral limit

Chern bands

FM in each Chern sector

Dispersion → AF superexchange

Bultinck, Khalaf, SC *et al*, PRX (2020)
Lian, Bernevig *et al*, TBG series (2020)

Ground state: easy-plane AF or KIVC

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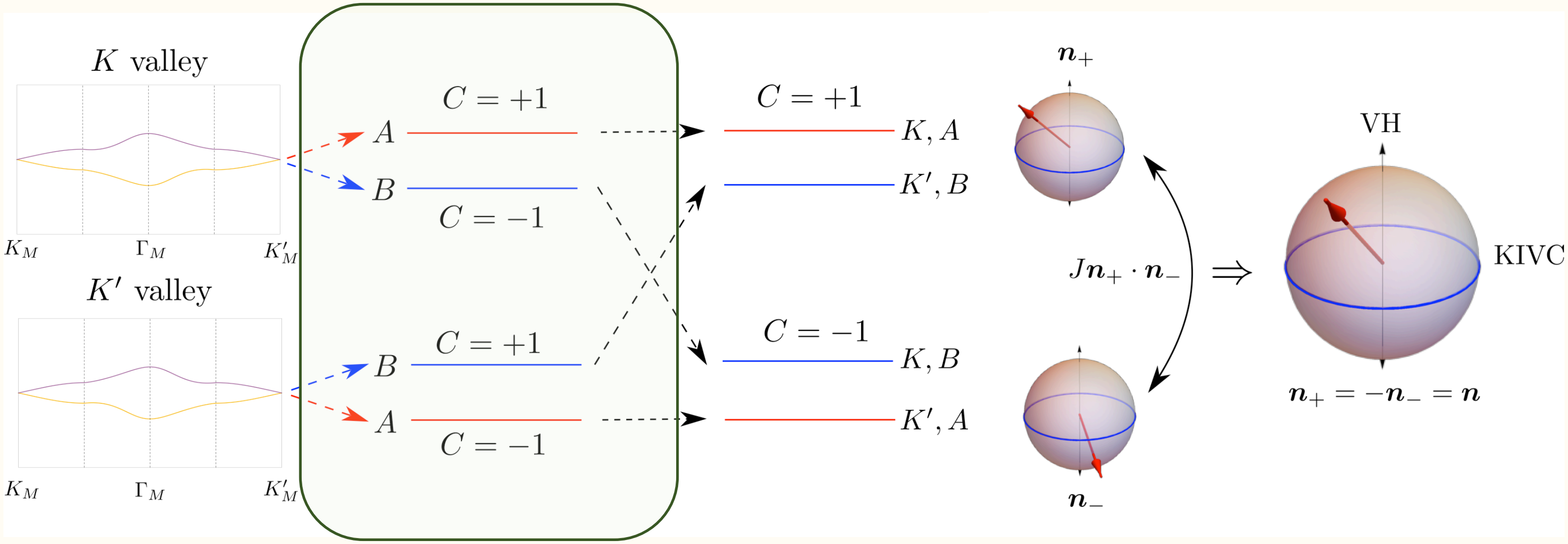

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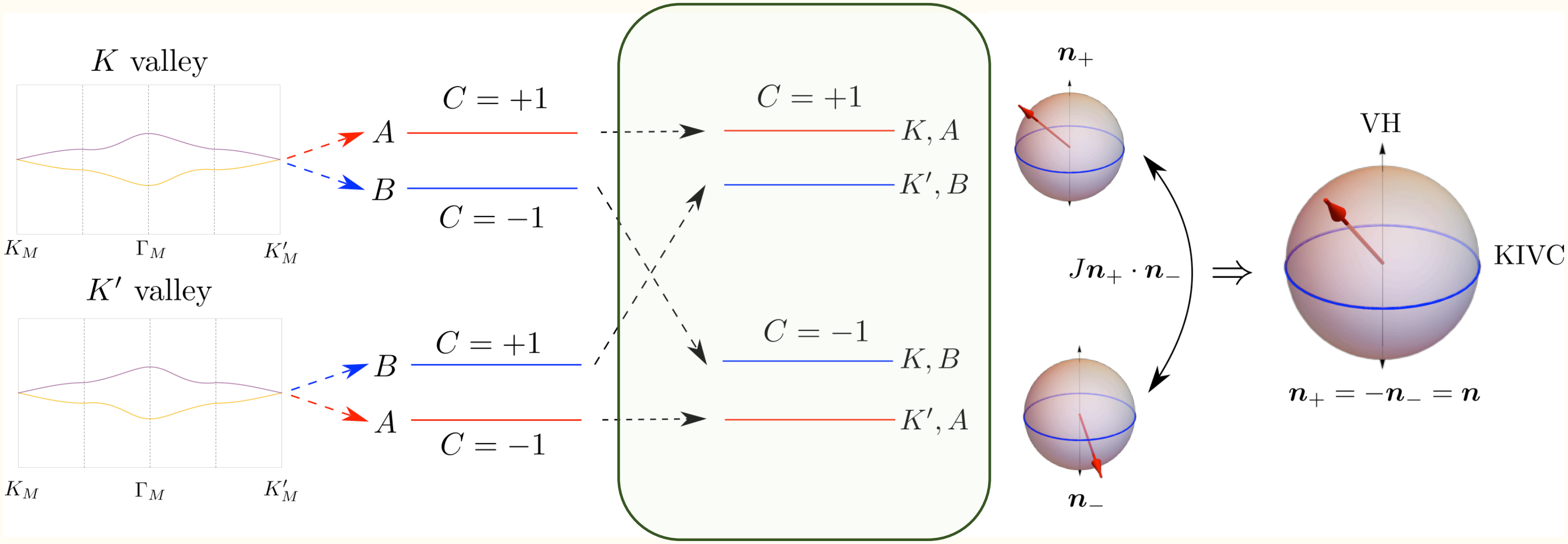

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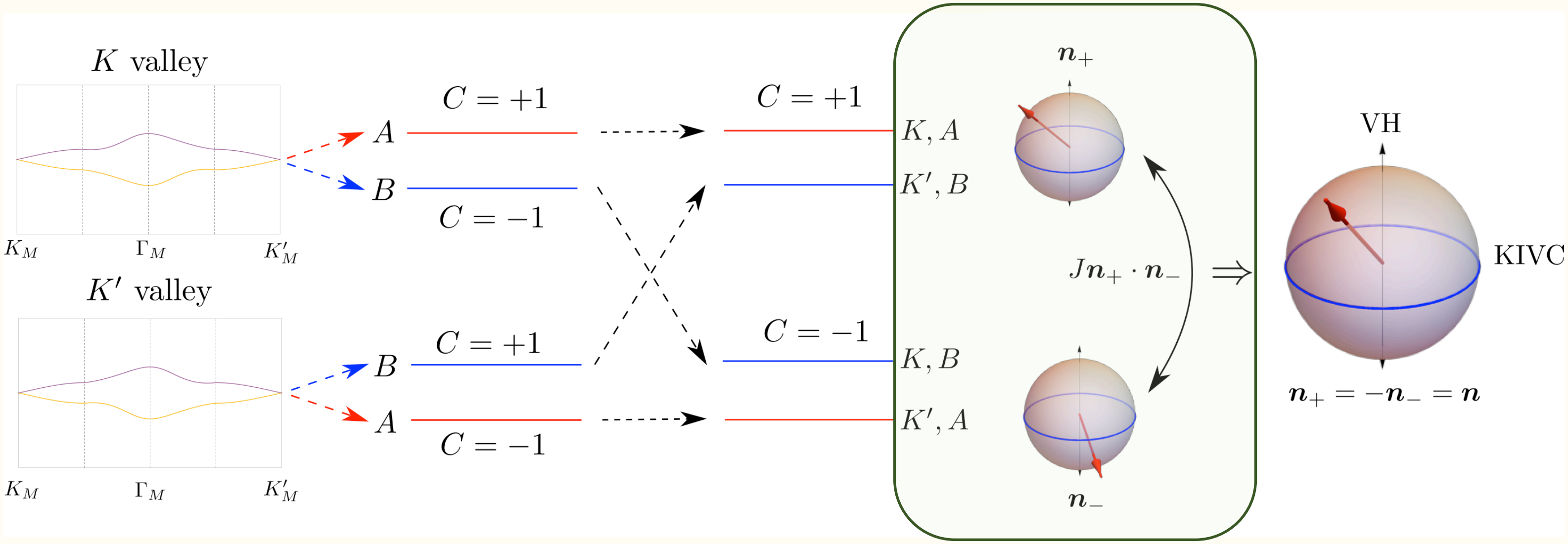

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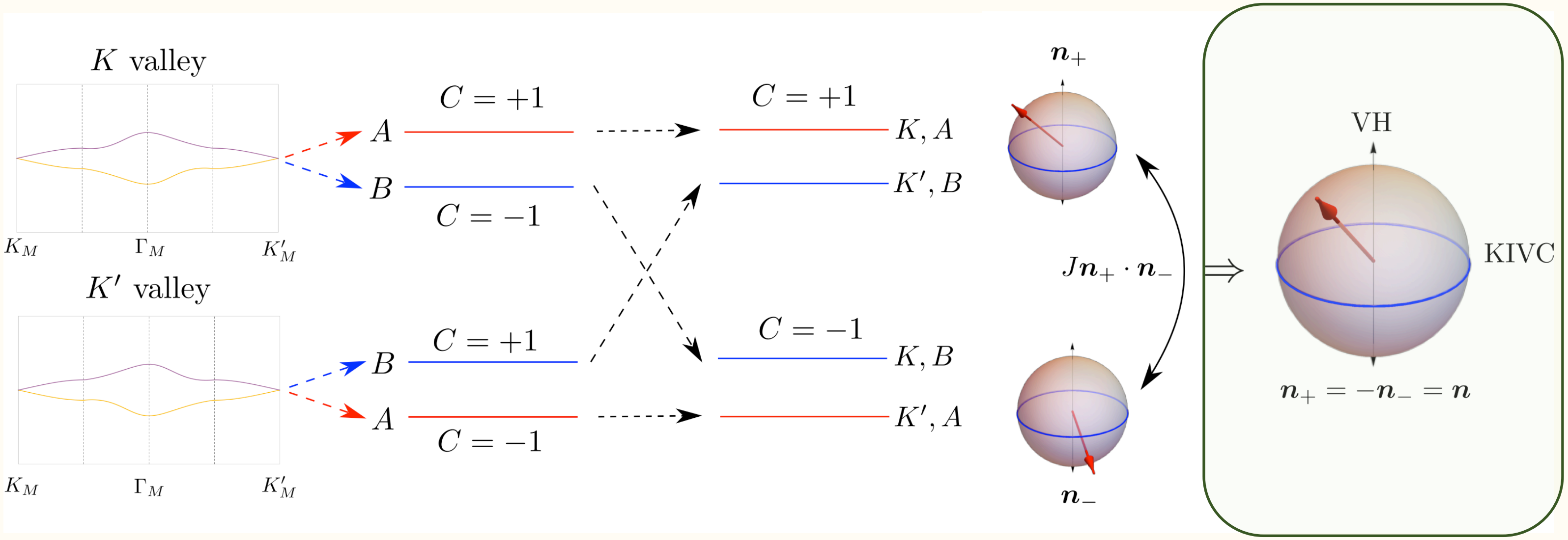

MAG flat bands

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Ground state: easy-plane AF or KIVC

Conclusions and Outlook

- Numerically established skyrmion-antiskyrmion pair condensation as a viable mechanism for superconductivity

- Band topology plays a crucial role (not seen in bands with same C)

- MATBG has the right physical ingredients to realize this mechanism: required band topology and low iso-spin stiffness ~ 1 meV (perhaps mirror symmetric MATLG too?)

Saito *et al*, arXiv:2008:10830
Park *et al*, arXiv:2012.01434
Hao *et al*, arXiv:2012.01434

- Open questions --- Effects of:

1. Non-uniform Berry curvature
2. Disorder
3. Spin-orbit coupling

Thank you for your attention!