Inter-valley coherent order and isospin fluctuation mediated superconductivity in rhombohedral trilayer graphene

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EP2DS-24/MSS-20

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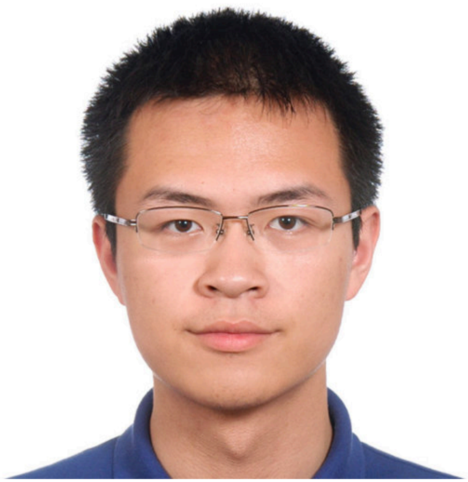
In collaboration with:



Taige Wang
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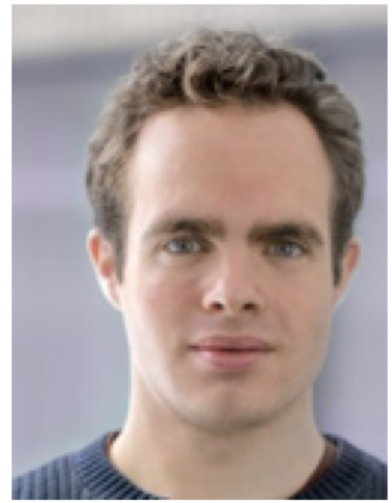
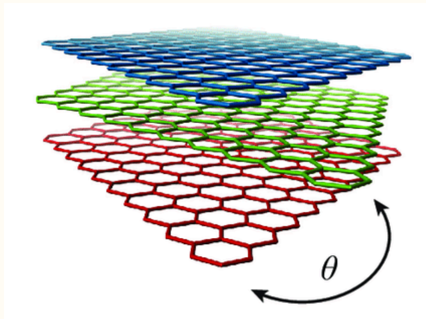


Mike Zaletel
UC Berkeley

SC, T. Wang, E. Berg and M. P. Zaletel, arXiv:2009.00002

Correlated physics in ABC trilayer graphene

- Superconductivity was discovered in several distinct moire graphene platforms

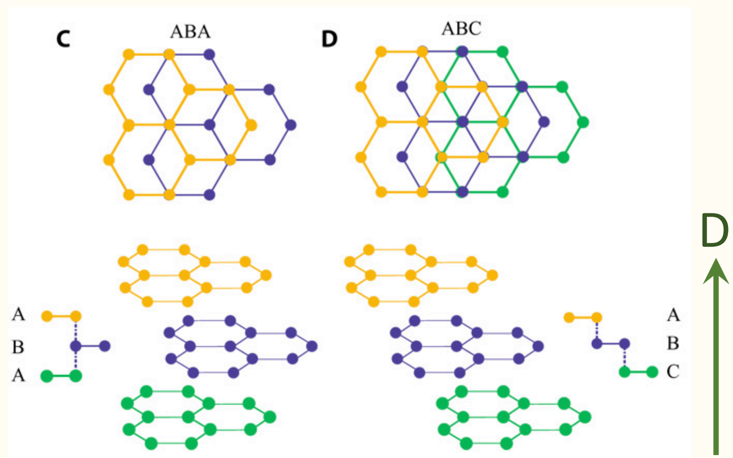


Cao *et al,* Nature (2018), Lu *et al,* Nature (2019), Yankowitz *et al*, Science (2019), Chen *et al*, Nature (2019) Several others…

Figures: Carr *et al*, Nano Letters (2020)
Shan et al, Sci. Adv. (2018)

- Recently, correlated behavior and superconductivity have also been found in *non-moire* ABC trilayer graphene under a strong perpendicular electric field D

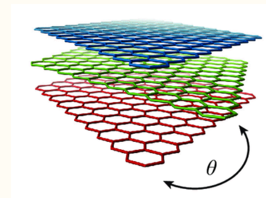
Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)



- Are these related?

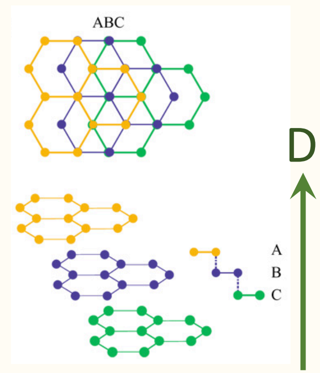
Correlated physics in ABC trilayer graphene

- Origin of strong correlations: Van Hove singularity in DOS



1. In twisted graphene platforms: quantum interference in minibands due to moire superlattice results in band flattening

   Bistritzer, MacDonald, PNAS (2011), Liu *et al*, PRB (2019), Tarnopolsky *et al*, PRL (2019)



2. In ABC trilayer graphene, flat dispersion near conduction band minima/valence band maxima due to electric field D

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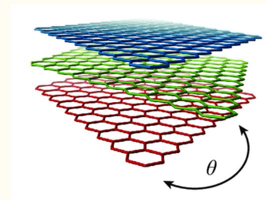

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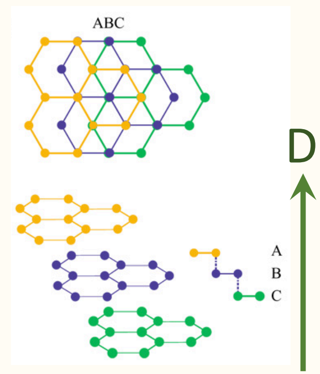

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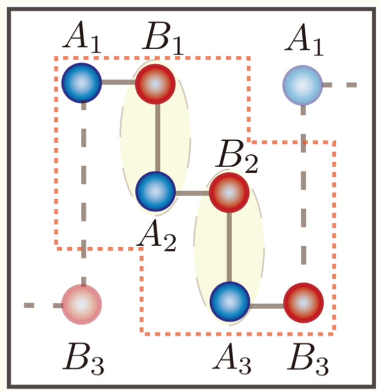

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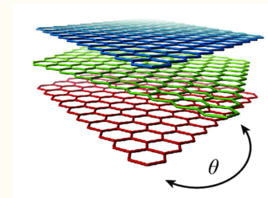

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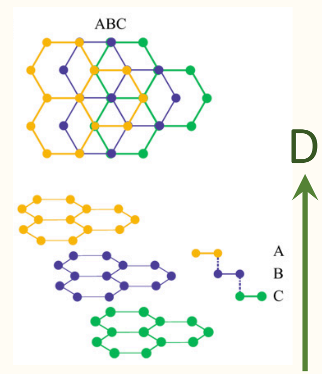

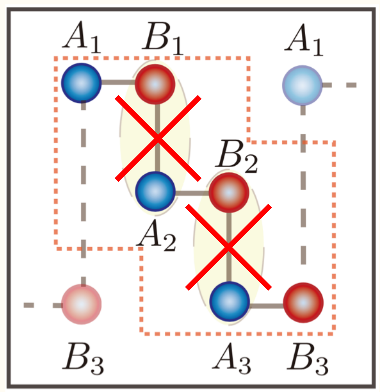

Active sites for low-energy physics: A_1 and B_3

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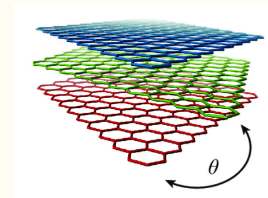

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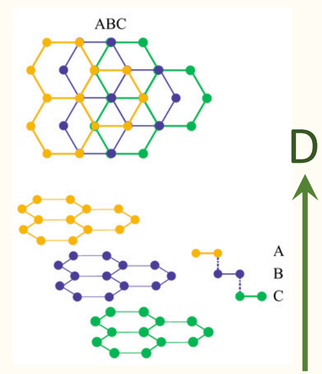

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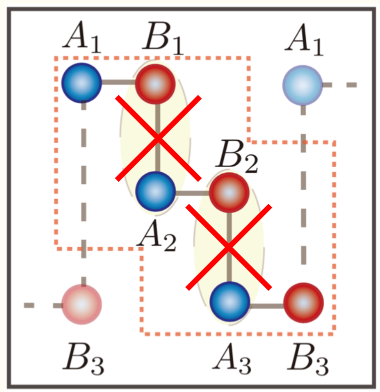
$$h \sim \begin{pmatrix} D & k_+^3 \\ k_-^3 & -D \end{pmatrix}$$



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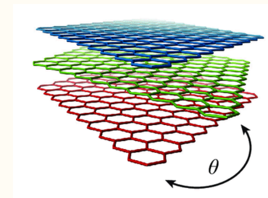

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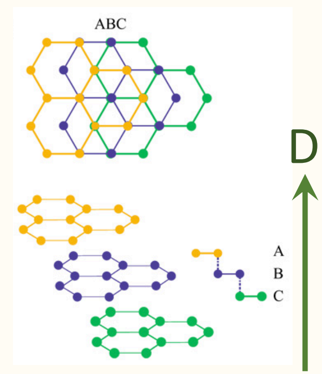

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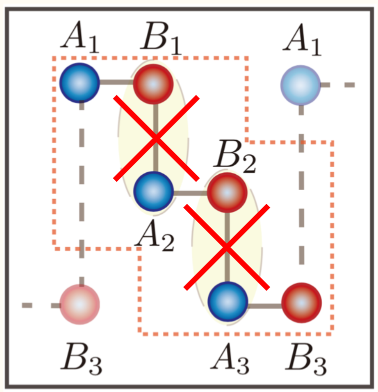

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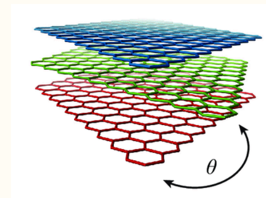

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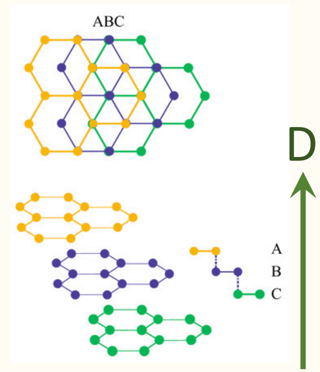

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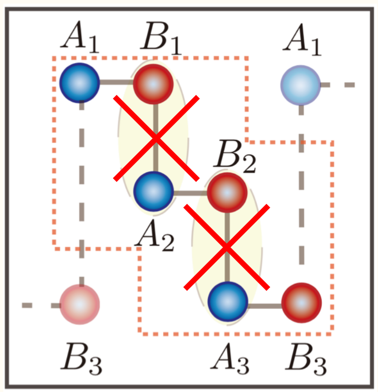

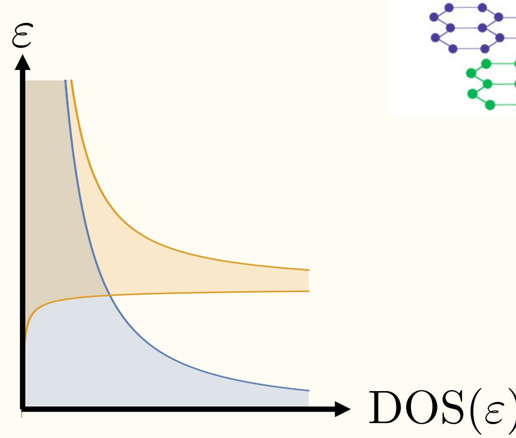

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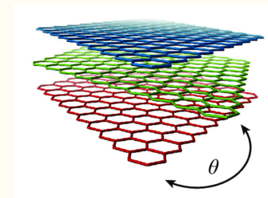

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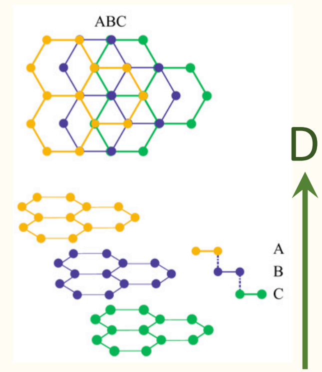
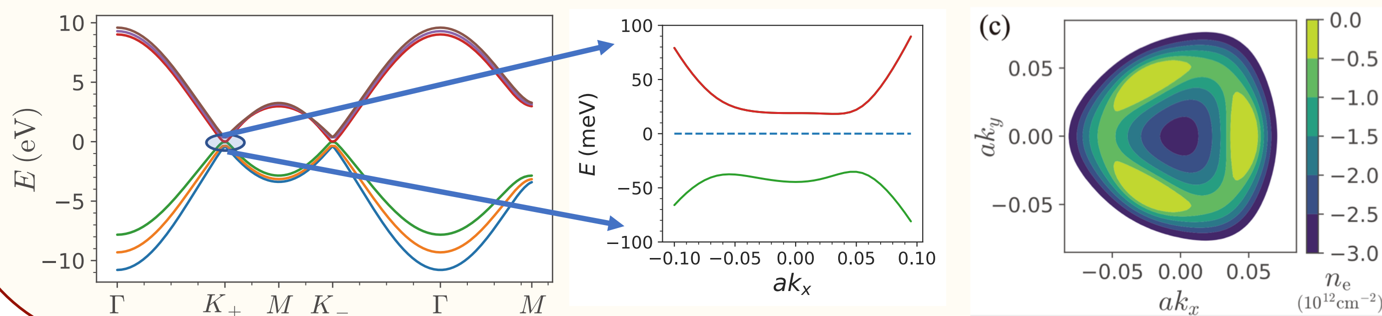
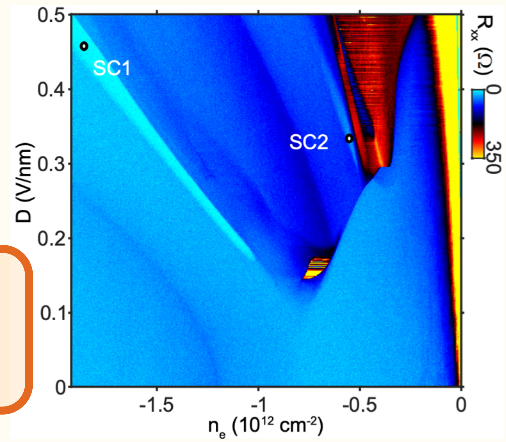




Zhang *et al*, PRB (2010)

Correlated physics in ABC trilayer graphene

- The metallic states at low doping show signatures of iso-spin (spin/valley) symmetry breaking in quantum oscillations under large D



Large electric field, correlated physics

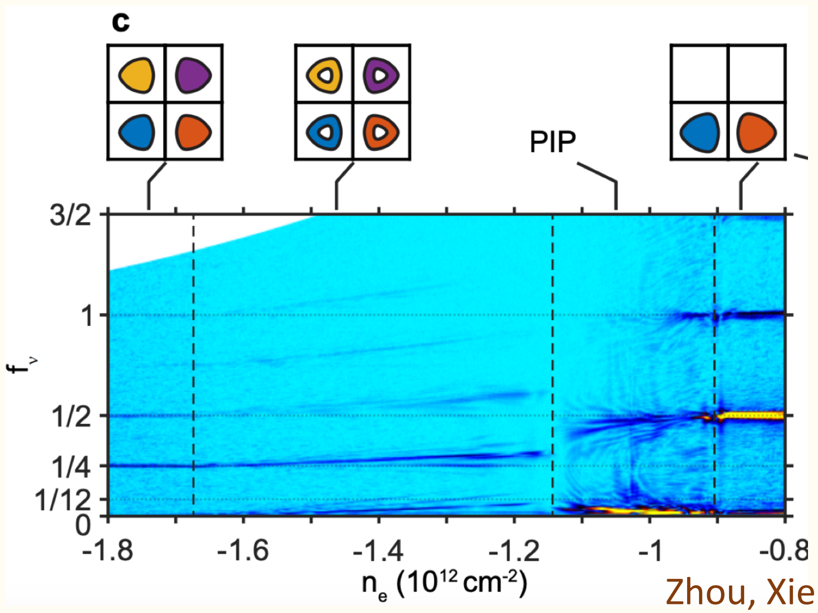
Small electric field, single-particle physics

Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)

- Interestingly, superconductivity appears twice in the hole-doped phase diagram, both times on the cusp of iso-spin symmetry breaking

Correlated physics in ABC trilayer graphene

- Like monolayer graphene: ABC trilayer graphene has both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D



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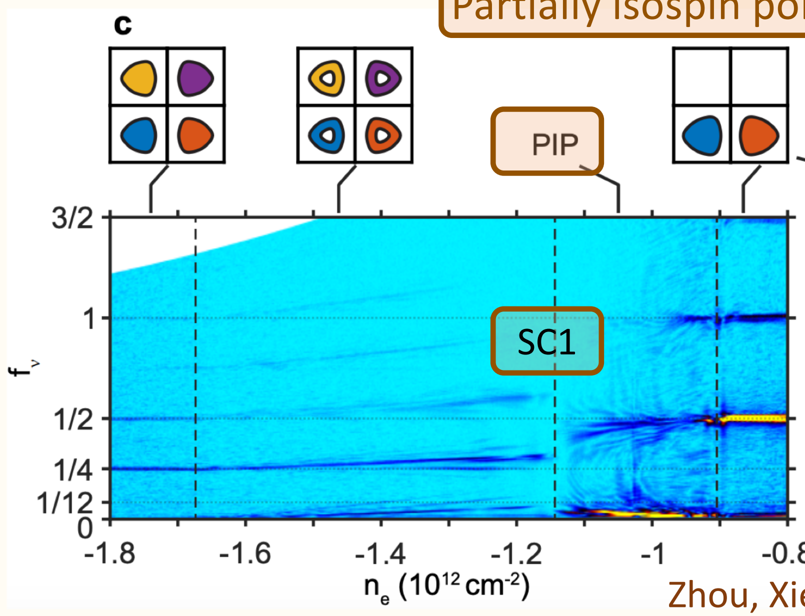
Partially isospin polarized, $R_{xy} = 0$



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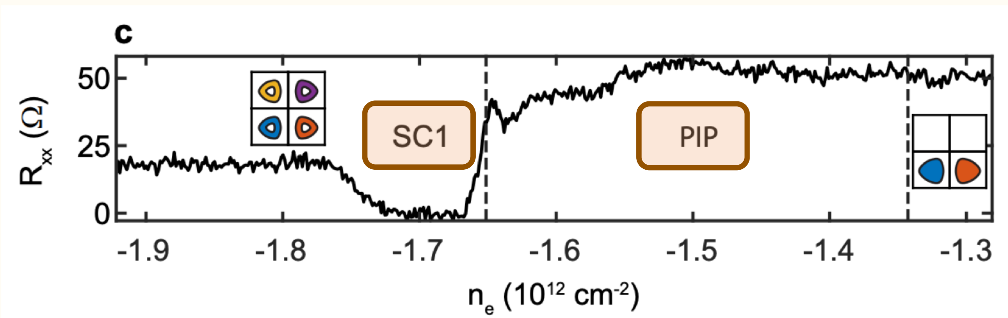
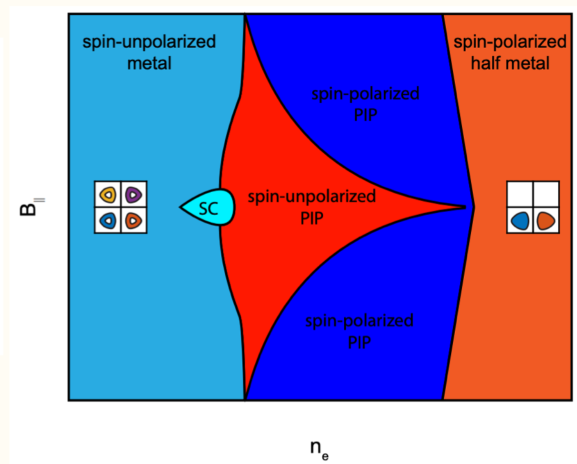
- SC1 appears at higher hole-doping, adjacent to a PIP phase. SC1 obeys Pauli limit, $\mu_B B_{\parallel,c} \sim k_B T_c$.





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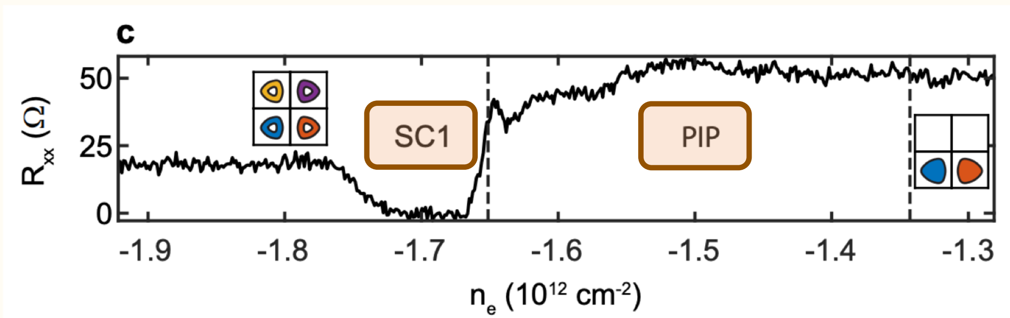

Spin-singlet SC with ferromagnetic inter-valley Hund's coupling?

Zhou, Xie *et al*, Nature (2021)
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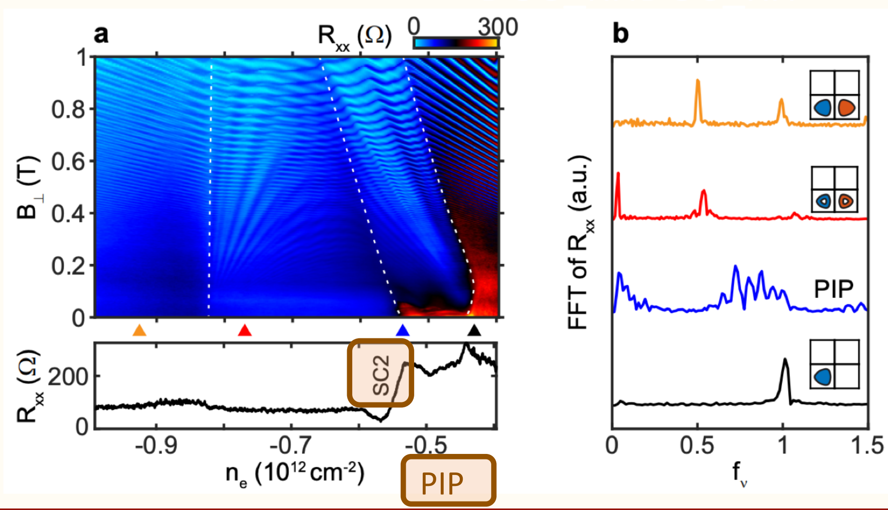
Correlated physics in ABC trilayer graphene

- Like monolayer graphene: ABC trilayer graphene have both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D
- SC2 appears at lower hole-doping, and adjacent to spin-polarized phases. SC2 strongly violates the Pauli limit, $\mu_B B_{\parallel,c} \gg k_B T_c$



Spin-triplet SC ✓

Zhou, Xie *et al*, Nature (2021)
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Correlated physics in ABC trilayer graphene

- What are the broken symmetries in the PIP phases?

- What are the pairing symmetries of SC1 and SC2 that arise at the boundaries of PIP phases?

- Do electronic correlations play any role in superconductivity?

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Inter-valley coherent order – spin-triplet SDW or spin-polarized CDW

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- Do electronic correlations play any role in superconductivity?

Possibly, near critical IVC fluctuations can act as pairing glue

Hamiltonian and symmetries

- Interacting Hamiltonian in band-basis:

$$H = \sum_{n,\tau,s,\mathbf{k}} \varepsilon_{n,\tau,\mathbf{k}} \psi^\dagger_{n,\tau,s,\mathbf{k}} \psi_{n,\tau,s,\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{q}} V_c(\mathbf{q}) : \rho(\mathbf{q}) \rho(-\mathbf{q}) :$$

$$\rho(\mathbf{q}) = \sum_{\tau,s,\mathbf{k}} \psi^\dagger_{n,\tau,s,\mathbf{k}} [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} \psi_{n',\tau,s,\mathbf{k}+\mathbf{q}}, \quad [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} = \langle u_{n,\tau,\mathbf{k}} | u_{n',\tau,\mathbf{k}} \rangle$$

- Symmetries: $U(1)_c$, $U(1)_v$, C_3 , M_x , T, $SU(2)_s \rightarrow SU(2)_+ \times SU(2)_-$

No Hund's coupling

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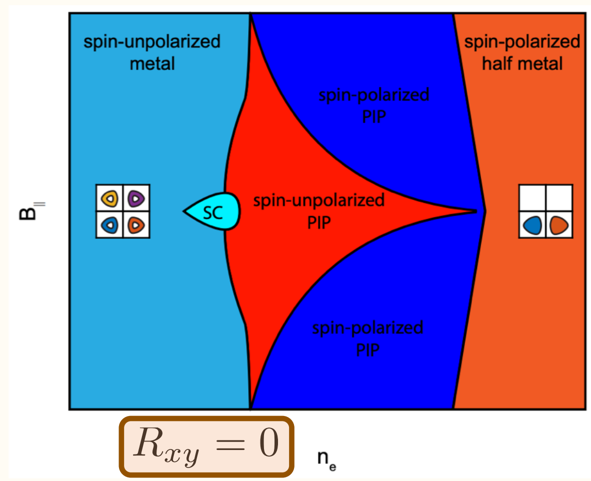
No Hund's coupling

- PIP phase near SC1:
  - (i) No spin-polarization,
  - (ii) No anomalous Hall effect (not valley-polarized)

- Inter-valley coherent (IVC) phase

$$\langle \psi^{\dagger}_{+,s,\mathbf{k}} \psi_{-,s',\mathbf{k}} \rangle \neq 0$$



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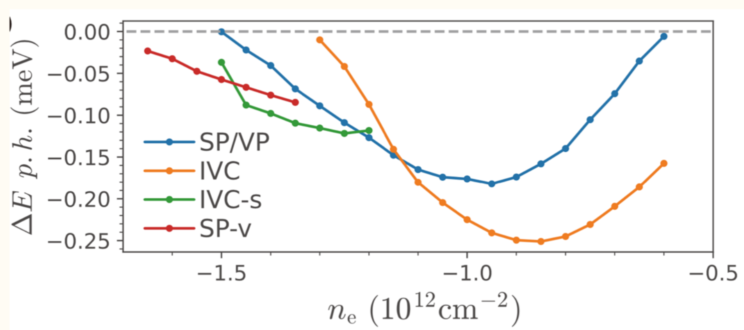
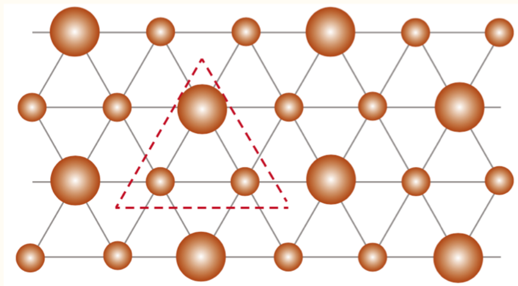
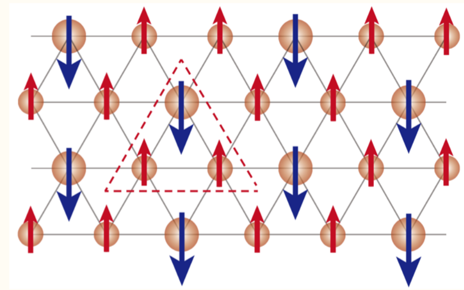


Hartree-Fock energetics

Inter-valley coherent phases

- IVC states form a U(2) manifold - Inter-valley Hund's term picks either spin-singlet or triplet.





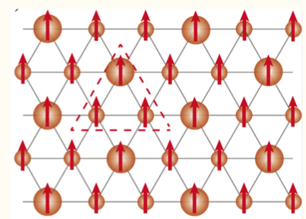
Singlet IVC: CDW $n_{\mathrm{S}}^{\mathrm{IV}} = \sum_{\mathbf{R}} e^{i(\mathbf{K}-\mathbf{K}')\cdot\mathbf{R}} \rho(\mathbf{R})$

Triplet IVC: SDW $\mathbf{n}_{\mathrm{T}}^{\mathrm{IV}} = \sum_{\mathbf{R}} e^{i(\mathbf{K}-\mathbf{K}')\cdot\mathbf{R}} \mathbf{s}(\mathbf{R})$

- Landau theory allows term of the form $\mathbf{n_T(Q).n_T(Q)}\ \rho(\mathbf{-2Q})$ $\rightarrow$ weak CDW is nucleated by SDW IVC

Zachar *et al*, PRB (1998)

- PIP phase near SC2 $\rightarrow$ spin-polarized IVC



FM CDW

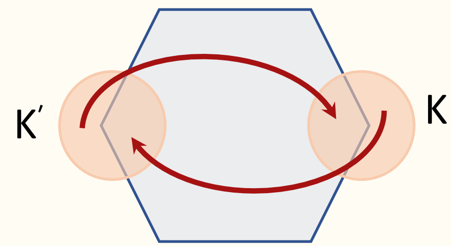
Aleiner *et al*, PRB (2007)
Cvetkovic, Vafek, arXiv:1210.4923

Inter-valley coherent phases

- Hund's coupling arises from inter-valley scattering of electrons

$$H_{\mathrm{Hund's}} = -J_H \sum_{\mathbf{q}} \mathbf{s}_{+-}(\mathbf{q}) \cdot \mathbf{s}^{\dagger}_{+-}(\mathbf{q})$$

$$\mathbf{s}_{+-}(\mathbf{q}) = \sum_{\mathbf{k}} \lambda^{+-}_{\mathbf{q}}(\mathbf{k}) \psi^{\dagger}_{+,s,\mathbf{k}} \mathbf{s}_{ss'} \psi_{-,s',\mathbf{k}+\mathbf{q}}$$



- Surprisingly, distinct from usually assumed (symmetry-allowed) form:

$$\tilde{H}_{\mathrm{Hund's}} = -\tilde{J}_H \sum_{\mathbf{q}} \mathbf{s}_{+}(\mathbf{q}) \cdot \mathbf{s}_{-}(-\mathbf{q}) \qquad \mathbf{s}_{\tau}(\mathbf{q}) = \sum_{\mathbf{k}} \lambda^{\tau\tau}_{\mathbf{q}}(\mathbf{k}) \psi^{\dagger}_{\tau,s,\mathbf{k}} \mathbf{s}_{ss'} \psi_{\tau,s',\mathbf{k}+\mathbf{q}}$$

- Directly favors spin-triplet IVC or SDW IVC when ferromagnetic

$$\mathbf{s}_{+-}(\mathbf{q}=0) = \mathbf{n}_{\mathrm{T}}$$

- Local repulsive Hubbard U gives rise to ferromagnetic $J_H > 0$, but local repulsive interactions disfavor a CDW/singlet-IVC

IVC fluctuation mediated superconductivity

- IVC order parameter fluctuations can lead to superconductivity

$$H^{\text{eff}}_{\text{IVC}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \text{Tr}[n^{\text{IV}}(\mathbf{q})[n^{\text{IV}}(\mathbf{q})]^{\dagger}], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\text{IVC}}^{-2}}$$

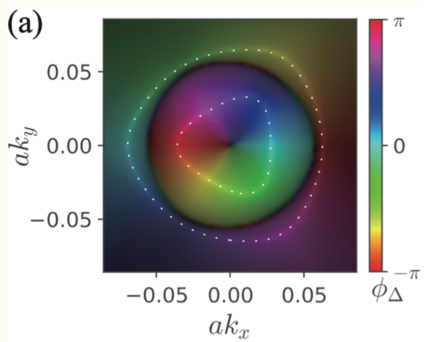
- Unconventional superconductivity is preferred by this mechanism

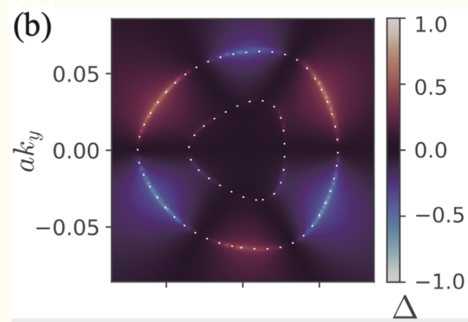
$$\langle \psi_{+,s,\mathbf{k}} \psi_{-,s',\mathbf{k}} \rangle = \varepsilon_{ss'} f_{\mathbf{k}} \qquad \langle H^{\text{eff}}_{\text{IVC}} \rangle \approx \sum_{\mathbf{k},\mathbf{k}'} g_{\mathbf{q}=-\mathbf{k}-\mathbf{k}'} |\lambda^{+-}_{\mathbf{q}=-\mathbf{k}-\mathbf{k}'}(\mathbf{k})|^2 \; f^*_{\mathbf{k}} f_{\mathbf{k}'}$$

$q \to 0$ limit

$$f_{-\mathbf{k}} = -f_{\mathbf{k}}$$

- Precise channel depends on the IVC correlation length ξ_{IVC}





Large ξ_{IVC} : chiral gapped, $p_x \pm i\, p_y$ around K

Small ξ_{IVC} : nodal, $\text{Im}[(p_x + i\, p_y)^3]$ around K

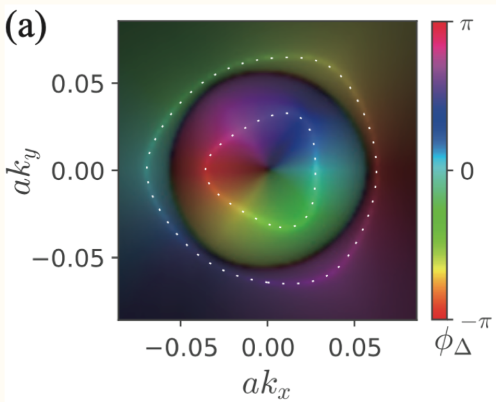
Distinct from s-wave via $M_x T$ (not C_3)

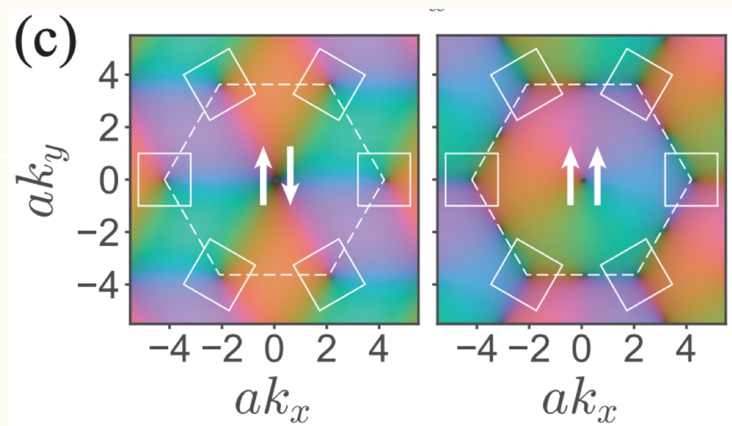
IVC fluctuation mediated superconductivity

- IVC order parameter fluctuations can lead to superconductivity

$$H^{\text{eff}}_{\text{IVC}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \text{Tr}[n^{\text{IV}}(\mathbf{q})[n^{\text{IV}}(\mathbf{q})]^\dagger], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\text{IVC}}^{-2}}$$

- What is the pairing symmetry in the full BZ? $\langle \psi_{s,\mathbf{k}} \psi_{s',\mathbf{k}} \rangle = ?$





Large ξ_{IVC} : chiral gapped

Spin-singlet: d+id

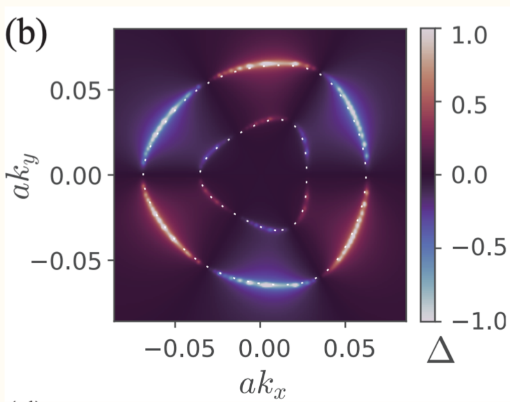
Spin-triplet: p+ip

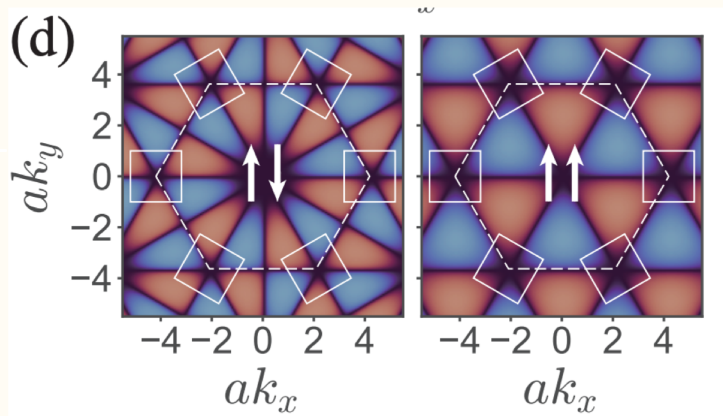
IVC fluctuation mediated superconductivity

- IVC order parameter fluctuations can lead to superconductivity

$$H_{\mathrm{IVC}}^{\mathrm{eff}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \mathrm{Tr}[n^{\mathrm{IV}}(\mathbf{q})[n^{\mathrm{IV}}(\mathbf{q})]^{\dagger}], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\mathrm{IVC}}^{-2}}$$

- What is the pairing symmetry in the full BZ? $\langle \psi_{s,\mathbf{k}} \psi_{s',\mathbf{k}} \rangle = ?$





Small ξ_{IVC} : non-chiral, nodal

Spin-singlet: i-wave

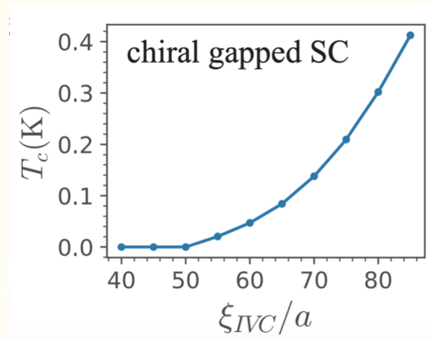
Spin-triplet: f-wave

IVC fluctuation mediated superconductivity

- What is T_c for unconventional superconductivity?

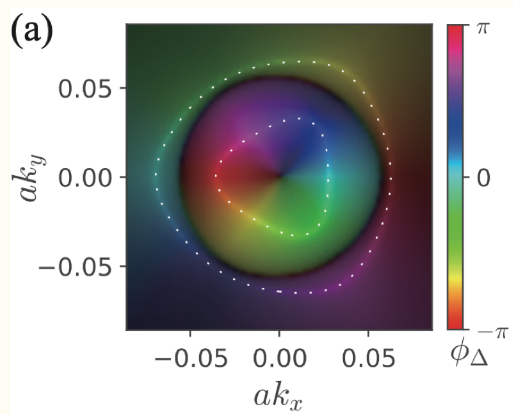
$$H^{\mathrm{eff}}_{\mathrm{IVC}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \mathrm{Tr}[n^{\mathrm{IV}}(\mathbf{q})[n^{\mathrm{IV}}(\mathbf{q})]^{\dagger}], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\mathrm{IVC}}^{-2}}$$

$$g \sim \frac{\text{hole-density } n_h}{\text{DOS at Fermi surface}}$$



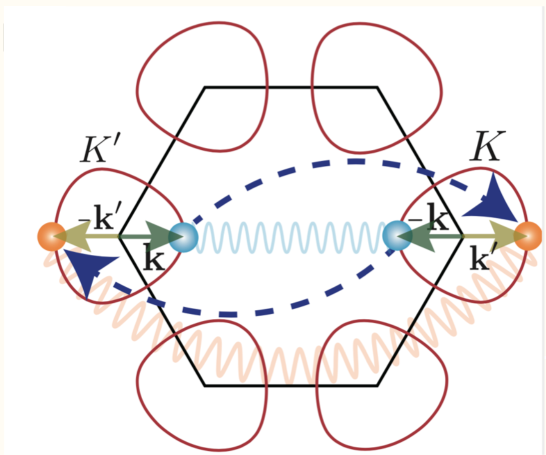
- Including Coulomb repulsion, prefers opposite sign on outer and inner Fermi surfaces

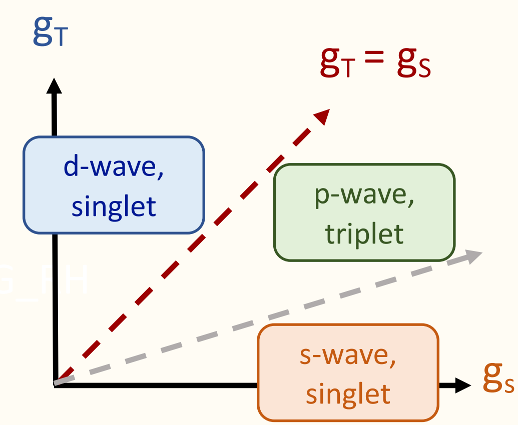
- Spin-singlet and spin-triplet superconductors are degenerate in the $SU(2)_+ \times SU(2)_-$ limit



IVC fluctuation mediated superconductivity

- FM Hund's coupling amplifies SDW fluctuations → leads to *spin-singlet* superconductivity: consistent with SC1





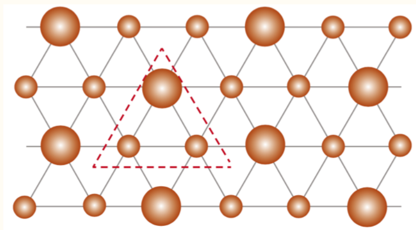
Scalapino, Phys. Reports (1995)

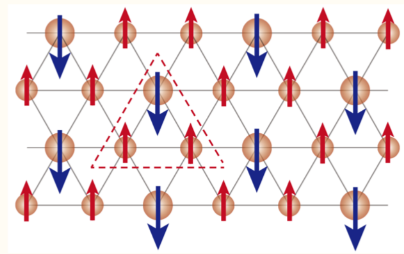
Nandkishore *et al* , Nat. Phys. (2012), Black-Schaffer, Honercamp, J. Phys. CM (2014)

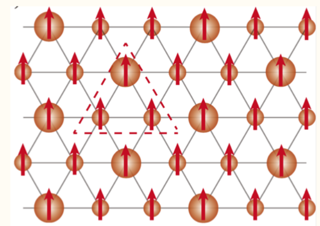
- Spin-polarized IVC can lead to non-unitary spin-triplet p/f-wave superconductor by similar mechanism (consistent with SC2)

Experimental probes

- For IVC states, STM or its spin-polarized cousin to resolve charge (spin) density at the atomic scale







Experimental probes

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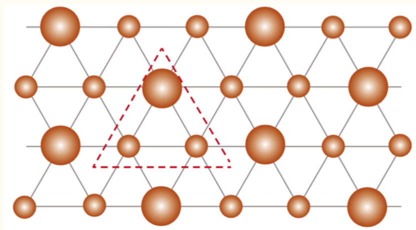
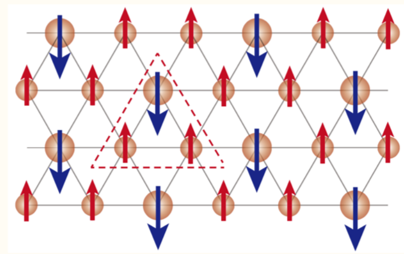
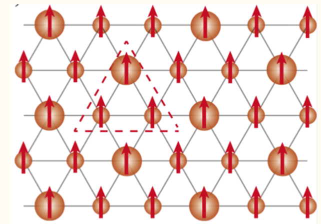






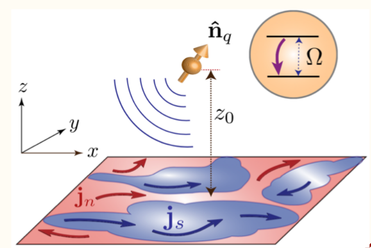
- Unconventional superconductors are susceptible to disorder

Abrikosov, Gorkov, JETP (1961)

- Measure spontaneous edge currents in chiral SC via nano-SQUIDs

Furusaki *et al*, PRB (2001)

- Measurement of magnetic noise by nearby single-spin qubit probes can distinguish gapped and gapless order parameters



SC, Dolgirev *et al*,
arXiv: 2106.03859, 2106.05283

Outlook

- Alternative theoretical predictions:
  - (i) Phonons (s or f-wave)

    Chou, Wu *et al*, PRL (2021)
  - (ii) Electronic interactions: eg: Kohn-Luttinger mechanism – unconventional (higher angular momentum), RG study – conventional s-wave spin-singlet (AF Hund's), Collective modes – unconventional (p-wave?)

    Ghazaryan, Holder *et al*, arXiv: 2009.00011, Cea *et al*, arXiv: 2109.04535, You, Vishwanath, arXiv: 2109.04669, Dong, Levitov arXiv:2109.01133
- Open questions:
  - Connections to more complicated systems like moire graphene
  - Possible realization of SC in related few-layer graphene platforms (e.g., Bernal graphene) under perpendicular electric field

    Zhou *et al*, arXiv:2110.1137

Thank you for your attention!