Inter-valley coherence and fluctuation mediated superconductivity in rhombohedral trilayer graphene

Shubhayu Chatterjee

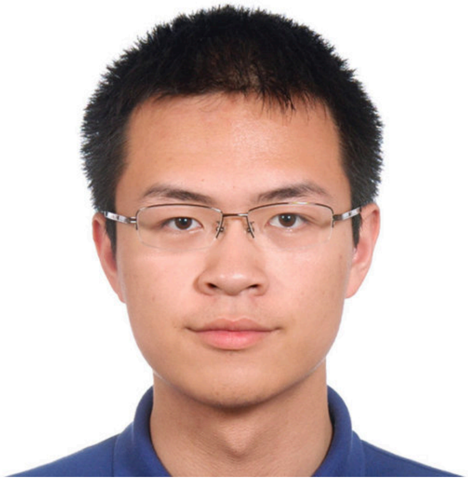
University of California, Berkeley

Quantum Materials 290K Seminar

University of California, Berkeley

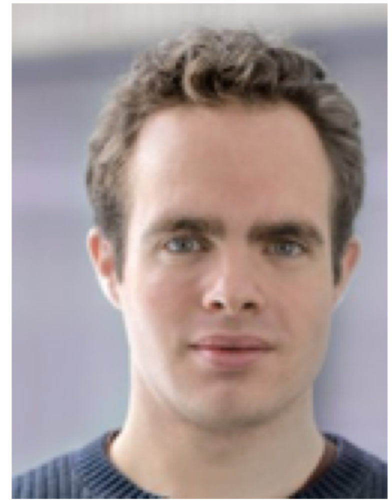
September 8, 2021

In collaboration with:



Taige Wang
UC Berkeley

Erez Berg
Weizmann Inst.

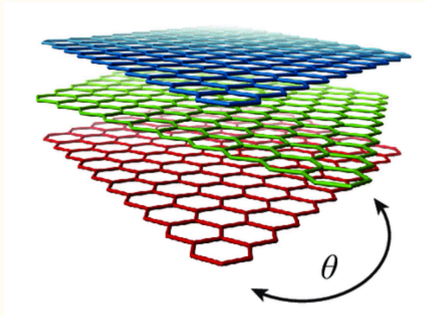


Mike Zaletel
UC Berkeley

SC, T. Wang, E. Berg and M. P. Zaletel, arXiv:2009.00002

Correlated physics in ABC trilayer graphene

- Superconductivity was discovered in several distinct moire graphene platforms

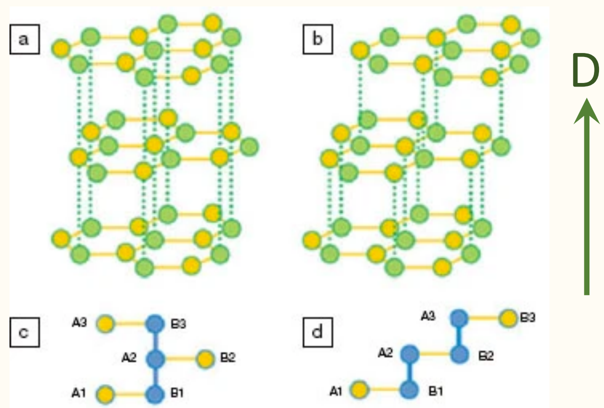


Cao *et al,* Nature (2018), Lu *et al,* Nature (2019), Yankowitz *et al*, Science (2019), Chen *et al*, Nature (2019) Several others…

Figure: Carr *et al*, Nano Letters (2020)

- Recently, correlated behavior and superconductivity have also been found in *non-moire* ABC trilayer graphene under a strong perpendicular electric field

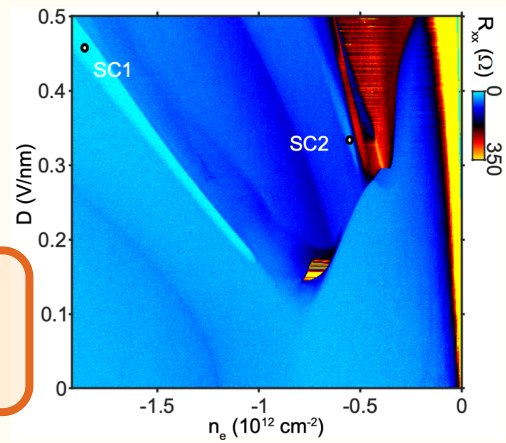
Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)



- Natural question: Are these related?

Correlated physics in ABC trilayer graphene

- Like monolayer graphene: ABC trilayer graphene has both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D
- Interestingly, superconductivity appears twice in the hole-doped phase diagram, both times on the cusp of iso-spin symmetry breaking



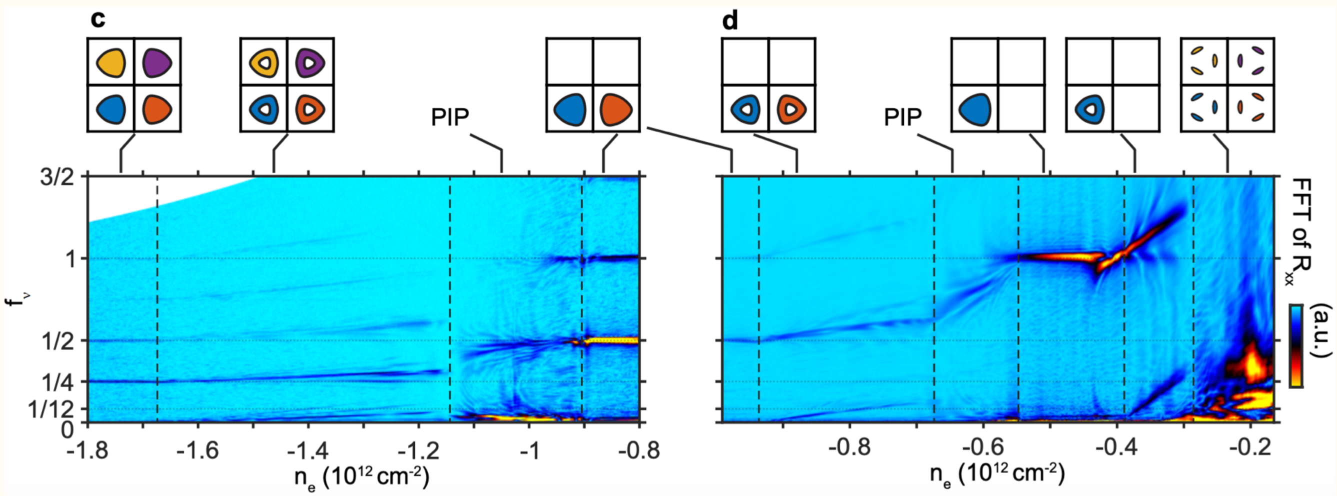
Large electric field, correlated physics

Small electric field, single-particle physics

Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)

Correlated physics in ABC trilayer graphene

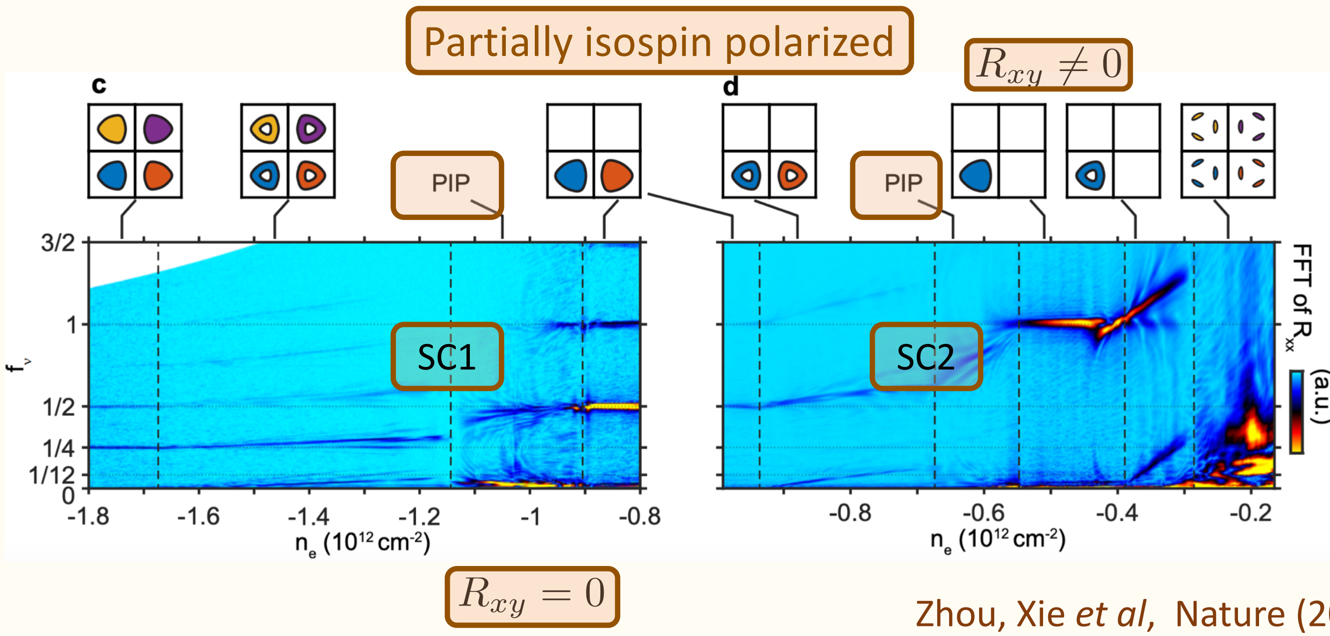
- Like monolayer graphene: ABC trilayer graphene have both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D



Zhou, Xie *et al*, Nature (2021)

Correlated physics in ABC trilayer graphene

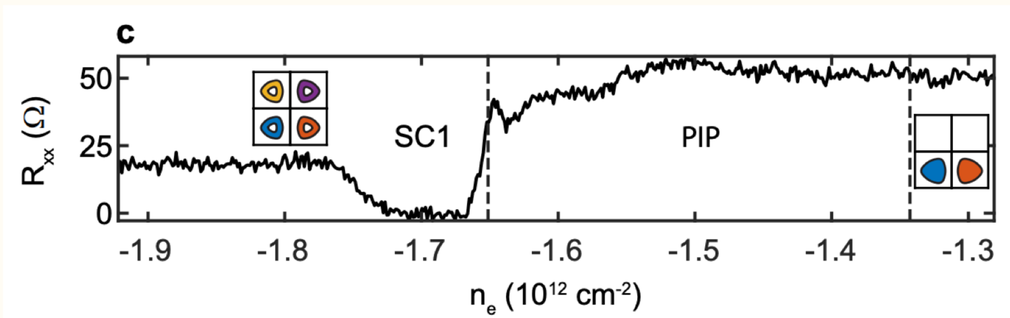
- Like monolayer graphene: ABC trilayer graphene have both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D

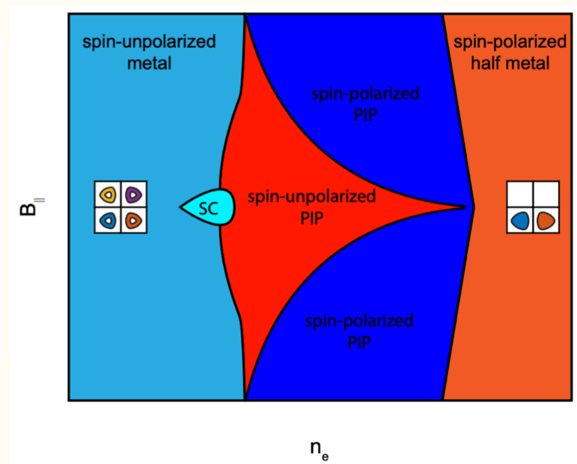


Zhou, Xie *et al*, Nature (2021)

Correlated physics in ABC trilayer graphene

- Like monolayer graphene: ABC trilayer graphene have both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D
- SC1 appears at higher hole-doping, adjacent to a spin-unpolarized PIP phase. SC1 obeys Pauli limit, $\mu_B B_{\parallel,c} \sim k_B T_c$.

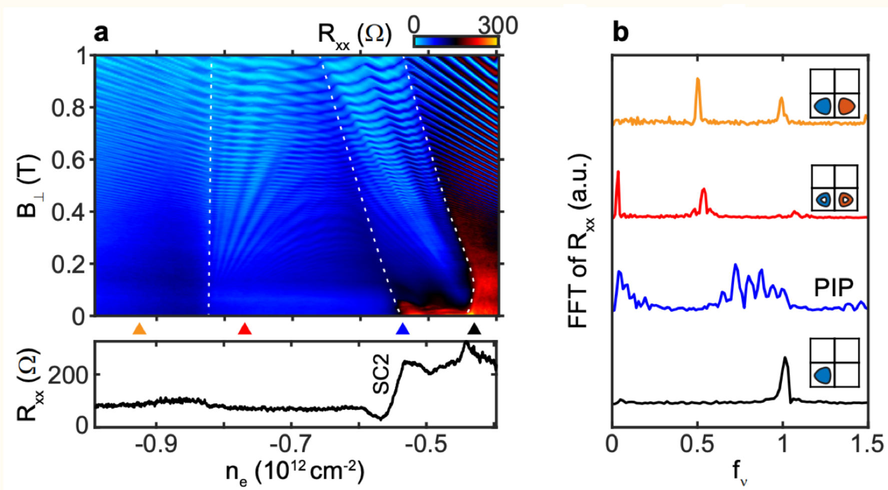




Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)

Correlated physics in ABC trilayer graphene

- Like monolayer graphene: ABC trilayer graphene have both valley and spin degrees of freedom (iso-spin)
- The metallic states at low doping show signatures of iso-spin symmetry breaking in quantum oscillations under large D
- SC2 appears at lower hole-doping, and adjacent to spin-polarized phases. SC2 strongly violates the Pauli limit, $\mu_B B_{\parallel,c} \gg k_B T_c$



Zhou, Xie *et al*, Nature (2021)
Zhou *et al*, Nature (2021)

Correlated physics in ABC trilayer graphene

- What are the broken symmetries in the PIP phases?
- What are the pairing symmetries of SC1 and SC2 that arise at the boundaries of PIP phases?

- Do electronic correlations play any role in superconductivity?

Correlated physics in ABC trilayer graphene

- What are the broken symmetries in the PIP phases?

Inter-valley coherent order – spin-triplet SDW or spin-polarized CDW

- What are the pairing symmetries of SC1 and SC2 that arise at the boundaries of PIP phases?

- Do electronic correlations play any role in superconductivity?

Correlated physics in ABC trilayer graphene

- What are the broken symmetries in the PIP phases?

Inter-valley coherent order – spin-triplet SDW or spin-polarized CDW

- What are the pairing symmetries of SC1 and SC2 that arise at the boundaries of PIP phases?

SC1: Spin-singlet, gapped chiral p + i p or nodal f-wave
SC2: Spin-polarized, non-unitary p-wave/f-wave

- Do electronic correlations play any role in superconductivity?

Correlated physics in ABC trilayer graphene

- What are the broken symmetries in the PIP phases?

Inter-valley coherent order – spin-triplet SDW or spin-polarized CDW

- What are the pairing symmetries of SC1 and SC2 that arise at the boundaries of PIP phases?

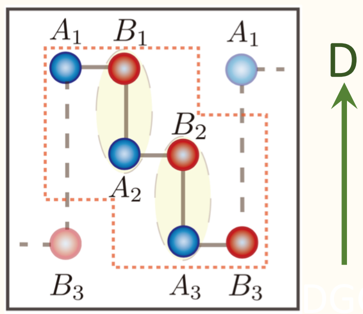
SC1: Spin-singlet, gapped chiral p + i p or nodal f-wave
DGG, RHDGG, RH
SC2: Spin-polarized, non-unitary p-wave/f-wave

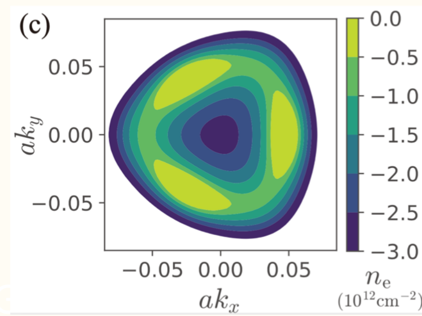
- Do electronic correlations play any role in superconductivity?

Possibly, near critical IVC fluctuations can act as pairing glue

Hamiltonian and symmetries

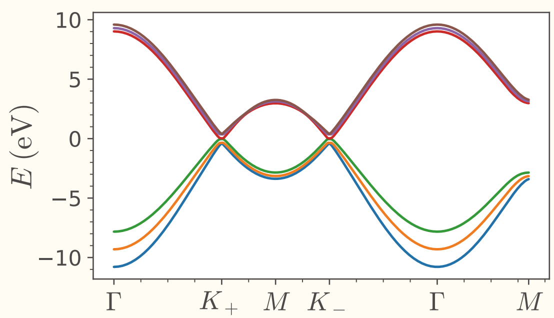
- Band structure of ABC trilayer graphene: Cubic band touching at K/K′ → 3 Dirac cones (per valley/spin) → gapped out by D





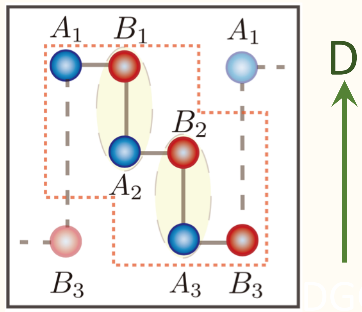
Zhang *et al*, PRB (2010)

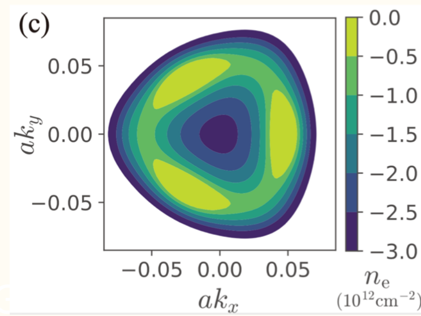
- DOS is enhanced by D at low doping



Hamiltonian and symmetries

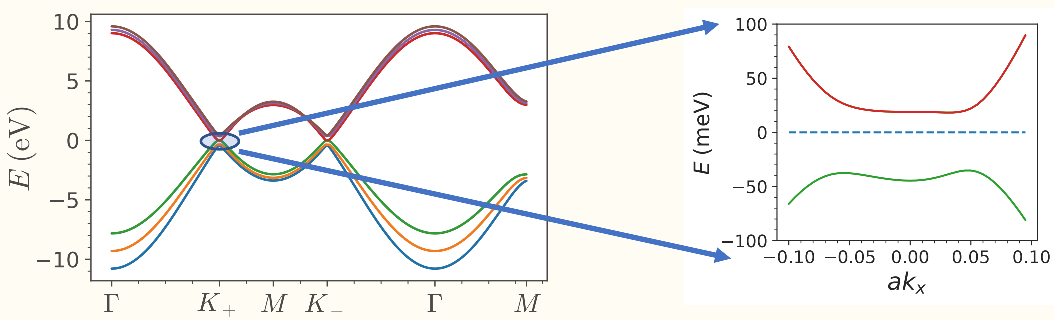
- Band structure of ABC trilayer graphene: Cubic band touching at K/K′ → 3 Dirac cones (per valley/spin) → gapped out by D





Zhang *et al*, PRB (2010)

- DOS is enhanced by D at low doping



Hamiltonian and symmetries

- Interacting Hamiltonian in band-basis:

$$H = \sum_{n,\tau,s,\mathbf{k}} \varepsilon_{n,\tau,\mathbf{k}} \psi^{\dagger}_{n,\tau,s,\mathbf{k}} \psi_{n,\tau,s,\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{q}} V_c(\mathbf{q}) : \rho(\mathbf{q}) \rho(-\mathbf{q}) :$$

$$\rho(\mathbf{q}) = \sum_{\tau,s,\mathbf{k}} \psi^{\dagger}_{n,\tau,s,\mathbf{k}} [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} \psi_{n',\tau,s,\mathbf{k}+\mathbf{q}}, \quad [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} = \langle u_{n,\tau,\mathbf{k}} | u_{n',\tau,\mathbf{k}} \rangle$$

- Symmetries: $U(1)_c$, $U(1)_v$, C_3 , M_x , T, $SU(2)_s \rightarrow SU(2)_+ \times SU(2)_-$

No Hund's coupling

Hamiltonian and symmetries

- Interacting Hamiltonian in band-basis:

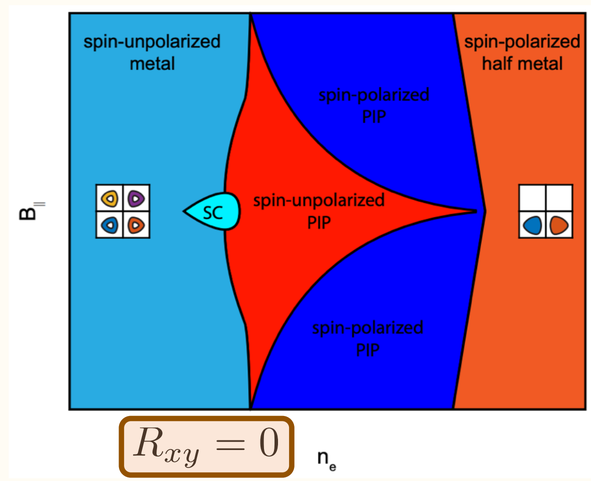
$$H = \sum_{n,\tau,s,\mathbf{k}} \varepsilon_{n,\tau,\mathbf{k}} \psi^\dagger_{n,\tau,s,\mathbf{k}} \psi_{n,\tau,s,\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{q}} V_c(\mathbf{q}) : \rho(\mathbf{q})\rho(-\mathbf{q}) :$$

$$\rho(\mathbf{q}) = \sum_{\tau,s,\mathbf{k}} \psi^\dagger_{n,\tau,s,\mathbf{k}} [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} \psi_{n',\tau,s,\mathbf{k}+\mathbf{q}}, \quad [\lambda_{\mathbf{q}}(\mathbf{k})]^{nn'} = \langle u_{n,\tau,\mathbf{k}} | u_{n',\tau,\mathbf{k}} \rangle$$

- Symmetries: $U(1)_c$, $U(1)_v$, C_3 , M_x , T, $SU(2)_s \rightarrow SU(2)_+ \times SU(2)_-$

No Hund's coupling

- PIP phase near SC1:
  - (i) No spin-polarization,
  - (ii) No anomalous Hall effect (not valley-polarized)

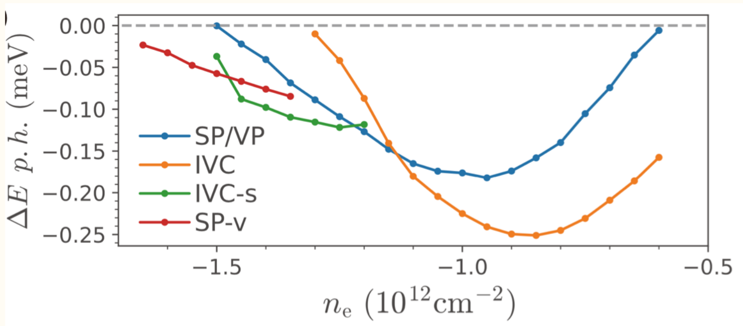


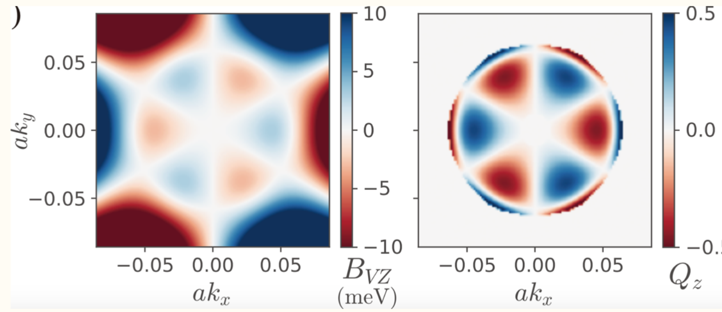
- Inter-valley coherent (IVC) phase

$$\langle \psi^\dagger_{+,s,\mathbf{k}} \psi_{-,s',\mathbf{k}} \rangle \neq 0$$

Inter-valley coherent phases

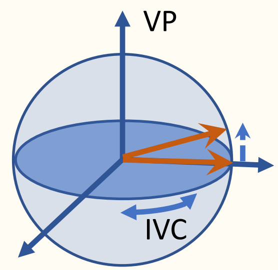
- Hartree-Fock numerics shows IVC is competitive





$B_{VZ}(\mathbf{k})$ $\langle\tau_z(\mathbf{k})\rangle_{IVC}$

- From time-reversal $\varepsilon_+(k) = \varepsilon_-(-k)$, but generally $\varepsilon_+(k) \neq \varepsilon_-(k) \rightarrow$ Local valley Zeeman-field $B_{VZ}(k) = \varepsilon_+(k) - \varepsilon_-(k)$



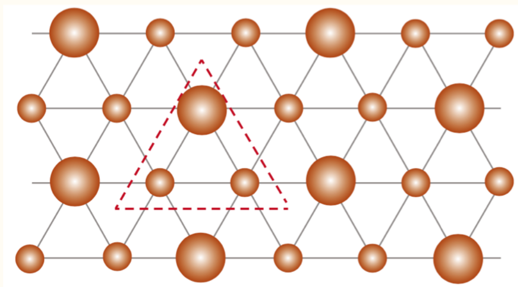
- IVC phase can gain kinetic energy by local *canting*

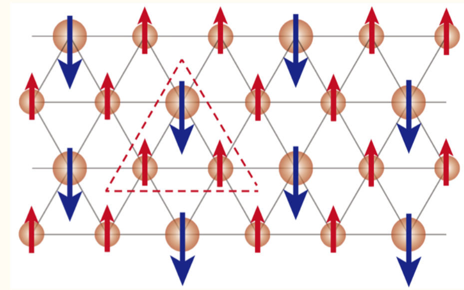
- Can compensate for energy penalty due to phase winding

Bultinck, SC *et al*, PRL (2020)

Inter-valley coherent phases

- IVC states form a U(2) manifold - Inter-valley Hund's term picks either spin-singlet or triplet.





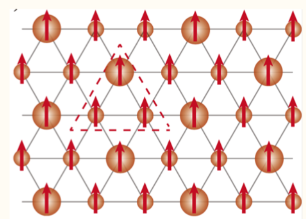
Singlet IVC: CDW $n_{\mathrm{S}}^{\mathrm{IV}} = \sum_{\mathbf{R}} e^{i(\mathbf{K}-\mathbf{K}')\cdot\mathbf{R}} \rho(\mathbf{R})$

Triplet IVC: SDW $\mathbf{n}_{\mathrm{T}}^{\mathrm{IV}} = \sum_{\mathbf{R}} e^{i(\mathbf{K}-\mathbf{K}')\cdot\mathbf{R}} \mathbf{s}(\mathbf{R})$

- Landau theory allows term of the form $\mathbf{n_T}(\mathbf{Q}).\mathbf{n_T}(\mathbf{Q})\ \rho(\text{-}2\mathbf{Q})$
  $\rightarrow$ weak CDW is nucleated by SDW IVC

Zachar *et al*, PRB (1998)

- PIP phase near SC2 $\rightarrow$ ferromagnetic IVC



FM CDW

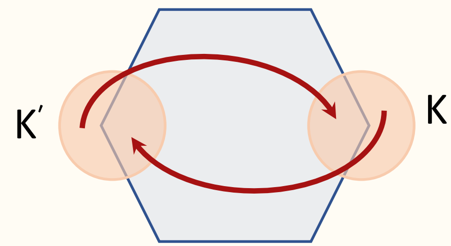
Aleiner *et al*, PRB (2007)
Cvetkovic, Vafek, arXiv:1210.4923

Inter-valley coherent phases

- Hund's coupling arises from inter-valley scattering of electrons

$$H_{\mathrm{Hund's}} = -J_H \sum_{\mathbf{q}} \mathbf{s}_{+-}(\mathbf{q}) \cdot \mathbf{s}^{\dagger}_{+-}(\mathbf{q})$$

$$\mathbf{s}_{+-}(\mathbf{q}) = \sum_{\mathbf{k}} \lambda^{+-}_{\mathbf{q}}(\mathbf{k}) \psi^{\dagger}_{+,s,\mathbf{k}} \mathbf{s}_{ss'} \psi^{\dagger}_{-,s',\mathbf{k}+\mathbf{q}}$$



- Surprisingly, distinct from usually assumed (symmetry-allowed) form:

$$\tilde{H}_{\mathrm{Hund's}} = -\tilde{J}_H \sum_{\mathbf{q}} \mathbf{s}_{+}(\mathbf{q}) \cdot \mathbf{s}_{-}(-\mathbf{q}) \qquad \mathbf{s}_{\tau}(\mathbf{q}) = \sum_{\mathbf{k}} \lambda^{\tau\tau}_{\mathbf{q}}(\mathbf{k}) \psi^{\dagger}_{\tau,s,\mathbf{k}} \mathbf{s}_{ss'} \psi^{\dagger}_{\tau,s',\mathbf{k}+\mathbf{q}}$$

- Directly favors spin-triplet IVC or SDW IVC when ferromagnetic

$$\mathbf{s}_{+-}(\mathbf{q}=0) = \mathbf{n}_{\mathrm{T}}$$

- Local repulsive Hubbard U gives rise to ferromagnetic $J_H > 0$, but local repulsive interactions disfavor a CDW/singlet-IVC

IVC fluctuation mediated superconductivity

- IVC order parameter fluctuations can lead to superconductivity

$$H_{\mathrm{IVC}}^{\mathrm{eff}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \mathrm{Tr}[n^{\mathrm{IV}}(\mathbf{q})[n^{\mathrm{IV}}(\mathbf{q})]^\dagger], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\mathrm{IVC}}^{-2}}$$

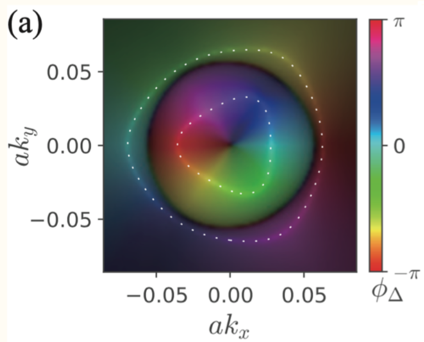
- Unconventional superconductivity is preferred by this mechanism

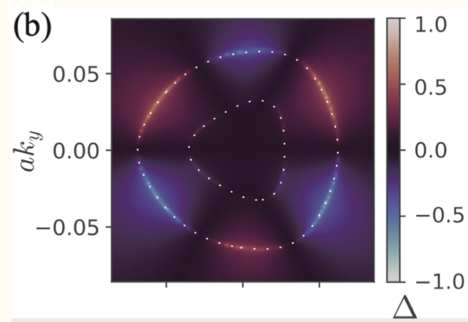
$$\langle \psi^\dagger_{+,s,\mathbf{k}} \psi_{-,s',\mathbf{k}} \rangle = \varepsilon_{ss'} f_{\mathbf{k}} \qquad \langle H_{\mathrm{IVC}}^{\mathrm{eff}} \rangle \approx \sum_{\mathbf{k},\mathbf{k}'} g_{\mathbf{q}=-\mathbf{k}-\mathbf{k}'} |\lambda^{+-}_{\mathbf{q}=-\mathbf{k}-\mathbf{k}'}(\mathbf{k})|^2 \; f^*_{\mathbf{k}} f_{\mathbf{k}'}$$

$q \to 0$ limit

$$f_{-\mathbf{k}} = -f_{\mathbf{k}}$$

- Precise channel depends on the IVC correlation length ξ_{IVC}





Large ξ_{IVC} : chiral gapped $p_x \pm i\, p_y$

Small ξ_{IVC} : nodal f-wave, $\mathrm{Im}[(p_x + i\, p_y)^3]$

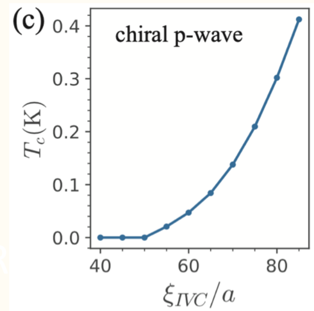
Distinct from s-wave via $M_x T$ (not C_3)

IVC fluctuation mediated superconductivity

- What is T_c for unconventional superconductivity?

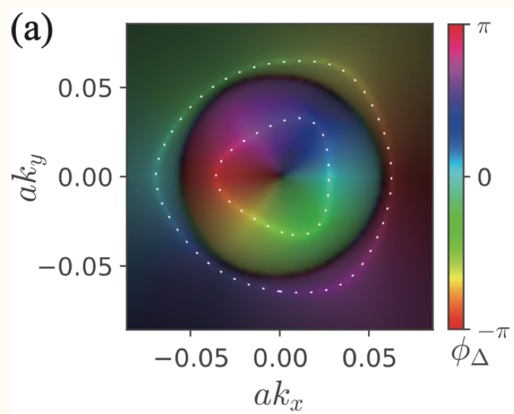
$$H^{\text{eff}}_{\text{IVC}} = -\sum_{\mathbf{q}} g_{\mathbf{q}} \text{Tr}[n^{\text{IV}}(\mathbf{q})[n^{\text{IV}}(\mathbf{q})]^\dagger], \quad g_{\mathbf{q}} = \frac{g}{q^2 + \xi_{\text{IVC}}^{-2}}$$

$$g \sim \frac{\text{hole-density } n_h}{\text{DOS at Fermi surface}}$$



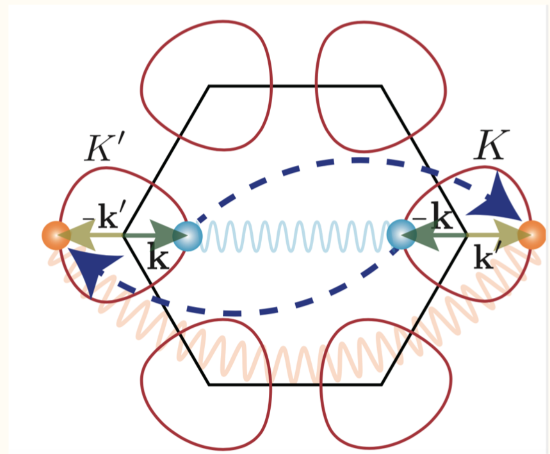
- Including Coulomb repulsion, prefers opposite sign on outer and inner Fermi surfaces

- Spin-singlet and spin-triplet superconductors are degenerate in the $SU(2)_+ \times SU(2)_-$ limit

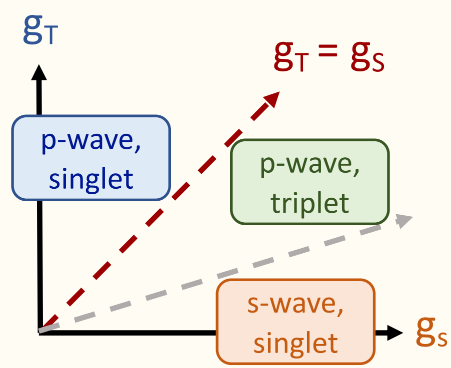


IVC fluctuation mediated superconductivity

- FM Hund's coupling amplifies SDW fluctuations → leads to *spin-singlet* superconductivity: consistent with SC1
- AFM Hund's coupling amplifies CDW fluctuations and leads to *spin-triplet* superconductivity perturbatively
- At large CDW fluctuations, *spin-singlet* s-wave superconductivity is preferred
- Spin-polarized IVC can lead to non-unitary spin-triplet p/f-wave superconductor by similar mechanism (consistent with SC2)

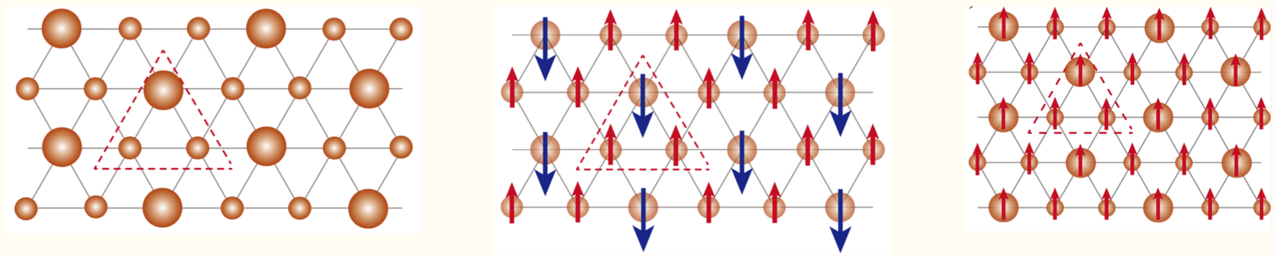


Scalapino, Phys. Reports (1995)



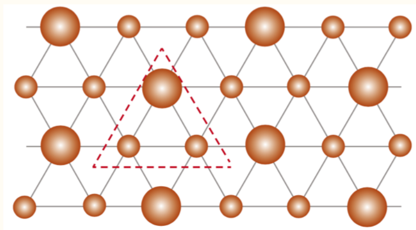
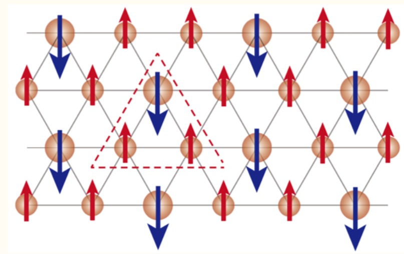
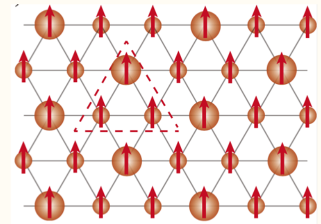
Experimental probes

- For IVC states, STM or its spin-polarized cousin to resolve charge (spin) density at the atomic scale



Experimental probes

- For IVC states, STM or its spin-polarized cousin to resolve charge (spin) density at the atomic scale

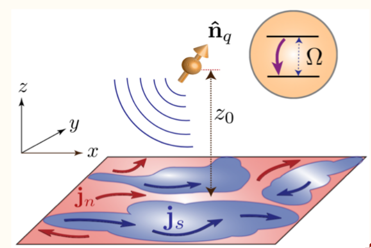
- Unconventional superconductors are susceptible to disorder

Abrikosov, Gorkov, JETP (1961)

- p + i p superconductors have spontaneous edge currents

Furusaki *et al*, PRB (2001)

- Measurement of magnetic noise by nearby single-spin qubit probes can distinguish gapped and gapless order parameters



SC, Dolgirev *et al*,
arXiv: 2106.03859, 2106.05283

Outlook

- Other theoretical predictions:
  - (i) Phonons (s or f-wave) — Chou, Wu *et al*, arXiv:2106.13231
  - (ii) Kohn Luttinger mechanism: chiral p-wave, also d/s wave gap changes signs between inner and outer Fermi surfaces

    Ghazaryan, Holder *et al*, arXiv: 2009.00011
  - (iii) Collective modes – unconventional (p-wave?)

    Dong, Levitov arXiv:2109.01133
- Open questions:
  - Connections to more complicated systems like moire graphene
  - Possible realization of similar physics in related few-layer graphene platforms under perpendicular electric field

Thank you for your attention!