01/08/2009 – Review for Test

When integrating two unrelated functions, integration by parts is used.

$$\int[u\,dv] = uv - \int[v\,du]$$

The function representing u is determined by descending through the following list, with the mnemonic device "LIPET" for "Long Island PET."

- Logarithms
- Inverse trigonometric functions
- Polynomials (i.e. algebraic functions)
- e^u (functions ...)
- trigonometric functions → It may be necessary to repeat the process. Ex. on next page

Tabular integration can also be used when one function can be differentiated in some degree to zero and the other function is easy to integrate.

Ex. $\int[x^3 \cos 2x \, dx] \rightarrow \frac{x^3 \sin 2x}{2} + \frac{3x^2 \cos 2x}{4} - \frac{3x \sin 2x}{4} - \frac{3 \cos 2x}{8}$

x^3	$+$	$\cos 2x$
$3x^2$	$-$	$\frac{1}{2}\sin 2x$
$6x$	$+$	$-\frac{1}{4}\cos 2x$
6	$-$	$-\frac{1}{8}\sin 2x$
0		$\frac{1}{16}\cos 2x$

Study Guide

Study rules for differentiation/integration

a^x, inverse trig

u-sub $\boxed{+C}$ $\boxed{dx}$

log/ln/e rules

completing the square

Integration by Parts

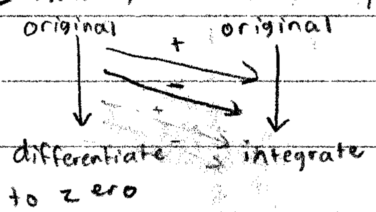
$$\int [u\,dv] = uv - \int [v\,du]$$

LIPET (u in ascending order)

- Logarithmic functions
- Inverse trigonometric functions
- Polynomials (algebraic functions)
- e^u functions
- Trigonometric functions

Tabular Integration

one function can be differentiated in some degree to zero and other function is easy to integrate



Variable Separable

treat $\frac{dy}{dx}$ as a fraction

Partial Fractions

Euler's Method

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} \Rightarrow \Delta y = \frac{dy}{dx}\Delta x$$

PRGM: EULER

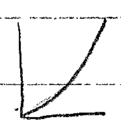
Definite Integral

can use graphing calculator

Unrestricted Growth

$$\frac{dy}{dt} = ky \Rightarrow y = Ce^{kt}$$

C: initial



Newton's Law of Cooling

$$\frac{dy}{dt} = \overset{k(A-y)}{-k(y-A)} \Rightarrow y = A + Ce^{-kt}$$

A: temperature of surrounding

C: initial temperature − surrounding temperature

Logistic Growth

$$\frac{dy}{dt} = \frac{ky}{m}(m-y) \Rightarrow y = \frac{m}{1+Ce^{-kt}}$$

m: carrying capacity

rate of increase is greatest between y=0 and y=m

02/25/2009 - Review for L'Hôpital's - Rates of Growth - Improper Integrals - Direct Compariso[n]

L'Hôpital's Rule

For the limits of fractions in the indeterminable form $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

If $f(a)=g(a)=0$ or ∞ and $f'(a)$ and $g'(a)$ both exist, then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Special Cases of L'Hôpital's Rules

For the limits of an indeterminable form $0^0, \infty^0, 1^\infty$:

$$\lim_{x\to a} f(x) = e^{\lim_{x\to a} \ln[f(x)]}$$

Rates of Growth

From greatest order to least: $x!, a^x, x^a, x, \ln x$ / $\log x$, where $a>1$

- greater values of a correspond to higher rates of growth

Let $f(x)$ and $g(x)$ be positive for x sufficiently large, - f grows faster than g and g grows slower than f if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$ or $\lim_{x\to\infty}\frac{g(x)}{f(x)}=0$

- f and g grow at the same rate if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=L$, where L is a non-zero constant

Transitivity of the rate of growth: When x approaches infinity, if f grows at the same rate as g and g grows at the same rate as h, then f grows at the same rate as h.

$f=og$: f is of lower order than g $\left(\lim_{x\to\infty}\frac{f(x)}{g(x)}=0\right)$

$f=Og$: f is at most (of lower or equal order) the order of g $\left(\lim_{x\to\infty}\frac{f(x)}{g(x)}=M\right.$, where M is a finite constant$\left.\right)$

Improper Integrals

Definite integrals with a "problem spot" are expressed as integrals with the "problem spots" as limits in the interval. If the "problem spot" occurs in the middle of the interval, one-sided limits are used.

Converge/Diverge

Converge: function "totals up" to a finite number

diverges: Function does not "total up" to a finite number; infinity; does not exist

$\int_a^b \frac{1}{x^p}$ converges $a<p<b$?
diverges $0<p\le a$ and $p\ge b$

<u>Direct Comparison Test</u>

Let f and g be continuous on $[a,\infty]$ with $0\le f(x)\le g(x)$,
For all $x\ge a$

- $\int_a^\infty f(x)\,dx$ converges if $\int_a^\infty g(x)\,dx$ converges
- $\int_a^\infty g(x)\,dx$ diverges if $\int_a^\infty f(x)\,dx$ diverges

03/23/2009 - HARDEST LESSON OF THE YEAR

Aim: How do we estimate the remainder of a Taylor Polynomial?

Finding error bound of the sum of an alternating series: estimate is $\pm$ within the first neglected term

Estimate $-$ first neglected term $\leq$ Actual Sum $\leq$ Estimate $+$ first neglected term

Taylors Theorem

If a function f has derivatives of all orders in an open interval I containing c, then for each positive integer n and for each x in I, the following is true:

$$f(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^n(a)(x-a)^n}{n!} + R_n(x),$$

$n+1$ = first neglected term

where $R_n(x) = \dfrac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}$ for some c between a and x

$\therefore f(x) = P_n(x) + R_n(x)$ Exact Value = Taylor Estimate + Remainder

Remainder Estimation Theorem (La Grange Error Bound)

$$|R_n(x)| \leq \max f^{n+1}(c) \left| \frac{(x-a)^{n+1}}{(n+1)!} \right|$$

Vectors

- a segment with magnitude (size) and direction, a directed line segment

Two vectors are equal if they have the same magnitude and direction.

Component form of vector ($\langle x, y \rangle$) has its initial point at the origin and terminal point at (x, y).

Linear Combination: $2i + 5j = \langle 2, 5 \rangle$

Unit Vector: vector with magnitude of one

Polar

$(r, \theta) = (-r, \pi + \theta)$

$(x, y) = (\sqrt{x^2+y^2}, \tan^{-1}\frac{y}{x})$ check reference angle

$(r, \theta) = (r\cos\theta, r\sin\theta)$

To differentiate polar equations, separate $r(\theta)$ into $x = r\cos\theta$ and $y = r\sin\theta$; differentiate parametrically with $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

Formulas

$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$ (used for differentiating $\cos^2\theta$)

$$A_k = \int_\alpha^\beta \frac{1}{2} r^2 \, d\theta \quad \text{(Area - polar)}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{dx}{dt}}$$

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \, d\theta \quad \text{(Arc Length - parametric)}$$

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta \quad \text{(Arc Length - polar)}$$

Study and practice finding the area between two polar curves, including determining the limits of the definite integral by setting $r = 0$ or $r_1 = r_2$.