STATISTICS SECTION 10.4

Inference as a Decision

Congratulations, you've gotten to the last section on the statistics midterm, that is of course if you are studying sections sequentially, and not in reverse order for some reason.

Sometimes a decision as a result of an inference (inference as a decision; p. 567–579) is preferred over measuring the strength of evidence.

acceptance sampling: accepting or rejecting an entire group based on the outcome of a sample
example: a batch of food meeting (H_0) or not meeting (H_a) a standard

Type I and II Errors

http://www.homebirthdebate.com/errorsb.gif

There a two ways for a decision between H_0 and H_a to be wrong.

Type I error: rejecting H_0 (accepting H_a) when H_0 is true (i.e. observing a difference when one does not exist; false positive)
example: rejecting a good batch
Type II error: accepting H_0 (rejecting H_a) when H_a is true (i.e. failing to observe a difference when one exists; false negative)
example: accepting a bad batch

	H_0 true	H_a true
Reject H_0	Type I error	correct decision
Accept H_0	correct decision	Type II error

Calculating Error

Type I error.

$$p = \alpha$$

The significance level α is the probability that the test will reject the null hypothesis when it is in fact true.

Type II error.

$$p = p\left(\mu_0 - z^*\left(\frac{\sigma}{\sqrt{n}}\right) \le \bar{x} \le \mu_0 + z^*\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$\begin{aligned} \bar{x}_l &= \mu_0 - z^*\left(\frac{\sigma}{\sqrt{n}}\right) \\ \bar{x}_u &= \mu_0 + z^*\left(\frac{\sigma}{\sqrt{n}}\right) \end{aligned}$$

$$p = p\left(\frac{\bar{x}_l - \mu_a}{\sigma/\sqrt{n}} \le Z \le \frac{\bar{x}_u - \mu_a}{\sigma/\sqrt{n}}\right)$$

example. $\mu_0 = 5$ vs. $\mu_a = 5.2$ (alternative)
$\alpha = 0.05$; $z^* = 1.96$
$\sigma = 0.1$
$n = 5$

$$\begin{aligned} p &= p\left(5 - 1.96\left(\frac{0.1}{\sqrt{5}}\right) \le \bar{x} \le 5 + 1.96\left(\frac{0.1}{\sqrt{5}}\right)\right) \\ p &= p(3.6852 \le \bar{x} \le 6.3148) \end{aligned}$$

$$\begin{aligned} \bar{x}_l &= 4.9123 \\ \bar{x}_u &= 5.0877 \end{aligned}$$

$$\begin{aligned} p &= p\left(\frac{4.9123 - 5.2}{0.1/\sqrt{5}} \le Z \le \frac{5.0877 - 5.2}{0.1/\sqrt{5}}\right) \\ p &= p(-6.4332 \le Z \le -2.5111) \\ p &= 0.006 \end{aligned}$$

Power.

power: probability that test will reject the null hypothesis when a particular alternative value is true
$\text{power} = 1 - p(\text{Type II error})$

Dara Adib.