

# Math Trigonometry Review

## CHSN Review Project



This review guide was written by Dara Adib. It was designed for a test on trigonometry administered by Jeanine Lennon to her Math 12H (4H/Precalculus) class on February 12, 2008, but also applies to trigonometry material in Math 11H (3H).

This is a development version of the text that should be considered a work-in-progress.

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# Angles and Sectors of Circles

## Radians

- To convert degrees to radians, multiply by  $\frac{\pi}{180}$ .
- To convert radians to degrees, multiply by  $\frac{180}{\pi}$ .
- One radian equals  $\frac{180}{\pi}$  or approximately  $57.296^\circ$ .

## Coterminal Angles

**coterminal angles** different angles that have the same initial and terminal ray

The differences between coterminal angles are multiples of  $360^\circ$ .

Examples:  $60^\circ$ ,  $-300^\circ$ ,  $420^\circ$

## Sectors

**sector** part of a circle formed by two radii and an arc

$s = r\theta$  (arc length = radius  $\times$  angle)

The angle must be represented in radians.

**apparent size** the angle that an object subtends at one's eyes; this explains why an object appears to be smaller when farther away

# Trigonometric Functions

## Reciprocal Functions

The following are reciprocal functions.

- $\csc x = \frac{1}{\sin x}$
- $\sec x = \frac{1}{\cos x}$
- $\cot x = \frac{1}{\tan x}$

As a result, the following are true as well.

- $\sin x = \frac{1}{\csc x}$
- $\cos x = \frac{1}{\sec x}$
- $\tan x = \frac{1}{\cot x}$

$\theta$ (degrees)	$\theta$ (radians)	Point	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ$	0	(1,0)	0	1	0
$90^\circ$	$\frac{\pi}{2}$	(0,1)	1	0	undefined
$180^\circ$	$\pi$	(-1,0)	0	-1	0
$270^\circ$	$\frac{3\pi}{2}$	(0,-1)	-1	0	undefined

  

$\theta$ (degrees)	$\theta$ (radians)	Point	$\cot \theta$	$\sec \theta$	$\csc \theta$
$0^\circ$	0	(1,0)	undefined	1	undefined
$90^\circ$	$\frac{\pi}{2}$	(0,1)	0	undefined	1
$180^\circ$	$\pi$	(-1,0)	undefined	-1	undefined
$270^\circ$	$\frac{3\pi}{2}$	(0,-1)	0	undefined	-1

Table 1: quadrantal angles

## Cofunctions

The following are cofunctions.

- $\sin x = \cos(90 - x)$
- $\tan x = \cot(90 - x)$
- $\sec x = \csc(90 - x)$

As a result, the following are true as well.

- $\cos x = \sin(90 - x)$
- $\cot x = \tan(90 - x)$
- $\csc x = \sec(90 - x)$

## Special Angles

### Quadrantal Angles

**quadrantal angles** angles that have a terminal side coinciding with a coordinate axis

The value of the trigonometric function (i.e. sine, cosine, tangent) is determined by the coordinates of the points on a unit circle. By definition, the point  $(x, y)$  on a unit circle corresponds to  $(\cos \theta, \sin \theta)$ .

As a result, quadrantal angles can be determined easily. Table 1 lists quadrantal angles and the values of trigonometric functions. Since  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ , the tangent function of an angle can be found by dividing the sine function of the angle by the cosine function of the angle.

Quadrant	Reference Angle ( $\beta$ )
I	$\beta = \theta$
II	$\beta = 180^\circ - \theta$
III	$\beta = \theta - 180^\circ$
IV	$\beta = 360^\circ - \theta$

Table 2: reference angles

## Angles greater than $90^\circ$

Angles greater than  $90^\circ$  may be in different quadrants. The trigonometric functions vary over whether the trigonometric function of a certain angle is positive or negative. It is useful to remember the mnemonic device "All Students Take Calculus."

- All trigonometric functions are positive in the first quadrant
- Sine and cosecant are positive in the second quadrant
- Tangent and cotangent are positive in the third quadrant
- Cosine and secant are positive in the fourth quadrant
- The remaining trigonometric functions in each quadrant are negative.

## Reference Angles

The use of reference angles is a way to simplify the calculation of the values of trigonometric functions in different quadrants. Refer to Table 2, where  $\theta$  is the angle (in degrees) and  $\beta$  is the reference angle.

## Sum and Difference Formulas

### Sum Formulas

#### Sine

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

#### Cosine

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

#### Tangent

The derivation for the tangent of the sum of two angles follows.

$$\tan(A + B)$$

$$\frac{\sin(A + B)}{\cos(A + B)}$$

$$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$\frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B}$$

## Difference Formulas

### Sine

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

### Cosine

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

### Tangent

The derivation for the tangent of the difference of two angles follows.

$$\tan(A - B)$$

$$\frac{\sin(A - B)}{\cos(A - B)}$$

$$\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

$$\frac{\frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}}$$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Identities

### Pythagorean Identities

1.  $\sin^2 \theta + \cos^2 \theta = 1$
2.  $1 + \tan^2 \theta = \sec^2 \theta$
3.  $1 + \cot^2 \theta = \csc^2 \theta$

## Quotient Identities

$$1. \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$2. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Graphing Trigonometric Functions

### General Form

The general form of a trigonometric function is one of the following (sine or cosine).

$$y = a \sin b(x \pm c) \pm d$$

$$y = a \cos b(x \pm c) \pm d$$

In other words, it can be understood to mean the following.

$$y = \text{amplitude} \times \sin [\text{frequency} (x - \text{horizontal translation})] + \text{vertical translation}$$

$$y = \text{amplitude} \times \cos [\text{frequency} (x - \text{horizontal translation})] + \text{vertical translation}$$

A negative amplitude means a reflection over the  $x$ -axis. The vertical translation may be kept in front only to prevent the ambiguity that the number may be part of the trigonometric function.

### Frequency and Period

The following statements are true regarding the relationship between frequency and period.

$$\text{frequency} = \frac{2\pi}{\text{period}}$$

$$\text{period} = \frac{2\pi}{\text{frequency}}$$

### Sine and Cosine

The sine function follows the form zero, maximum, zero, minimum, before any translation.

The cosine function follows the form maximum, zero, minimum, zero, before any translation.

As a result, sine and cosine are horizontal translations of each other.

## Double and Half Angle Formulas

### Double Angle Formulas

The following are determined by plugging in an angle twice into the sum formulas.

### Sine

$$\sin 2A = 2 \sin A \cos A$$

### Cosine

$$\cos 2A = \cos^2 A - \sin^2 A$$

**More Cosine Formulas** However, more formulas for the double of an angle with cosine can be determined since  $\sin^2 A + \cos^2 A = 1$  (first Pythagorean identity).

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

### Tangent

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

## Half Angle Formulas

### Sine

$$\sin \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

### Cosine

$$\cos \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

### Tangent

$$\tan \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$