AP Physics C Review Mechanics

CHSN Review Project

This review guide was written by Dara Adib based on inspiration from Shelun Tsai's review packet. It is designed as preparatory information for the AP[1] Physics C Mechanics Exam on May 11, 2009, but may still be useful for other purposes as well.

This is a development version of the text that should be considered a work-in-progress.

This review guide is developed by the CHSN Review Project. To download this review guide and other review guides, visit chsntech.org.

[1] AP is a registered trademark of the College Board, which was not involved in the production of, and does not endorse, this product.

"Why do we love ideal worlds? ... I've been doing this for 38 years and school is an ideal world." — Steven Henning

Contents

Graphic Credits

- Figure 1 on page 3 is based off a public domain graphic by Concordia College and vectorized by Stannered: http://en.wikipedia.org/wiki/File:Incline.svg.
- Figure 2 on page 3 is based off a public domain graphic by Mpfiz: http://en.wikipedia.org/wiki/File:AtwoodMachine.svg.
- Figure 5 on page 7 is a public domain graphic by Rsfontenot: http://en.wikipedia.org/wiki/File:Reference_line.PNG.
- Figure 6 on page 7 was drawn by Enoch Lau and vectorized by Stannered: http://en.wikipedia.org/wiki/File:Angularvelocity.svg. It is licensed under the Creative Commons Attribution-Share Alike 2.5 license: http://creativecommons.org/licenses/by-sa/2.5/.
- Figure 7 on page 8 is based off a public domain graphic by Mazemaster: http://en.wikipedia.org/wiki/File:Simple_Harmonic_Motion_Orbit.gif.
- Figure 8 on page 9 is a public domain graphic by Chetvorno: http://en.wikipedia.org/wiki/File:Simple_gravity_pendulum.svg.

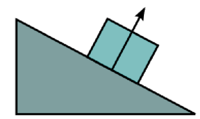

Figure 1: Normal Force

Kinematic Equations

$$\Delta x = \frac{1}{2}at^2 + v_0 t$$

$$\Delta v = at$$

$$(v)^2 - (v_0)^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v_0 + v}{2} \times t$$

Free Body Diagrams

N Normal Force
f Frictional Force
T Tension
mg Weight

$$F = ma$$

In a particular direction:

$$\Sigma F = (\Sigma m)a$$

Atwood's Machine[2]

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2}$$

[2] Pulley and string are assumed to be massless.

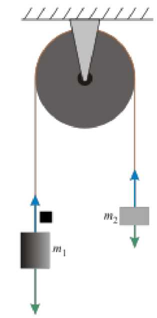

Figure 2: Atwood's Machine

Figure 3: Draw a banked curve diagram

Pulled Weights

$$a = \frac{F - f}{\Sigma m}$$

$$T = ma$$

Elevator

Normal force acts upward, weight acts downward.

- Accelerating upward: $N = |ma| + |mg|$
- Constant velocity: $N = |mg|$
- Accelerating downward: $N = |mg| - |ma|$

Banked Curve

Friction can act up the ramp (minimum velocity when friction is maximum) or down the ramp (maximum velocity when friction is maximum).

$$v_{ideal} = \sqrt{rg\tan\theta}$$

$$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{\mu\tan\theta + 1}}$$

$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$$

Projectile Motion

Position

$$\Delta x = v_x t$$

$$\Delta y = -\frac{1}{2}gt^2 + (v_y)_0 t$$

Velocity

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

$$(v_x)_0 = v_0\cos\theta$$

$$(v_y)_0 = v_0\sin\theta$$

$$\Delta v_x = 0$$

$$\Delta v_y = -gt$$

Height

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

Starting from a height of $x = 0$:

$$y_{max} = \frac{(v_0\sin\theta)^2}{2g}$$

Range

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

Starting from a height of $x = 0$:

$$x_{max} = \frac{(v_0)^2\sin 2\theta}{g}$$

Circular Motion

Centripetal (radial)

Centripetal acceleration and force is directed towards the center. It refers to a change in direction.

$$a_c = \frac{v^2}{r}$$

$$F_c = ma_c = \frac{mv^2}{r}$$

Tangential

Tangential acceleration is tangent to the object's motion. It refers to a change in speed.

$$a_t = \frac{d|v|}{dt}$$

Combined

$$a_{total} = \sqrt{(a_c)^2 + (a_t)^2}$$

Vertical loop

In a vertical loop, the centripetal acceleration is caused by a normal force and gravity (weight).

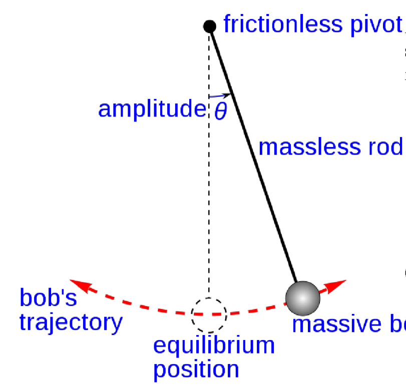

Figure 8: Simple Pendulum

Pendulum

Simple

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

Compound

A cable with a moment of inertia swings back and forth. d represents the distance from the pendulum's pivot to its center of mass.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

Torsional

A horizontal mass with a moment of inertia is suspended from a cable and swings back and forth.

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\omega = \frac{k}{I}$$

Gravity

$$F = \frac{-Gm_1m_2}{R^2}$$

$$G \approx 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$

Kepler's Laws

1. All orbits are elliptical.
2. Law of Equal Areas.
3. $T^2 = \frac{4\pi^2}{GM}R^3 = K_sR^3$, where K_s is a uniform constant for all satellites/planets orbiting a specific body

Energy

$$U = \frac{-Gm_1m_2}{R}$$

$$E = \frac{-GMm}{2r}$$

$$v = \frac{2\pi R}{T}$$

$$v_{escape} = \sqrt{\frac{2GM}{r_e}}$$

For orbits around the earth, r_e represents the radius of the earth.

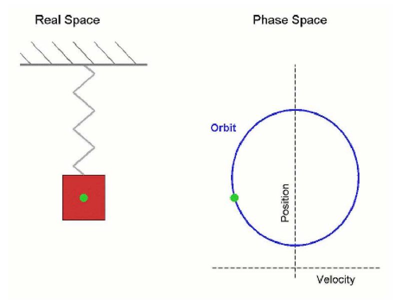

Figure 7: Simple Harmonic Motion

Angular Momentum

$$L = I\omega$$

$$L = r \times p = rp\sin\theta = rmv\sin\theta$$

$$\tau = \frac{dL}{dt}$$

Total angular momentum is always conserved when there are no external torques ($\tau = \frac{dL}{dt} = 0$).

Simple Harmonic Motion

Simple harmonic motion is the projection of uniform circular notion on to a diameter. Likewise, uniform circular motion is the combination of simple harmonic motions along the x-axis and y-axis that differ by a phase of 90°.

amplitude (A) maximum magnitude of displacement from equilibrium

cycle one complete vibration

period (T) time for one cycle

frequency (f) cycles per time

angular frequency (ω) radians per time

$$x = A\cos(\omega t + \phi)$$

$$v = -\omega A\sin(\omega t + \phi)$$

$$a = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

$$A = \sqrt{(x_0)^2 + \left(\frac{v_0}{\omega}\right)^2}$$

$$\phi = \arctan\left(\frac{-v_0}{\omega x_0}\right)$$

$$E = \frac{1}{2}kA^2$$

Spring

$$F_s = -kx$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega_s = \sqrt{\frac{k}{m}}$$

Top

$$\begin{aligned} F &= ma \\ N + mg &= m \times \frac{v^2}{r} \\ N &= \frac{mv^2}{r} - mg \end{aligned}$$

Bottom

$$\begin{aligned} F &= ma \\ N - mg &= m \times \frac{v^2}{r} \\ N &= \frac{mv^2}{r} + mg \end{aligned}$$

Friction

Friction converts mechanical energy into heat. Static friction (at rest) is generally greater than kinematic friction (in motion).

$$f_{max} = \mu N$$

Momentum-Impulse

$$p = mv$$

$$F = \frac{dp}{dt}$$

$$I = \int F dt = \bar{F}\Delta t = \Delta p = m\Delta v$$

Collisions

Total momentum is always conserved when there are no external forces ($F = \frac{dp}{dt} = 0$).

Elastic

Kinetic energy is conserved.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$-(v_2' - v_1') = v_2 - v_1$$

Inelastic

Kinetic energy is not conserved.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

Center of Mass

$$r_{cm} = \frac{\Sigma mr}{\Sigma m} = \frac{1}{\Sigma m}\int r dm = \frac{1}{\Sigma m}\int x\lambda dx$$

$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$\Sigma m = \int dm = \int \lambda dx$$

$$(\Sigma m)v_{CM} = \Sigma mv = \Sigma p$$

$$F_{net} = (\Sigma m)a_{CM}$$

Energy

Work

$$W = \int F dx = \Delta K$$

Power

$$P_{avg} = \frac{W}{t} = \frac{Fx}{t}$$

$$P_{instant} = \frac{dW}{dt} = Fv$$

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Potential Energy

$$F = -\frac{dU}{dx}$$

$$\Delta U = -\int_{x_i}^{x_f} F_C dx = -W_C$$

$$U_{Hooke} = -\int F_{Hooke} dx = -\int -kx dx = \frac{1}{2}kx^2$$

$$U_g = mgh$$

equilibrium point $F = -\frac{du}{dx} = 0$ (extrema)

stable equilibrium U is a minimum

unstable equilibrium U is a maximum

Total

$$E = K + U$$

$$E_i + W_{NC} = E_f$$

W_{NC} represents non-conservative work that converts mechanical energy into other forms of energy. For example, friction converts mechanical energy into heat.

Linear	Angular
$v = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$	$\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t}$
$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{d\omega}{dt} = \frac{\Delta\omega}{\Delta t}$
$\Delta x = \frac{1}{2}at^2 + v_0 t$	$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t$
$\Delta v = at$	$\Delta\omega = \alpha t$
$(v)^2 - (v_0)^2 = 2a(\Delta x)$	$(\omega)^2 - (\omega_0)^2 = 2\alpha(\Delta\omega)$
$\Delta x = \frac{v_0 + v}{2} \times t$	$\Delta\theta = \frac{\omega_0 + \omega}{2} \times t$
$F = ma$	$\tau = I\alpha$
$W = \int_{x_0}^{x} F dx$	$W_{rot} = \int_{\theta_0}^{\theta} \tau d\theta$
$W = \frac{1}{2}mv^2 - \frac{1}{2}m(v_0)^2$	$W_{rot} = \frac{1}{2}m\omega^2 - \frac{1}{2}m(\omega_0)^2$
$P = Fv$	$P_{rot} = \tau\omega$
$p = mv$	$L = I\omega$
$F = \frac{dp}{dt}$	$\tau = \frac{dL}{dt}$

Figure 4: Rotational Motion

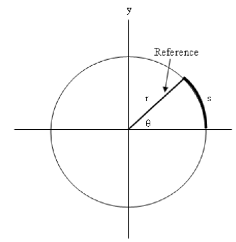

Figure 5: Arc Length

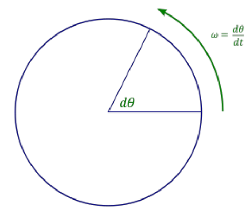

Figure 6: Angular Velocity

Rotational Motion

The same equations for linear motion can be modified for use with rotational motion (Figure 4 on the previous page).

Angular Motion

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a_t}{r}$$

$$a_t = r\sqrt{\alpha^2 + \omega^4}$$

$$a_c = \omega^2 r$$

$$K_{rolling} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Torque

$$\tau = r \times F = rF\sin\theta$$

$$\tau = I\alpha$$

Moment of Inertia

$$I = \Sigma mr^2 = \int r^2 dm$$

$$I = I_{cm} + Mh^2$$

(h represents the distance from the center)

Values

rod (center) $\frac{1}{12}ml^2$

rod (end) $\frac{1}{3}ml^2$

hollow hoop/cylinder mr^2

solid disk/cylinder $\frac{1}{2}mr^2$

hollow sphere $\frac{2}{3}mr^2$

solid sphere $\frac{2}{5}mr^2$

Atwood's Machine

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2 + \frac{1}{2}M}$$