

# AP Physics C Review Mechanics

CHSN Review Project



This is a review guide designed as preparatory information for the AP<sup>1</sup> Physics C Mechanics Exam on May 11, 2009. It may still, however, be useful for other purposes as well. Use at your own risk. I hope you find this resource helpful. Enjoy!

This review guide was written by Dara Adib based on inspiration from Shelun Tsai's review packet.

This is a development version of the text that should be considered a work-in-progress.

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“Why do we love ideal worlds? ... I’ve been doing this for 38 years and school is an ideal world.” — Steven Henning

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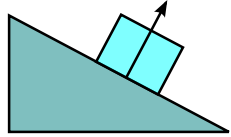


Figure 1: Normal Force

## Kinematic Equations

$$\Delta x = \frac{1}{2}at^2 + v_0t$$

$$\Delta v = at$$

$$(v)^2 - (v_0)^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v_0 + v}{2} \times t$$

## Free Body Diagrams

N Normal Force

f Frictional Force

T Tension

mg Weight

$$F = ma$$

In a particular direction:

$$\Sigma F = (\Sigma m)a$$

## Atwood's Machine<sup>2</sup>

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2}$$

<sup>2</sup>Pulley and string are assumed to be massless.

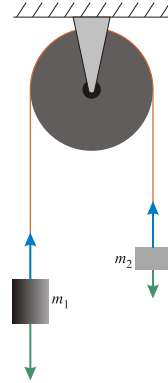


Figure 2: Atwood's Machine

Figure 3: Draw a banked curve diagram

## Pulled Weights

$$a = \frac{F - f}{\Sigma m}$$

$$T = ma$$

## Elevator

Normal force acts upward, weight acts downward.

- Accelerating upward:  $N = |ma| + |mg|$
- Constant velocity:  $N = |mg|$
- Accelerating downward:  $N = |mg| - |ma|$

## Banked Curve

Friction can act up the ramp (minimum velocity when friction is maximum) or down the ramp (maximum velocity when friction is maximum).

$$v_{ideal} = \sqrt{rg \tan \theta}$$

$$v_{\min} = \sqrt{\frac{rg(\tan \theta - \mu)}{\mu \tan \theta + 1}}$$

$$v_{\max} = \sqrt{\frac{rg(\tan \theta + \mu)}{1 - \mu \tan \theta}}$$

## Projectile Motion

### Position

$$\Delta x = v_x t$$

$$\Delta y = -\frac{1}{2}gt^2 + (v_y)_0 t$$

### Velocity

$\theta$  represents the smaller angle from the  $x$ -axis to the direction of the projectile's initial motion.

$$(v_x)_0 = v_0 \cos \theta$$

$$(v_y)_0 = v_0 \sin \theta$$

$$\Delta v_x = 0$$

$$\Delta v_y = -gt$$

### Height

$\theta$  represents the smaller angle from the  $x$ -axis to the direction of the projectile's initial motion.

Starting from a height of  $x = 0$ :

$$y_{\max} = \frac{(v_0 \sin \theta)^2}{2g}$$

## Range

$\theta$  represents the smaller angle from the  $x$ -axis to the direction of the projectile's initial motion.

Starting from a height of  $x = 0$ :

$$x_{\max} = \frac{(v_0)^2 \sin 2\theta}{g}$$

## Circular Motion

### Centripetal (radial)

Centripetal acceleration and force is directed towards the center. It refers to a change in direction.

$$a_c = \frac{v^2}{r}$$

$$F_c = ma_c = \frac{mv^2}{r}$$

### Tangential

Tangential acceleration is tangent to the object's motion. It refers to a change in speed.

$$a_t = \frac{d|v|}{dt}$$

### Combined

$$a_{\text{total}} = \sqrt{(a_c)^2 + (a_t)^2}$$

### Vertical loop

In a vertical loop, the centripetal acceleration is caused by a normal force and gravity (weight).

## Top

$$\begin{aligned}
 F &= ma \\
 N + mg &= m \times \frac{v^2}{r} \\
 N &= \frac{mv^2}{r} - mg
 \end{aligned}$$

## Bottom

$$\begin{aligned}
 F &= ma \\
 N - mg &= m \times \frac{v^2}{r} \\
 N &= \frac{mv^2}{r} + mg
 \end{aligned}$$

## Friction

Friction converts mechanical energy into heat. Static friction (at rest) is generally greater than kinematic friction (in motion).

$$f_{\max} = \mu N$$

## Momentum-Impulse

$$p = mv$$

$$F = \frac{dp}{dt}$$

$$I = \int F dt = \bar{F} \Delta t = \Delta p = m \Delta v$$

## Collisions

Total momentum is always conserved when there are no external forces ( $F = \frac{dp}{dt} = 0$ ).

## Elastic

Kinetic energy is conserved.

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$-(v'_2 - v'_1) = v_2 - v_1$$

## Inelastic

Kinetic energy is not conserved.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

## Center of Mass

$$r_{\text{cm}} = \frac{\sum m r}{\sum m} = \frac{1}{\sum m} \int r dm = \frac{1}{\sum m} \int x \lambda dx$$

$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$\sum m = \int dm = \int \lambda dx$$

$$(\sum m) v_{\text{CM}} = \sum m v = \sum p$$

$$F_{\text{net}} = (\sum m) a_{\text{CM}}$$

## Energy

### Work

$$W = \int F dx = \Delta K$$

## Power

$$P_{\text{avg}} = \frac{W}{t} = \frac{Fx}{t}$$

$$P_{\text{instant}} = \frac{dW}{dt} = Fv$$

## Kinetic Energy

$$K = \frac{1}{2}mv^2$$

## Potential Energy

$$F = -\frac{dU}{dx}$$

$$\Delta U = -\int_{x_i}^{x_f} F_C dx = -W_C$$

$$U_{\text{Hooke}} = -\int F_{\text{Hooke}} dx = -\int -kx dx = \frac{1}{2}kx^2$$

$$U_g = mgh$$

**equilibrium point**  $F = -\frac{du}{dx} = 0$  (extrema)

**stable equilibrium**  $U$  is a minimum

**unstable equilibrium**  $U$  is a maximum

## Total

$$E = K + U$$

$$E_i + W_{\text{NC}} = E_f$$

$W_{\text{NC}}$  represents non-conservative work that converts mechanical energy into other forms of energy. For example, friction converts mechanical energy into heat.

Linear	Angular
$v = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$	$\omega = \frac{d\theta}{dt} = \frac{\Delta \theta}{\Delta t}$
$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{d\omega}{dt} = \frac{\Delta \omega}{\Delta t}$
$\Delta x = \frac{1}{2}at^2 + v_0t$	$\Delta \theta = \frac{1}{2}\alpha t^2 + \omega_0t$
$\Delta v = at$	$\Delta \omega = \alpha t$
$(v)^2 - (v_0)^2 = 2a(\Delta x)$	$(\omega)^2 - (\omega_0)^2 = 2\alpha(\Delta \theta)$
$\Delta x = \frac{v_0 + v}{2} \times t$	$\Delta \theta = \frac{\omega_0 + \omega}{2} \times t$
$F = ma$	$\tau = I\alpha$
$W = \int_{x_0}^x F dx$	$W_{\text{rot}} = \int_{\theta_0}^{\theta} \tau d\theta$
$W = \frac{1}{2}mv^2 - \frac{1}{2}m(v_0)^2$	$W_{\text{rot}} = \frac{1}{2}m\omega^2 - \frac{1}{2}m(\omega_0)^2$
$P = Fv$	$P_{\text{rot}} = \tau\omega$
$p = mv$	$L = I\omega$
$F = \frac{dp}{dt}$	$\tau = \frac{dL}{dt}$

Figure 4: Rotational Motion

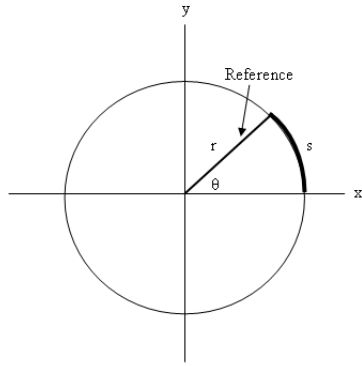


Figure 5: Arc Length

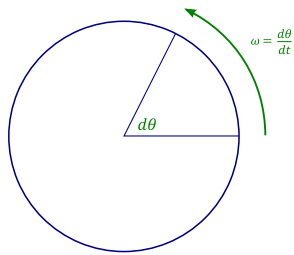


Figure 6: Angular Velocity

## Rotational Motion

The same equations for linear motion can be modified for use with rotational motion (Figure 4 on the previous page).

### Angular Motion

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a_t}{r}$$

$$a_t = r\sqrt{\alpha^2 + \omega^4}$$

$$a_c = \omega^2 r$$

$$K_{\text{rolling}} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

## Torque

$$\tau = \mathbf{r} \times \mathbf{F} = rF \sin \theta$$

$$\tau = I\alpha$$

## Moment of Inertia

$$I = \sum mr^2 = \int r^2 dm$$

$$I = I_{\text{cm}} + Mh^2$$

(h represents the distance from the center)

## Values

**rod (center)**  $\frac{1}{12}ml^2$

**rod (end)**  $\frac{1}{3}ml^2$

**hollow hoop/cylinder**  $mr^2$

**solid disk/cylinder**  $\frac{1}{2}mr^2$

**hollow sphere**  $\frac{2}{3}mr^2$

**solid sphere**  $\frac{2}{5}mr^2$

## Atwood's Machine

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2 + \frac{1}{2}M}$$

## Angular Momentum

$$L = I\omega$$

$$L = \mathbf{r} \times \mathbf{p} = rp \sin \theta = rmv \sin \theta$$

$$\tau = \frac{dL}{dt}$$

Total angular momentum is always conserved when there are no external torques ( $\tau = \frac{dL}{dt} = 0$ ).

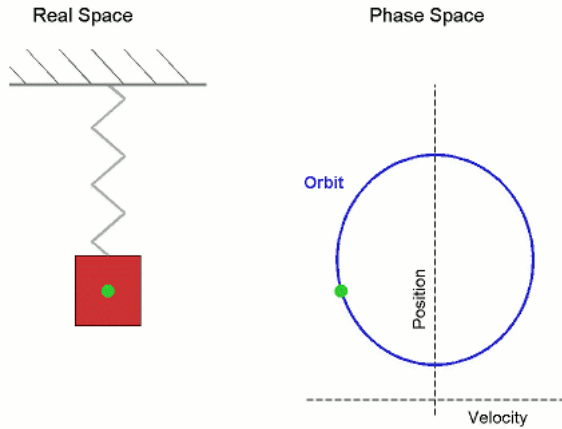


Figure 7: Simple Harmonic Motion

## Simple Harmonic Motion

Simple harmonic motion is the projection of uniform circular motion on to a diameter. Likewise, uniform circular motion is the combination of simple harmonic motions along the  $x$ -axis and  $y$ -axis that differ by a phase of  $90^\circ$ .

**amplitude (A)** maximum magnitude of displacement from equilibrium

**cycle** one complete vibration

**period (T)** time for one cycle

**frequency (f)** cycles per time

**angular frequency ( $\omega$ )** radians per time

$$x = A \cos(\omega t + \phi)$$

$$v = -\omega A \sin(\omega t + \phi)$$

$$a = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

## Spring

$$\omega = 2\pi f$$

$$A = \sqrt{(x_0)^2 + \left(\frac{v_0}{\omega}\right)^2}$$

$$\phi = \arctan\left(\frac{-v_0}{\omega x_0}\right)$$

$$E = \frac{1}{2} k A^2$$

$$F_s = -kx$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$\omega_s = \sqrt{\frac{k}{m}}$$

## Pendulum

### Simple

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

### Compound

A cable with a moment of inertia swings back and forth.  $d$  represents the distance from the pendulum's pivot to its center of mass.

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

$$\omega = \sqrt{\frac{mgd}{I}}$$



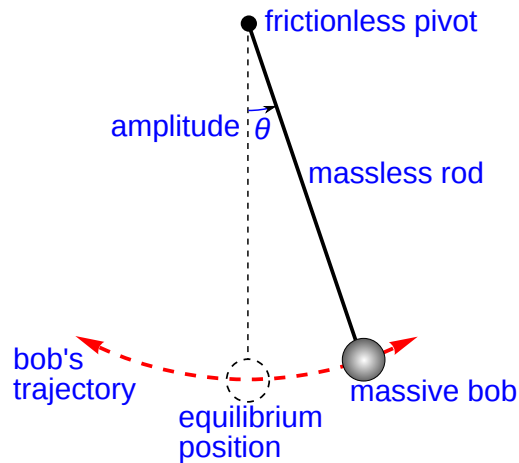


Figure 8: Simple Pendulum

## Energy

$$U = \frac{-Gm_1m_2}{R}$$

$$E = \frac{-GMm}{2r}$$

$$v = \frac{2\pi R}{T}$$

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r_e}}$$

## Torsional

A horizontal mass with a moment of inertia is suspended from a cable and swings back and forth.

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\omega = \frac{k}{I}$$

For orbits around the earth,  $r_e$  represents the radius of the earth.

## Gravity

$$F = \frac{-Gm_1m_2}{R^2}$$

$$G \approx 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

## Kepler's Laws

1. All orbits are elliptical.
2. Law of Equal Areas.
3.  $T^2 = \frac{4\pi^2}{GM} R^3 = K_s R^3$ , where  $K_s$  is a uniform constant for all satellites/planets orbiting a specific body