

EW MBA 296 (Fall 2015)

Section 0

GSI: Fenella Carpena

October 15, 2015

Agenda for Today

- ▶ Housekeeping: GSI and Section Information
- ▶ Chapters 1-12 and Quiz # 1 Review
 - ▶ Random Variables and their Probability Distributions
 - ▶ The Normal Distribution
- ▶ Practice Problems
 - ▶ Quiz 1, 2014
 - ▶ Stine and Foster, Ch. 12, Question 46 (a) and (b)
 - ▶ Quiz 1, 2013
- ▶ Note: I have also posted section notes online at bCourses.

GSI and Section Information

- ▶ Introductions: Hi! I'm really excited to be your GSI this semester!
- ▶ Section Goals: Review materials from the previous week's lecture, work on practice problems, prepare for quizzes/exams.
- ▶ Section Times: Every Thursday (except Thanksgiving)
 - ▶ 6:30PM-7:30PM (Axe)
 - ▶ 7:30PM-8:30PM (Oski)
 - ▶ All sections in Adobe Connect
- ▶ Recordings of section meetings will also be made available (takes a few days)
- ▶ Office Hours
 - ▶ In-person: Saturdays, 8-9AM and 1-2PM in F421.
 - ▶ Online: By appointment at Adobe Connect. Please email me to set up a time.
- ▶ Email: fcarpena+ewmba296@berkeley.edu

Random Variables

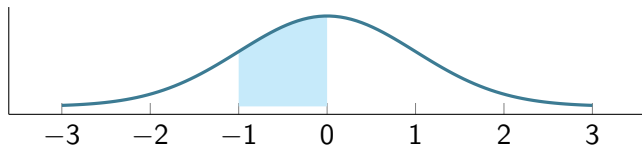
- ▶ A **random variable** represents the outcome of a random process.
- ▶ They can be either **discrete** or **continuous**.
- ▶ We denote a r.v. using capital letters X , Y , Z , etc.
- ▶ Examples:
 - ▶ The # obtained from rolling a fair die (discrete)
 - ▶ Total # of “Heads” when a coin is tossed 10 times (discrete)
 - ▶ Consumption of natural gas (continuous)
 - ▶ Weight of a bag of potato chips (continuous)
- ▶ **Question:** *(Use the chat window to share your answers)* Can you provide an example of a random variable? Is the example you provided discrete or continuous?

Probability Distribution

- ▶ Each r.v. is associated with a function which maps every potential outcome to a probability.
- ▶ This function is called **probability mass function (pmf)** (discrete) or **probability distribution function (pdf)** (continuous).
- ▶ Example of a pmf (tabular format): Let X be the number obtained from rolling a fair die. Then the pmf of X is:

x	1	2	3	4	5	6
$P(X = x)$	1/6	1/6	1/6	1/6	1/6	1/6

- ▶ Example of a pdf (graphical format): Let Z be a standard normal r.v. The area under the pdf gives us the probability.



- ▶ **Question:** (Poll) What is the probability given by the shaded area in the above graph?

The Normal Distribution

- ▶ Really important for this course!
- ▶ If X is a **normally distributed r.v.**, we write $X \sim \mathcal{N}(\mu, \sigma^2)$.
- ▶ A special case of the normal distribution occurs when $\mu = 0$ and $\sigma^2 = 1$. This is called the **standard normal distribution**.
- ▶ **Important property:** Any normal r.v. $X \sim \mathcal{N}(\mu, \sigma^2)$ can be transformed to a standard normal r.v. using the formula

$$Z = \frac{X - \mu}{\sigma}$$

- ▶ Why do we care? This property allows us to find the probabilities of any normal r.v. using the standard normal table (*Z Table*) at the back of the textbook.

Quiz 1, 2014 - "Potato Chips"

The weight of a single 9-ounce bag of Lay's Classic potato chips has a normal distribution with a mean of 9.12 ounces and a standard deviation of 0.15 ounces.

Part A (5 points). What is the probability that a randomly selected bag weighs less than 9 ounces?

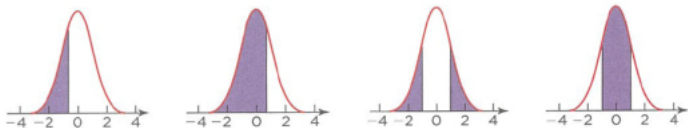
Z-TABLE Percentiles of the normal distribution.



z	$P(Z \leq -z)$	$P(Z \leq z)$	$P(Z > z)$	$P(Z \leq z)$
0	0.5	0.5	1	0
0.1	0.4602	0.5398	0.9203	0.0797
0.2	0.4207	0.5793	0.8415	0.1585
0.3	0.3821	0.6179	0.7642	0.2358
0.4	0.3446	0.6554	0.6892	0.3108
0.5	0.3085	0.6915	0.6171	0.3829
0.6	0.2743	0.7257	0.5485	0.4515
0.7	0.2420	0.7580	0.4839	0.5161
0.8	0.2119	0.7881	0.4237	0.5763
0.9	0.1841	0.8159	0.3681	0.6319
:	:	:	:	:
2.9	0.001866	0.998134	0.003732	0.996268
3	0.001350	0.998650	0.002700	0.997300
3.1	0.0009676	0.9990324	0.001935	0.998065
3.2	0.0006871	0.9993129	0.001374	0.998626
3.3	0.0004834	0.9995166	0.0009668	0.9990332
3.4	0.0003369	0.9996631	0.0006739	0.9993261
3.5	0.0002326	0.9997674	0.0004653	0.9995347

- Step 1: Define r.v., its mean/sd, etc.
- Step 2: Write in mathematical terms what we are asked to solve.
- Step 3: Solve for the expression from previous step.

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Quiz 1, 2014 - "Potato Chips"

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Part B (5 points). Suppose you would like to guarantee that only .1% (i.e. $1/10^{th}$) percent of all bags weigh less than 9 ounces. Holding the standard deviation fixed, how much would you need to increase the mean weight of the bags to honor this guarantee?

Quiz 1, 2014 - "Potato Chips"

The weight of a single 9-ounce bag of Lay's Classic potato chips has a normal distribution with a mean of 9.12 ounces and a standard deviation of 0.15 ounces.

Part C (10 points). Go back to the original mean and the standard deviation (9.12 ounces and 0.15 ounces, respectively). What is the probability that the total weight of three randomly selected bags is less than 27 ounces? Assume that the weights of the three bags are independent.

Quiz 1, 2014 - "Potato Chips"

The weight of a single 9-ounce bag of Lay's Classic potato chips has a normal distribution with a mean of 9.12 ounces and a standard deviation of 0.15 ounces.

Extra Question for Discussion. Part C assumed that the weight of 3 randomly selected bags are independent. Do you think this is a reasonable assumption? (*Raise your hand or use chat if you would like to share your answer.*)

Stine and Foster, Chapter 12, Question 46, parts (a) & (b)

A tire manufacturer warrants its tires to last at least 20,000 miles or “you get a new set of tires.” In its experience, a set of these tires lasts on average 26,000 miles with SD 5,000 miles. Assume that wear is normally distributed. The manufacturer profits \$200 on each set sold, and replacing a set costs the manufacturer \$400.

Part (a) What is the probability that a set of tires wears out before 20,000 miles?

Part (b) What is the probability that the manufacturer turns a profit on selling a set to one customer?

Quiz 1, 2014 - "PG&E"

PG&E is one of the largest investor-owned utilities in the US. Homes in the PG&E territory use annually an average of 12,000 kWh of electricity, and 85 thousand cubic feet (85 MCF) of natural gas. The standard deviation of annual electricity use is 2,000 kWh and the standard deviation of annual natural gas use is 15 MCF.

Part A (5 points). Let the r.v. X denote the electricity use of a home in kWh, and Y denote the use of natural gas in MCF. Do you think that PG&E should model these two variables as independent? Explain why or why not.

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Part B (5 points). It does not make sense to add X and Y because these variables have different units. However, we can convert them both to dollars, and then add the dollar amounts. If electricity costs \$0.09 per kWh, and natural gas costs \$10 per MCF, then how much on average do households spend on electricity and natural gas?

Quiz 1, 2014 - "PG&E" (continued)

PG&E is one of the largest investor-owned utilities in the US. Homes in the PG&E territory use annually an average of 12,000 kWh of electricity, and 85 thousand cubic feet (85 MCF) of natural gas. The standard deviation of annual electricity use is 2,000 kWh and the standard deviation of annual natural gas use is 15 MCF.

Part C (5 points). If the correlation between electricity use and natural gas use is 0.35, then what is the correlation between expenditure on electricity and natural gas?

Quiz 1, 2014 - "PG&E" (continued)

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Part D (5 points). If the correlation between electricity use and natural gas use is 0.35, then what is the covariance between electricity use and natural gas use?

Quiz 1, 2014 - "PG&E" (continued)

PG&E is one of the largest investor-owned utilities in the US. Homes in the PG&E territory use annually an average of 12,000 kWh of electricity, and 85 thousand cubic feet (85 MCF) of natural gas. The standard deviation of annual electricity use is 2,000 kWh and the standard deviation of annual natural gas use is 15 MCF.

Part E (10 points). Find the standard deviation of the total annual bill paid by these households. Continue to assume that the correlation between electricity use and natural gas use is 0.35.

Final Points

- ▶ Don't stress out about the quiz!
 - ▶ You can drop your lowest quiz.
 - ▶ You can bring a single-sided, 8.5 x 11 sheet of notes.
 - ▶ You can use a calculator.
 - ▶ You will be provided with the normal table if necessary.
- ▶ If you still have questions about the material, please feel free to come to my office hours on Saturday, or email me to set up a meeting online.
- ▶ The section notes posted in bCourses has more practice problems.
- ▶ Please fill out the (very short!) feedback form at
<http://tinyurl.com/ewmba296feedback>
so that we can improve our section meetings moving forward.
Thank you!