

EW MBA 296 (Fall 2015)  
Section 3

GSI: Fenella Carpena

November 5, 2015

# Agenda for Today

- ▶ Basic Concepts of Hypothesis Testing
- ▶ One-Sided Hypothesis Tests
  - ▶ Using a Test Statistic (z-stat or t-stat)
  - ▶ Using the p-value

**I HAVE NO IDEA**

**WHAT I'M DOING**

quickmeme.com

# Basic Concepts of Hypothesis Testing

- ▶ **Hypothesis testing** refers to set of procedures used to weigh the “strength” of statistical evidence, i.e. whether the evidence from our sample is “sufficiently strong” to support one hypothesis over another.
- ▶ The approach requires that we designated a **null hypothesis**  $H_0$  and an **alternative hypothesis**  $H_a$ .
  - ▶  $H_0$  is the hypothesis associated with preserving the status quo, i.e. taking no action.
  - ▶  $H_a$  is the hypothesis that contradicts  $H_0$ .
  - ▶ Important:  $H_0$  is given the benefit of the doubt and  $H_a$  has the burden of proof, so  $H_0$  is the default course of action unless there is strong evidence in favor of  $H_a$ .

# Basic Concepts of Hypothesis Testing

## Exercise 1.1. from Section Notes

Suppose that Gollum (a bald-headed character from the Lord of the Rings) is interested in using Rogaine, a product that promotes hair growth. However, Gollum is skeptical about Rogaine's effectiveness, and he would like to conduct an experiment to determine if the product works as advertised. State in words what Gollum's null and alternative hypotheses are.

# Basic Concepts of Hypothesis Testing

Stine and Foster, Chapter 16, Question 27

A pharmaceutical company is testing a newly developed therapy. If the therapy lowers the blood pressure of a patient by more than 10 mm it is deemed effective. What are the natural hypotheses to test in a clinical study of this new therapy?

# Basic Concepts of Hypothesis Testing

- ▶ The conclusion of a hypothesis test is either: (1) we **reject**  $H_0$ , or (2) we **fail to reject**  $H_0$ . This wording is important.
- ▶ Because every hypothesis concludes by either rejecting or failing to reject  $H_0$ , there are 2 ways we can make an error.
  - ▶ **Type I**: unnecessary action (reject  $H_0$  when  $H_0$  is true)
  - ▶ **Type II**: failing to take action (fail to reject  $H_0$  when  $H_a$  is true)
- ▶ What are the Type I and Type II errors in Gollum's case?  
Recall:  $H_0$ : Rogaine is not effective, and  $H_a$ : Rogaine is effective
- ▶ How do we conclude whether we reject or fail to reject  $H_0$ ?  
We use a **test statistic** ( $z$ -stat,  $t$ -stat) or a **p-value**. Both methods will arrive at the same conclusion. We'll discuss next how to use each of these for a hypothesis test.

# One-Sided Hypothesis Test: Formulas

	Proportion	Mean	Differences of Proportions	Differences of Means
Population parameter	$p$	$\mu$	$p_1, p_2$	$\mu_1, \mu_2$
Sample statistic	$\hat{p}$	$\bar{X}$	$\hat{p}_1 - \hat{p}_2$ Note: The order of subtraction follows the same order of the parameters in the null hypothesis.	$\bar{X}_1 - \bar{X}_2$
Estimated standard error (se) of sample statistic	$\sqrt{\frac{p_0(1-p_0)}{n}}$ Note: $p_0$ is the "cutoff" value	$s / \sqrt{n}$	$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Test statistic (z-stat or t-stat)	$z = \frac{\hat{p} - p_0}{se(\hat{p})}$ Note: $p_0$ is the "cutoff" value	$t = \frac{\bar{X} - \mu_0}{se(\bar{X})}$ Note: $\mu_0$ is the "cutoff" value	$z = \frac{\hat{p}_1 - \hat{p}_2 - D_0}{se(\hat{p}_1 - \hat{p}_2)}$ Note: $D_0$ is the "cutoff" value	$t = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{se(\bar{X}_1 - \bar{X}_2)}$
Conditions required to allow use of the Z-Table	(1) SRS (2) SSC: $np_0 > 10,$ $n(1 - p_0) > 10$	(1) SRS (2) SSC: $n > 10(K_3)^2,$ $n > 10 \cdot  K_4 $	(1) SRS (2) SSC: $n\hat{p}_1 > 10,$ $n\hat{p}_2 > 10,$ $n(1 - \hat{p}_1) > 10,$ $n(1 - \hat{p}_2) > 10$	(1) SRS (2) SSC: same as those for the mean but should hold for both groups.
Other checks	NA	NA	No obvious lurking variable	No obvious lurking variable; Similar variance.

# One-Sided Hypothesis Test: Using a Test Statistic

A **test statistic** (*z*-stat or *t*-stat) is the number of standard errors that separate the sample statistic from the “cutoff” value in the hypothesis test. Here is a general “recipe” for conducting a hypothesis test using *z*-stat/*t*-stat.

- ▶ **Step 0:** Check that SRS and SSC hold.
- ▶ **Step 1:** Choose a level of significance  $\alpha$  (or it is given to you)
- ▶ **Step 2:** Formulate the null and alternative hypothesis.
  - ▶ Important:  $H_0$  always includes the “cutoff” value;
  - ▶ Important:  $H_0$  and  $H_a$  involve the population parameters NOT sample statistics.
- ▶ **Step 3:** Calculate the test statistic (see above table for the formulas). Note: the formula for *z*-stat/*t*-stat follows the following format:

$$\frac{\text{sample statistic} - \text{“cutoff” value from hypothesis}}{\text{se(sample statistic)}}$$

- ▶ **Step 4:** Find the “one-sided critical value”  $z_\alpha$  or  $t_{\alpha,n-1}$  depending on whether the conditions from Step 0 hold.
- ▶ **Step 5:** Compare the *test statistic* (from Step 3) with the *one-sided critical value* (from Step 4), and apply the following rejection rule.

- ▶ If  $H_0$  has the format *population parameter*  $\leq$  “cutoff” value:

**Reject  $H_0$**  if  $\underbrace{\text{test statistic}}_{\text{Step 3}} > \underbrace{\text{one-sided critical value}}_{\text{Step 4}}$ .

Otherwise, we **fail to reject  $H_0$** .

- ▶ If  $H_0$  has the format *population parameter*  $\geq$  “cutoff” value:

**Reject  $H_0$**  if  $\underbrace{\text{test statistic}}_{\text{Step 3}} < -1 * \underbrace{\text{one-sided critical value}}_{\text{Step 4}}$ .

Otherwise, we **fail to reject  $H_0$** .

- ▶ Remember: The “rejection rule” has the opposite inequality sign as  $H_0$ .
- ▶ If we reject  $H_0$ , for example with  $\alpha = 0.05$ , then we say that our result is **statistically significant** at the 5% level.

## Exercise: Adapted from Lecture # 5 Slides

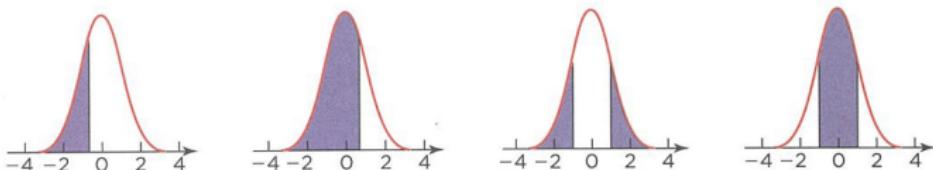
Your company, *Nordex*, is currently producing turbine blades using fiberglass. To reduce costs, you are considering switching to carbon fiber blades. You have calculated that the switch to carbon fiber blades would be profitable for the company if the proportion defective is less than 40%. From a simple random sample of 35 carbon fiber blades, you find that 12 are defective.

- (a) State the null and alternative hypothesis.

## Exercise: Adapted from Lecture # 5 Slides

Your company, *Nordex*, is currently producing turbine blades using fiberglass. To reduce costs, you are considering switching to carbon fiber blades. You have calculated that the switch to carbon fiber blades would be profitable for the company if the proportion defective is less than 40%. From a simple random sample of 35 carbon fiber blades, you find that 12 are defective.

- (b) Based on the results from this sample, should you switch to carbon fiber blades? Use the  $z$ -statistic and the significance level  $\alpha = 0.10$ .



$z$	$P(Z \leq -z)$	$P(Z \leq z)$	$P( Z  > z)$	$P( Z  \leq z)$
1.2816	0.10	0.90	0.20	0.80
1.3408	0.09	0.91	0.18	0.82
1.4051	0.08	0.92	0.16	0.84
1.4758	0.07	0.93	0.14	0.86
1.5548	0.06	0.94	0.12	0.88
1.6449	0.05	0.95	0.10	0.90
1.7507	0.04	0.96	0.08	0.92
1.8808	0.03	0.97	0.06	0.94
1.9600	0.025	0.975	0.05	0.95
2.0537	0.02	0.98	0.04	0.96
2.3263	0.01	0.99	0.02	0.98
2.5758	0.005	0.995	0.01	0.99
2.8070	0.0025	0.9975	0.005	0.995
3.0902	0.001	0.999	0.002	0.998
3.2905	0.0005	0.9995	0.001	0.999
3.7190	0.0001	0.9999	0.0002	0.9998
3.8906	0.00005	0.99995	0.0001	0.9999
4.2649	0.00001	0.99999	0.00002	0.99998
4.4172	0.000005	0.999995	0.00001	0.99999

## Exercise: Adapted from Lecture # 5 Slides

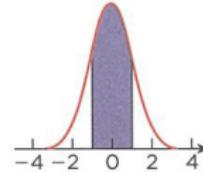
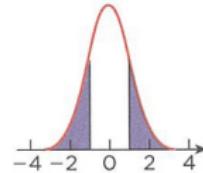
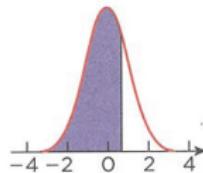
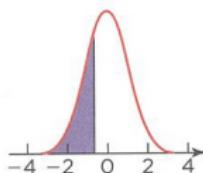
Your company, *Nordex*, is currently producing turbine blades using fiberglass. To reduce costs, you are considering switching to carbon fiber blades. You have calculated that the switch to carbon fiber blades would be profitable for the company if the proportion defective is less than 40%. From a simple random sample of 35 carbon fiber blades, you find that 12 are defective.

- (c) For what values of the sample proportion would the company reject  $H_0$  if the company adopts a level of  $\alpha = 0.05$ ?

## Exercise: Adapted from Lecture # 5 Slides

Your company, *Nordex*, is currently producing turbine blades using fiberglass. To reduce costs, you are considering switching to carbon fiber blades. You have calculated that the switch to carbon fiber blades would be profitable for the company if the proportion defective is less than 40%. From a simple random sample of 35 carbon fiber blades, you find that 12 are defective.

- (d) What is the probability of a Type II error using the procedure from part (c), if the true population proportion defective  $p$  is 0.20?



$z$	$P(Z \leq -z)$	$P(Z \leq z)$	$P( Z  > z)$	$P( Z  \leq z)$
0	0.50	0.50	1	0
0.0502	0.48	0.52	0.96	0.04
0.1004	0.46	0.54	0.92	0.08
0.1510	0.44	0.56	0.88	0.12
0.2019	0.42	0.58	0.84	0.16
0.2533	0.40	0.60	0.80	0.20
0.3055	0.38	0.62	0.76	0.24
0.3585	0.36	0.64	0.72	0.28
0.4125	0.34	0.66	0.68	0.32
0.4677	0.32	0.68	0.64	0.36
0.4959	0.31	0.69	0.62	0.38
0.5244	0.30	0.70	0.60	0.40
0.5828	0.28	0.72	0.56	0.44
0.6433	0.26	0.74	0.52	0.48
0.6745	0.25	0.75	0.50	0.50
0.7063	0.24	0.76	0.48	0.52
0.7388	0.23	0.77	0.46	0.54
0.7722	0.22	0.78	0.44	0.56
0.8064	0.21	0.79	0.42	0.58
0.8416	0.20	0.80	0.40	0.60
0.8779	0.19	0.81	0.38	0.62
0.9154	0.18	0.82	0.36	0.64
0.9542	0.17	0.83	0.34	0.66
0.9945	0.16	0.84	0.32	0.68
1.0364	0.15	0.85	0.30	0.70



# One-Sided Hypothesis Test: Using the p-value

The **p-value** is the smallest  $\alpha$  level at which  $H_0$  can be rejected. We can think of the p-value as the weight of evidence against  $H_0$ , that is, small p-values mean that  $H_0$  is not plausible. Here is a general “recipe” for conducting a hypothesis test using the p-value.

- ▶ **Step 0 to Step 3.** Follow the same steps as in the previous section. Make a note of the z-stat/t-stat from Step 3, we will need this in the next step.
- ▶ **Step 4.** Find the following probabilities, which give us the p-value.
  - ▶ If  $H_0$  has the format *population parameter*  $\leq$  “cutoff” value:  
find  $P(Z > \underbrace{\text{test statistic}}_{\text{Step 3}})$
  - ▶ If  $H_0$  has the format *population parameter*  $\geq$  “cutoff” value:  
find  $P(Z < \underbrace{\text{test statistic}}_{\text{Step 3}})$
  - ▶ Remember: we use the opposite inequality sign as  $H_0$ .
- ▶ **Step 5.** Compare the p-value (step 4) with the level of  $\alpha$  (step 1). We **reject**  $H_0$  if p-value  $< \alpha$ . Otherwise, we **fail to reject**  $H_0$ .

## Exercise: Adapted from Lecture # 5 Slides

*Continuing the turbine blades example:* Your company, Nordex, is currently producing turbine blades using fiberglass. To reduce costs, you are considering switching to carbon fiber blades. You have calculated that the switch to carbon fiber blades would be profitable for the company if the proportion defective is less than 40%. From a simple random sample of 35 carbon fiber blades, you find that 12 are defective.

Find the p-value of the test. Using  $\alpha = 0.10$ , would you reject  $H_0$ ?

## Final Points

- ▶ Section3\_Notes.pdf are posted online (contains solutions to exercises from today's section).
- ▶ Quiz 4 Logistics
  - ▶ Covers Ch. 16, 17.1, 17.2, 17.4 (Lecture 6 contains material from Ch. 17 that is not covered)
  - ▶ You can bring a single-sided, 8.5 x 11 sheet of notes and use a calculator, mobile phone/iPad.
- ▶ **Tip:** I would recommend memorizing (or writing on your sheet) the one-sided  $z$  values from the  $Z$ -table for  $\alpha$  levels 1%, 5% 10%, since these are the most common values of  $\alpha$  when carrying out a hypothesis test.
  - ▶  $\alpha = 0.10: z_{0.10} = 1.2816$
  - ▶  $\alpha = 0.05: z_{0.05} = 1.6449$
  - ▶  $\alpha = 0.01: z_{0.01} = 2.3263$
- ▶ Lecture this Saturday (11/7) will cover Chapter 19, 20.1, and 20.4 (Linear and Curved Patterns).
- ▶ Please bring laptops to lecture this Saturday.