Econ 140 – Spring 2016
Section 1
GSI: Fenella Carpena

0. GSI/Section information

Section Times: Section 105, Thursdays 9:30-11AM, 179 Stanley; Section 104, Thursdays 11AM-12:30PM, 87 Evans. If you wish to attend a section that you are not registered for, please only do so with my permission.

Office Hours: 536 Evans, Thursdays, 1-2PM and 3:30-4:30PM

Email: fenella+econ140@econ.berkeley.edu. Please use "[ECON 140]" (without quotes in the subject line.

Email Policy: For any questions that you may have, email is often a convenient way to communicate them to me. I will check my email daily and will respond to your email within 72 hours. However, I would request that you use email only for administrative matters or for questions that you think will require a short and straightforward answer, as it is often difficult to provide a careful explanation over email. For more detailed questions, I encourage you to ask them in-person during section or office hours. In this way, other students in the class will also benefit from the answer. Section will also be more interesting with your active participation.

1. Random variables and their probability distributions

Exercise 1.1. Let W be the total number of heads obtained from two tosses of a fair coin. Is W discrete or continuous?

Discrete, it can only take on the values 0, 1, 2

Exercise 1.2. Let Z be electricity consumption in kwH of a randomly selected household in Berkeley. Is Z discrete or continuous?

Continuous, it can take on any value from 0 to infinity

Exercise 1.3. What is the pdf of the r.v. W in Exercise 1.1?

w	0	1	2
f(w) = P(W=w)	1/4	2/4	1/4

Exercise 1.4. What is the cdf of the r.v. W in Exercise 1.1?

w	any # < 0	0	1	2	any # > 2
F(w) = P(W<=w)	0	1/4	3/4	1	1

2. Joint distributions and independence

Exercise 2.1. (Adapted From Section # 1 Notes) Let X and Y be two discrete random variables with the following distribution

X	$\rightarrow$	0	1	2	3
Y	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
$\downarrow$	1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

(a) What is P(X=0, Y=1)? What is P(X = 2, Y = 0)?
(b) Find the marginal probability of X and Y.
(c) Are X and Y independent?

(a) P(X=0, Y = 1) = 0, P(X = 2, Y= 0) = 1/8

(b) Note that the probabilities below sum to 1

x	0	1	2	3
f(x) = P(X=x)	1/8	3/8	3/8	1/8

y	0	1
f(y) = P(Y=y)	4/8	4/8

(c) Independence means that P(X = x, Y= y) = P(X = x)*P(Y = y) for all possible values x and y. Note that P(X = 1, Y = 1) = 1/8, but P(X=1)*P(Y = 1) = (3/8)*(4/8). Hence X and Y are not independent.

3. Features of probability distributions and joint distributions

Exercise 3.1. Consider again the random variables X and Y from Exercise 2.1.

X	$\rightarrow$	0	1	2	3
Y	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
$\downarrow$	1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

(a) Find the mean and variance of X and Y
(b) Calculate the covariance and correlation between X and Y
(c) Find the probability distribution and the mean of X | Y = 1.
(d) Calculate the variance of Y | X = 1.

See solutions in Section_1.pdf in bCourses, Exercise # 1.

Exercise 3.2. Let X_1, X_2, and X_3 be random variables representing the numbers of small, medium, and large pizzas, respectively sold during the day at a pizza parlor. Suppose $E(X_1) = 25$, $E(X_2) = 57$, and $E(X_3) = 40$. The prices of small, medium, and large pizzas are $5.50, $7.60, and $9.15, respectively. What is the expected revenue from pizza sales on a given day?

E(Revenue) = $E(5.5*X_1 + 7.6*X_2 + 9.15*X_3) = 5.5*E(X_1) + 7.6*E(X_2) + 9.15*E(X_3) = 936.70$

Exercise 3.3. Let X, Y, and Z be random variables with E(X) = 2, E(Y) = 5, E(Z) = 3, Var(X) = 4, Var(Y) = 9, Var(Z) = 1, Cov(Y,Z) = -3, X and Y independent, X and Z are independent. Calculate the following:

(a) E(8 + 3X – 2Y + 9Z)
(b) Var(2X + 3Y)
(c) Var(2Y – 3Z + 5)
(d) Cov(3+6Y, 5-7Z)
(e) Corr(3+6Y, 5-7Z)

(a) E(8 + 3X – 2Y + 9Z) = 8 + 3E(X) -2E(Y) + 9E(Z) = 8 + 3*2 – 2*5 + 9*3 = 31.

(b) Var(2X + 3Y) = Var(2X) + Var(3Y) + 2Cov(3X,2Y) = 4Var(X) + 9Var(Y) + 2*3*2*Cov(X, Y) = 4*4 + 9*9 = 16 + 81 = 97. Note that we used the fact that X and Y are independent, so Cov(X,Y) = 0.

(c) Var(2Y – 3Z + 5) = Var(2Y – 3Z) = Var(2Y) + Var(3Z) – 2Cov(2Y, 3Z) = 4Var(Y) + 9Var(Z) -2*2*3Cov(Y,Z) = 36 + 9 + 36 = 81.

(d) Cov(3+6Y, 5-7Z) = Cov(6Y, -7Z) = 6*-7*Cov(Y, Z) = -42*-3 = 126.

(e) Corr(3+6Y, 5-7Z) = Cov(3+6Y, 5-7Z)/[SD(3+6Y)*SD(5-7Z)] =126/(18*7) = 1