Econ 140 - Spring 2016
Section 3

GSI: Fenella Carpena

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1 Basic Concepts of Hypothesis Testing

Exercise 1.1. Suppose that you would like to investigate whether there is gender discrimination among workers in the US. (a) State the null and alternative hypothesis. (b) Describe the Type I and Type II errors.

Let μ_1 bethe population mean wages of men, and let μ_2 be the population mean wages of women.

(a) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$; $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

(b) Type I error: concluding that there is discrimination even if there isn't.
Type II error: concluding that there is no discrimination, but infact there is.

2 Two-Sided Hypothesis Tests

Exercise 2.1. (Adapted from Stock & Watson, Exercise 3.13.) Data on fifth-grade test scores (reading and mathematics) for 420 school districts in California yield $\overline{Y} = 646.2$ and standard deviation $s_Y = 19.5$.

(a) At the 5% significance level, can we reject the hypothesis that the mean test score in the population is 650? Carry out the test using a t-statistic. Let μ denote the population mean test score

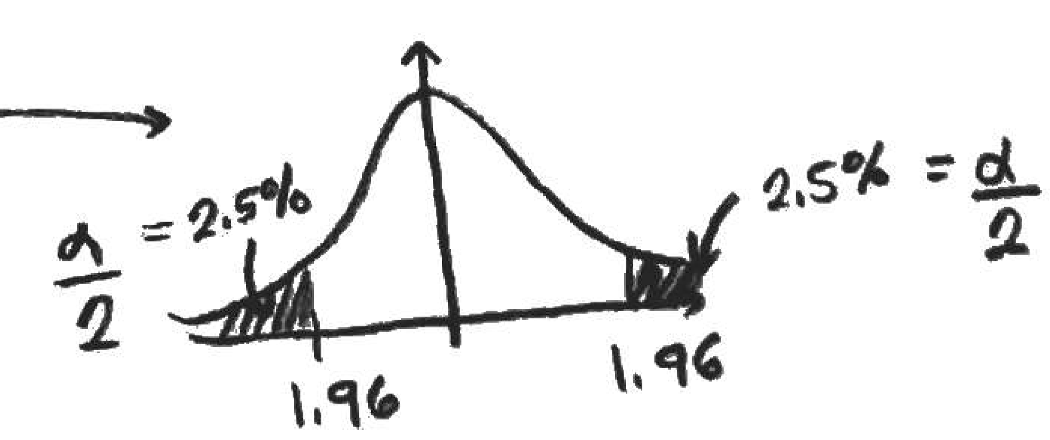
step 1: $\alpha = 5\%$ (given)

Step 2: $H_0: \mu = 650$
$H_1: \mu \neq 650$

Step 3: $t\text{-stat} = \dfrac{646.2 - 650}{19.5/\sqrt{420}} = \dfrac{-3.8}{0.952} = -4$

step 4: two sided critical value is 1.96

step 5: $|t\text{-stat}| = 4 > 1.96 \Rightarrow \boxed{\text{reject } H_0}$



(b) Carry out the same test as in part (a) but using a p-value.

Step 1 to 3: same as above

step 4: $p\text{-value} = 2 \cdot P(Z > |t\text{-stat}|) = 2 \cdot P(Z > 4) \approx 0$

step 5: $p\text{-value} < 0.05 \Rightarrow \boxed{\text{reject } H_0}$

Note that we got the same conclusion as in part (a).

(c) Without doing any calculation, is 650 contained in the 95% confidence interval for the population mean test score? Explain why or why not.

650 will not be contained in the 95% CI because as we found in part (a) and (b), we reject Ho that $\mu = 650$.

(d) Calculate the 95% confidence interval for the mean test score in the population, and verify your answer in part (c).

$\bar{Y} \pm 1.96\, SE(\bar{Y}) = 646.2 \pm 1.96(0.952) = (644, 648)$ → we note that 650 is not contained in this interval.

When districts were divided into districts with small classes (< 20 students per teacher) and large classes (≥ 20 students per teacher), the following results were found:

Class Size	Ave. Score	Standard Deviation	n
Small	657.4	19.4	238
Large	650.0	17.9	182

(e) At the 10% significance level, can we reject the null hypothesis that the mean test scores across the two groups is equal? (For practice, solve this problem using t-statistic, p-value, and confidence interval).

T-statistic:

Step 1: $\alpha = 10\%$

Step 2: $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$ } where μ_1 is the pop. mean score of small class, μ_2 is the pop. mean score of large class

Step 3: $\text{t-stat} = \dfrac{(657.4 - 650) - 0}{\sqrt{\frac{19.4^2}{238} + \frac{17.9^2}{182}}} = \dfrac{7.4}{1.83} = 4.04$

Step 4: critical value = 1.64

Step 5: $|4.04| > 1.64 \Rightarrow \boxed{\text{reject Ho}}$ @ 10% significance

p-value:

p-value $= 2 \cdot P(Z > 4) \approx 0 \Rightarrow \boxed{\text{reject Ho}}$ since $0 < 10\%$

confidence interval:

$7.4 \pm 1.96(1.83) = (3.8132, 10.9868)$ ⇒ 0 is not in the 90% CI

⇒ $\boxed{\text{reject Ho}}$ @ 10% significance