1 Population Regression vs. Sample Regression, Ordinary Least Squares (OLS) Estimation

Exercise 1.1. (Stock & Watson, Review the Concepts 4.1) Explain the difference between $\hat{\beta}_1$ and $\beta_1$; between the residual $\hat{u}_i$ and the regression error $u_i$; and between the OLS predicted value $\hat{Y}_i$ and $E(Y_i|X_i)$?

Exercise 1.2 (Final Exam, Fall 2014) Consider the population regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$.

(a) What minimization problem does OLS solve in order to estimate $\beta_0$ and $\beta_1$? Express the problem mathematically (there is no need to solve the minimization problem problem).

(b) The figure below shows data from a sample of 250 observations of $X$ and $Y$. One of the lines is the sample regression line, $\hat{\beta}_0 + \hat{\beta}_1 X_i$; the other is the population regression line $\beta_0 + \beta_1 X_i$. Is the sample regression line solid or dashed? Explain.
2 Interpreting OLS Regression Coefficients

Exercise 2.1  A regression of \emph{wage} (hourly wage, measured in 1979 dollars per hour) and \emph{educ} (years of schooling) using data from a random sample of 526 American workers yields the following:

\[ \hat{wage} = -0.90 + 0.54 \cdot educ \]

(a) Interpret the intercept of this regression.

(b) It turns out that all workers in the data have at least 8 years of education. Does this help reconcile your answer in part (a)?

(c) Interpret the slope of this regression.

(d) A worker has 16 years of education. What is the regression’s prediction for the worker’s hourly wage?

(e) Suppose a worker obtains 4 more years of schooling. What is the regression’s prediction for the change in the worker’s hourly wage?

3 Measures of Fit

Exercise 3.1  (Stock & Watson, Review the Concepts 4.3) \emph{SER} and \emph{R}^2 are “measures of fit” for a regression. Explain how the \emph{SER} measures the fit of a regression. What are the units of \emph{SER}? Explain how \emph{R}^2 measures the fit of a regression. What are the units of \emph{R}^2?
Exercise 3.2  (Stock & Watson, Review the Concepts 4.4) Sketch a hypothetical scatterplot of data for an estimated regression with $R^2 = 0.9$. Sketch a hypothetical scatterplot of data for a regression with $R^2 = 0.5$.

Exercise 3.3  Suppose that the $R^2$ from the regression in Exercise 2.1 is equal to 0.242. How would you interpret this $R^2$?

4  Least Squares Assumptions

Exercise 4.1.  (Stock & Watson, Chapter 4, Review the Concepts, Exercise 2) For each least squares assumption, provide an example in which the assumption is valid and then provide an example in which the assumption fails.

Exercise 4.2.  (From Section Notes 4 in bCourses (All Students) > Files > Discussion Section) A professor decides to run an experiment to measure the effect of time pressure on final exam scores. He gives each of the 400 students in his course the same final exam, but some students have 90 minutes to complete the exam while others have 120 minutes. Each student is randomly assigned one of the exam times based on the flip of a coin. Let $Y_i$ denote the number of points scored on the exam by the $i^{th}$ student ($0 \leq Y_i \leq 100$). Let $X_i$ denote the amount of time the student has to complete the exam ($X_i = \{90, 120\}$), and consider the regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$.

1. Explain what the term $u_i$ represents. Why will different students have different values of $u_i$?
2. Explain why $\mathbb{E}[u_i|X_i] = 0$ for this regression model.

3. Are the other least squares assumptions satisfied?

5 Regressions in Stata

Exercise 6.1  The table below shows regression output from Stata.

<table>
<thead>
<tr>
<th>Linear regression</th>
<th>Number of obs = 74</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F( 1, 72) = 17.28</td>
</tr>
<tr>
<td></td>
<td>Prob &gt; F = 0.0001</td>
</tr>
<tr>
<td></td>
<td>R-squared = 0.2196</td>
</tr>
<tr>
<td></td>
<td>Root MSE = 2623.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>Coef.</td>
</tr>
<tr>
<td>mpg</td>
<td>-238.8943</td>
</tr>
<tr>
<td>_cons</td>
<td>11253.06</td>
</tr>
</tbody>
</table>

Identify the following from the above regression output.

(a) Dependent and independent variables

(b) Sample size

(c) $R^2$

(d) SER

(e) $\beta_0$ and $\beta_1$