Econ 140 - Spring 2016 Section 7

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1 Hypothesis Testing in MRM: Overview

Туре	Example	Test Statistic
1. Individual		
2. Joint		
3. Linear		
3. Linear		

2 Hypothesis Testing in MRM: Single Coefficients and Joint Tests

Suppose we have the following model that explains baseball players' salaries.

$$salary_i = \beta_0 + \beta_1 years_i + \beta_2 gamesyr_i + \beta_3 bavg_i + \beta_4 hrunsyr_i + \beta_5 rbisyr_i + u_i$$
 (1)

where for each player i, salary is the salary in 1993, years is years in the league, gamesyr is average games played per year, bavg is the career batting average, hrunsyr is the number of home runs per year, rbisyr is runs batted in per year. Further, suppose that we estimated the above equation using data we have on hand, and that we obtained the following regression results

$$\widehat{salary} = \underbrace{11.10}_{(0.29)} + \underbrace{0.0689}_{(0.0121)} \cdot years + \underbrace{0.0126}_{(0.0026)} \cdot gamesyr + \underbrace{0.00098}_{(0.0010)} \cdot bavg + \underbrace{0.0144}_{(0.00161)} \cdot hrunsyr + \underbrace{0.0108}_{(0.0072)} \cdot rbisyr$$

$$N = 353, \ SSR = 183.186, \ R^2 = 0.6278$$

Example 2.1. What test statistic would we use to test the hypothesis $H_0: \beta_4 = 0, H_1: \beta_4 \neq 0$? Carry out this test at the 5% level?

Example 2.2. What test statistic would we use to test the hypothesis $H_0: \beta_2 = 0.1, H_1: \beta_2 \neq 0.1$? Carry out this test at the 10% level.

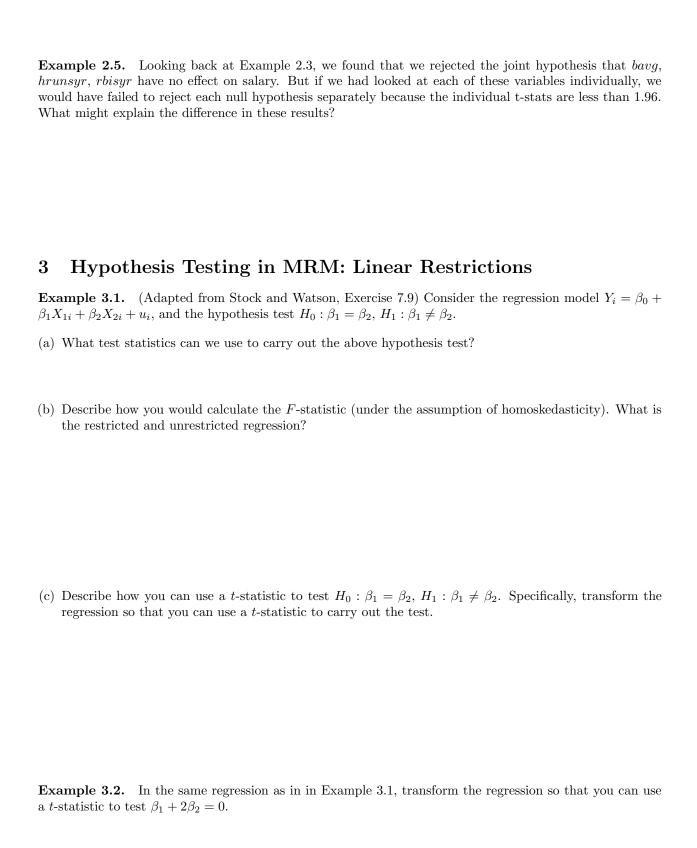
Example 2.3. A sports analyst hypothesizes that once years in the league and games per year have been controlled for, the variables *bavg*, *hrunsyr*, and *rbisyr* (which we can think of as measure of performance) have no effect on salary.

- (a) What is the null and alternative hypothesis?
- (b) What test statistic would we use to test the hypothesis in part (a)? Assuming that the population errors are homoskedastic, what is the formula for this test statistic?
- (c) What is the restricted regression?
- (d) What is the unrestricted regression?
- (e) What is q?
- (f) What is n?
- (g) What is k?
- (h) Suppose that a regression of salary on years and gamesyr yielded an SSR of 198.311. Calculate the F-statistic.
- (i) Given a significance level of 5%, what is the critical value from the F-distribution.
- (i) What is the conclusion of your hypothesis test?

Example 2.4. Let us consider again the joint hypothesis

$$H_0: \beta_3=0,\ \beta_4=0,\ \beta_5=0\ vs.\ H_1:$$
 at least one of β_3,β_4,β_5 is not equal to 0

that we tested in Example 2.3 using an F-test. Is it possible to carry out this joint hypothesis test using the 3 t-statistics from the following 3 individual tests: (1) $H_0: \beta_3 = 0, H_1: \beta_3 \neq 0$; (2) $H_0: \beta_4 = 0, H_1: \beta_4 \neq 0$; (3) $H_0: \beta_5 = 0, H_1: \beta_5 \neq 0$?



4 Hypothesis Testing in MRM: Stata

Example 4.1. Suppose we have 1980 census data on the 50 states recording the population size in each state (pop), the median age (medage), the number of deaths (death), the number of marriages, (marriage), and the number of divorces (divorce). We estimate the following regression:

. reg pop medage death marriage divorce

Source	SS	df	MS	Number of obs = 50
+-				F(4, 45) = 1299.46
Model	1.0800e+15	4	2.7000e+14	Prob > F = 0.0000
Residual	9.3500e+12	45	2.0778e+11	R-squared = 0.9914
+				Adj R-squared = 0.9907
Total	1.0893e+15	49	2.2232e+13	Root MSE = $4.6e+05$

pop	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
medage death marriage divorce _cons	-181303.8 91.30243 1.80206 39.80303 5241295	43749.97 4.137673 4.303597 8.146704 1272002	-4.14 22.07 0.42 4.89 4.12	0.000 0.000 0.677 0.000 0.000	-269420.8 82.96873 -6.865829 23.39473 2679350	-93186.87 99.63613 10.46995 56.21134 7803239

What Stata commands would you use to test the following hypothesis?

- (a) Test the individual hypothesis that the coefficient on medage is zero.
- (b) Test the joint hypothesis that the coefficients on all four regressors are simultaneously zero.
- (c) Test the linear hypothesis that the coefficient on death minus the coefficient on marriage is zero.

5 Additional Exercise: Spring 2014 MT2, Question 4

In this exercise we use a dataset containing information on 269 NBA basketball players including their salaries. Table 1 below (see next page) shows the results from 3 OLS regressions where heteroskedasticity-robust standard errors are given in square brackets. The dependent variable is salary (annual salary in thousands \$), and the explanatory variables are points (average points per game), rebounds (average rebounds per game), and assists (average assists per game).

(a) [6] What is the interpretation of the coefficient on points in all three regressions? Give the meaning of the OLSE of the points coefficient from the third regression.

Table 1: Regressions on NBA Salaries

	6		
Variables	Model 1	Model 2	Model 3
points	111.67	87.72	80.67
	[8.18]**	[9.43]**	[11.20]**
rebounds		86.44	93.36
		[20.62]**	[21.25]**
assists			26.08
			[21.56]
_cons	278.10	137.56	115.02
	[83.62]**	[83.09]	[86.09]
R2	0.43	0.47	0.48
N	269	269	269

Notes: * p-value<0.05; ** p-value<0.01.

- (b) [8] When the rebounds variable is added, both the R2 and the Adjusted R2 increase whereas when additionally assists variable is added, only the R2 increases. Does this indicate that rebounds should be in the regression and that assists should not? Explain.
- (c) [5] We wonder whether a player's position matters, thinking that different positions may be more valuable to a team than others. To investigate, we regresses salary on dummy indicators of each of the three possible court positions a player can have, center, forward, and guard (and a constant), with no other explanatory variables. Stata refuses to report OLSEs for all three regressor. Why does this happen? How would you solve this problem?
- (d) [3] Based on the results, it seems that the marginal effect of rebounds on salaries is higher than the marginal effect of points. So immediately after running the regression for model 3 we issue the command: test rebounds=points. What does this Stata command do?
- (e) [7] Stata generates output from the command in (d) that includes an F-statistic. Suppose that it takes the value: 4.60. Decide whether you reject the null at the 1% level of significance.

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2.89 2.50 2.51 2.40 2.17 2.67 2.60 1.55 1.91 1.87 1.84 2.7 4.21 3.45 2.65 2.73 2.57 2.46 2.37 2.31 2.31 2.31 2.31 2.31 2.31 2.32 2.30 2.32 2.32 2.32 2.32 2.32 2.32	421 335 296 273 257 246 237 231 225 220 27 788 549 460 460 440 341 333 299 2.00 255 245 236 229 224 219 28 764 545 457 451 418 333 299 2.00 255 242 235 229 228 22 218 7 76 545 457 451 411 332 299 2.00 255 242 233 227 218 210 210 210 210 210 210 210 210 210 210	4.14 3.82 3.59 3.42 3.29 3.18 3.09
2.89 2.90 2.29 2.16 2.06 2.00 1.94 1.90 1.87 1.84 2.8 4.20 3.34 2.95 2.71 2.56 2.45 2.20 2.24 2.19 1.97 1.84 2.89 2.90 2.39 2.90 2.55 2.45 2.50 2.29 2.24 2.10 1.95 1.89 1.86 1.83 2.90 4.18 3.33 2.99 2.70 2.55 2.42 2.35 2.28 2.24 2.20 2.94 1.99 1.99 1.80 1.97 1.71 1.71 1.71 1.71 1.71 1.71 1.71	4.20 3.34 2.95 2.71 2.56 2.45 2.26 2.29 2.24 2.19 2.8 7.64 5.45 4.57 4.18 3.32 2.92 2.05 2.45 2.35 2.27 2.18 2.9 7.66 5.47 4.54 4.54 4.17 3.32 2.92 2.87 2.42 2.33 2.27 2.13 2.16 2.9 7.66 5.39 4.54 4.54 4.10 3.15 2.76 2.25 2.27 2.17 2.10 2.04 1.99 60 7.08 4.08 4.08 4.13 3.10 2.04 2.05 2.05 2.05 2.05 2.05 2.05 2.05 2.05	4.11 3.78 3.56 3.39 3.26 3.15 3.06
2.89 2.50 2.38 2.14 2.06 1.99 1.95 1.89 1.86 1.87 2.9 4.18 3.33 2.95 2.70 2.55 2.45 2.38 2.22 2.18 2.25 2.24 2.05 1.95 1.99 1.99 1.99 1.99 1.99 1.99 1.9	4.18 332 293 270 255 244 235 228 218 22 7.06 547 454 417 332 292 2.07 255 242 233 227 212 216 70 756 549 454 4400 315 276 255 257 225 217 210 204 199 60 7.06 498 418 418	4.07 3.75 3.53 3.36 3.23 3.12 3.03
28 249 228 214 216 195 187 187 171 171 171 60 417 315 226 237 235 212 216 216 217 210 214 199	4.17 3.32 2.92 2.69 2.55 2.42 2.33 2.27 2.2.1 2.16 30 7.56 5.39 4.51 4.00 315 2.76 2.53 2.57 2.52 2.17 2.10 2.04 1.99 60 7.08 4.98 4.13	3.73 3.50 3.33 3.20 3.09
60 400 3.15 2.76 2.53 2.37 2.25 2.17 2.10 2.04 1.99	4.00 3.15 2.76 2.55 2.37 2.25 2.17 2.10 2.04 1.99 60 7.08 4.98 4.13	4.02 3.70 3.47 3.30 3.17 3.07 2.98
111 111 111 111 111 111 111 111 111 11		3.34 3.12 2.95 2.82 2.72
2.76 2.36 2.15 2.01 1.91 1.84 1.78 1.74 1.70 1.67 90 3.95 3.10 2.71 2.47 2.32 2.20 2.11 2.04 1.99 1.94	3.95 3.10 2.71 2.47 2.52 2.20 2.11 2.04 1.99 1.94	3.23 3.01 2.84 2.72 2.61
2.13 1.99 1.90 1.82 1.77 1.72 1.68 1.65 120 3.92 3.07 2.68 2.45 2.29 2.18 2.09 2.02 1.96	3.92 3.07 2.68 2.45 2.29 2.18 2.09 2.02 1.96 1.91	3,48 3.17 2.96 2.79 2.66 2.56 2.47
$2.30 2.08 1.94 1.48 1.77 1.72 1.67 1.69 \infty \qquad 3.84 3.00 2.60 2.57 2.21 2.10 2.01 1.94 1.88 1.83$	3.84 3.00 2.60 2.37 2.21 2.10 2.01 1.94 1.88 1.83	3.02 2.80 2.64 2.51 2.41