

Econ 140 - Spring 2016

Section 11

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1 IV Regression: Review the Concepts

Exercise 1.1. (Stock & Watson, Review the Concepts 12.1) Consider the problem of estimating the elasticity of demand for butter using the demand equation

$$\ln(Q_i^{butter}) = \beta_0 + \beta_1 \ln(P_i^{butter}) + u_i.$$

In this regression model, is $\ln(P_i^{butter})$ positively or negatively correlated with the error u_i ? If β_1 is estimated by OLS, would you expect the estimated value to be larger or smaller than the true value of β_1 ? Explain.

An increase in the regression error, u , shifts out the demand curve, leading to an increase in both price and quantity. Thus $\ln(P_i^{butter})$ is positively correlated with the regression error. Because of this positive correlation, the OLS estimator of β_1 is inconsistent and is likely to be larger than the true value of β_1 .

Exercise 1.2. (Stock & Watson, Review the Concepts 12.2) Consider the problem of estimating the elasticity of demand for cigarettes using the demand equation

$$\ln(Q_i^{cigs}) = \beta_0 + \beta_1 \ln(P_i^{cigs}) + u_i.$$

Suppose that we used as an instrument for $\ln(P_i^{cigs})$ the number of trees per capita in the state. Is this instrument relevant? Is it exogenous? Is it a valid instrument?

The number of trees per capita in the state is exogenous because it is plausibly uncorrelated with the error in the demand function. However, it probably is also uncorrelated with $\ln(P_i^{cigs})$, so it is not relevant. A valid instrument must be exogenous and relevant, so the number of trees per capita in the state is not a valid instrument.

Exercise 1.3. (Adapted from Stock & Watson, Review the Concepts 12.3) Consider a study on the effect of incarceration (imprisonment) on crime rates. Specifically, we want to examine whether putting criminals in jail reduces crime.

(a) One strategy for estimating this effect is to regress *crime_rates* (crimes per 100,000 member of the general population) against *incarceration_rate* (prisoners per 100,000) using annual data from U.S. states. Explain why this regression is subject to bias.

There are many potential reasons (e.g. OVB, etc.), but one serious issue is due to simultaneous causality. On the one hand, incarceration reduces crime rate. But on the other hand, if the crime rates go up and the police do their job, there will be more prisoners, so incarceration rates will go up.

(b) Suppose that we use the number of lawyers per capita as an instrument for *incarceration_rate*. Would this instrument be relevant? Would it be exogenous? Would it be a valid instrument?

The number of lawyers is arguably correlated with the incarceration rate, so it is relevant (although this should be checked in the data as well, using the methods in Section 12.3). However, states with higher than expected crime rates (with positive regression errors) are likely to have more lawyers (criminals must be defended and prosecuted), so the number of lawyers will be positively correlated with the regression error. This means that the number of lawyers is not exogenous. A valid instrument must be exogenous and relevant, so the number of lawyers is not a valid instrument.

Exercise 1.4. (Adapted from Stock & Watson, Review the Concepts 12.4) Does a new medical procedure (in this example, cardiac catheterization) prolong lives?

- (a) Suppose we answer the above question by comparing patients who received the treatment to those who did not. This leads to regressing the length of survival of the patient, *months_survived*, on a binary variable for whether the patient received the procedure, *got_treatment*. Explain why this regression is subject to bias.

There is sample selection bias the doctor and the patient decide to undertake the new procedure (e.g., they decide to do it if they believe it might be effective). If the healthiest patients are the ones who receive the treatment, then the OLS estimate will be biased and will appear more effective than it really is.

- (b) Suppose that we use as an instrument for *got_treatment*, the difference between the distance from the patient's home to the nearest cardiac catheterization hospital, and the distance to the nearest hospital of any sort (which did not offer the treatment); this instrument takes on the value zero if the nearest hospital is a cardiac catheterization hospital, and otherwise it is positive. How could you determine whether this instrument is relevant? How could you determine whether this instrument is exogenous?

If the difference in distance is a valid instrument, then it must be correlated with X , which in this case is a binary variable indicating whether the patient received cardiac catheterization. Instrument relevance can be checked by regressing *got_treatment* on "difference in distance." In this regression, if the F -stat testing that the coefficient on the "difference in distance" variable is less than 10, then the instrument is weak. Checking instrument exogeneity is more difficult. If there are more instruments than endogenous regressors, then joint exogeneity of the instruments can be tested using the J -test. However, if the number of instruments is equal to the number of endogenous regressors, then it is impossible to test for exogeneity statistically. In this case, we have one endogenous regressor (*got_treatment*) and one instrument (difference in distance), so the J -test cannot be used. Expert judgment is required to assess the exogeneity.

Exercise 1.5. (Stock & Watson, Exercise 12.7) In an IV regression model with one regressor, X_i , and two instruments, Z_{1i} and Z_{2i} , the value of the J -statistic is $J = 18.2$.

- (a) Does this suggest that $E(u_i|Z_{1i}, Z_{2i}) \neq 0$? Explain.

Yes. Under the null hypothesis of instrument exogeneity, the J statistic is distributed as a χ^2_{2-1} random variable, with a 1% critical value of 6.63. Thus we reject the null, the statistic is significant, and instrument exogeneity $E(u_i|Z_{1i}, Z_{2i}) = 0$ is rejected.

- (b) Does this suggest that $E(u_i|Z_{1i}) \neq 0$? Explain.

No. The J test suggests that $E(u_i|Z_{1i}, Z_{2i}) \neq 0$, but doesn't provide evidence about whether the problem is with Z_{1i} or Z_{2i} (or both).

Exercise 1.6. (Stock & Watson, Exercise 12.9) A researcher is interested in the effect of military service on human capital. He collects data from a random sample of 4000 workers aged 40 and runs the OLS regression $Y_i = \beta_0 + \beta_1 X_i + u_i$, where Y_i is the worker i 's annual earnings and X_i is a binary variable that is equal to 1 if the person served in the military and 0 otherwise.

- (a) Explain why the OLS estimates are likely to be unreliable. (*Hint*: Which variables are omitted from the regression? Are they correlated with military service?)

There are other factors that could affect both the choice to serve in the military and annual earnings. One example could be *education*, although this could be included in the regression as a control variable. Another variable is *ability* which is difficult to measure, and thus difficult to control for in the regression.

- (b) During the Vietnam War there was a draft, where priority for the draft was determined by a national lottery. (The days of the year were randomly reordered 1 through 365. Those with birthdates ordered first were drafted before those with birthdates ordered second, and so forth.) Explain how the lottery might be used as an instrument to estimate the effect of military service on earnings.

The draft was determined by a national lottery so the choice of serving in the military was random. Because it was randomly selected, the lottery number is uncorrelated with individual characteristics that may affect earnings and hence the instrument is *exogenous*. Because it affected the probability of serving in the military, the lottery number is *relevant*.

2 IV Regression: Stata Example

Exercise 2.1. (Adapted from Stock & Watson, Empirical Exercise 12.1) In this exercise, we will use Stata to estimate the effect of fertility on women's labor supply. The data set `fertility.dta` contains information on married women aged 21-35 with two or more children. The variables we will use are:

- `weeksworked`: mom's weeks worked in 1979
- `morekids`: dummy variable equal to 1 if mom had more than 2 kids
- `twoboys`: dummy variable equal to 1 if mom's first two kids are boys
- `twogirls`: dummy variable equal to 1 if mom's first two kids are girls
- `age`: age of mom at 1980 census

Using these data, we are interested in understanding the following question: how much does a woman's labor supply fall when she has an additional child? Hence, the regression we want to estimate is:

$$\text{weeksworked} = \beta_0 + \beta_1 \text{morekids} + \beta_2 \text{age} + u$$

- (a) Using Stata, implement an IV regression of `weeksworked` on `morekids` and `age`, and using `twoboys` and `twogirls` as instruments for `morekids`, by estimating each of the two stages using OLS. What are Y , W , X , and Z here?

The Y variable is `weeksworked`, W is `age`, X is `morekids`, and we have two instruments Z_1 and Z_2 for `twoboys` and `twogirls`, respectively. The commands for each stage are as follows:

- First stage: `reg morekids twoboys twogirls age, robust`
- Obtain predicted values for endogenous variable: `predict morekids_hat, xb`
- Second stage: `reg weeksworked morekids_hat age, robust`

```
. reg morekids twoboys twogirls age, robust
```

```
Linear regression                                Number of obs = 254654
                                                F( 3,254650) = 1342.66
                                                Prob > F      = 0.0000
                                                R-squared    = 0.0151
                                                Root MSE    = .48185
```

```
-----+-----
```

	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
morekids						
twoboys	.0578227	.0023012	25.13	0.000	.0533124	.0623329
twogirls	.0789634	.0023967	32.95	0.000	.0742659	.0836608
age	.0143633	.0002769	51.88	0.000	.0138206	.014906
_cons	-.0902769	.0084654	-10.66	0.000	-.1068689	-.073685

```
-----+-----
```

```
. predict morekids_hat, xb
```

```
. reg weeksworked morekids_hat age, robust
```

```
Linear regression                                Number of obs = 254654
                                                F( 2,254651) = 1729.59
                                                Prob > F      = 0.0000
                                                R-squared    = 0.0124
                                                Root MSE    = 21.731
```

```
-----+-----
```

	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
weeksworked						
morekids_hat	-5.58984	1.239716	-4.51	0.000	-8.01965	-3.160029
age	.7977256	.0215668	36.99	0.000	.7554553	.8399959
_cons	-3.099863	.3770051	-8.22	0.000	-3.838783	-2.360943

```
-----+-----
```

- (b) Perform the same IV regression as in part (a) but using Stata's `ivregress` command. Verify that the coefficients are the same you obtained are the same as in part (a), but the standard errors are different; why is this the case?

```
. ivregress 2sls weeksworked (morekids = twoboys twogirls) age, robust
```

```
Instrumental variables (2SLS) regression          Number of obs = 254654
                                                Wald chi2(2) = 3518.15
                                                Prob > chi2  = 0.0000
                                                R-squared   = 0.0295
                                                Root MSE   = 21.542
```

```
-----+-----
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
morekids	-5.58984	1.229054	-4.55	0.000	-7.998741	-3.180938
age	.7977256	.0213784	37.31	0.000	.7558246	.8396265

```
-----+-----
```

```

      _cons | -3.099863   .373689   -8.30   0.000   -3.83228   -2.367446
-----+-----

```

```

Instrumented:  morekids
Instruments:   age twoboys twogirls

```

Comparing the results we have here to those in part (a), we find that the coefficients are the same but the standard errors are different. The standard errors in part (a) are not correct because it does not take into account the fact that the regressor `morekids_hat` used in the second stage is coming from a first stage regression (so it is a random variable, because of sampling variability). In contrast, the `ivregress 2sls` *does* take it into account, so the standard errors from this command are correct.

(c) For the Stata command in part (b):

(i) Explain each part of this Stata command.

`ivregress 2sls` tells Stata to run a two-stage least-squares regression. Then, `ivregress 2sls` is followed by the outcome variable Y , which in this case is `weeksworked`. In the part where we have `(morekids = twoboys twogirls)`, this says that the endogenous variable X is `morekids`, and `twoboys`, `twogirls` are the two instruments Z_1 and Z_2 , respectively. Finally, `age` is a control variable here, which corresponds with W in our notation.

Note that when executing the code in part (b), the first stage regression is not shown in the Stata output. However if we wanted Stata to show the output for the first stage, we just need to add the word `first` at the end of the command, i.e., `ivregress 2sls weeksworked (morekids = twoboys twogirls) age, robust first`. Executing this command in Stata will show the following output, which includes the results from the first stage regression before showing the second stage results:

```
. ivregress 2sls weeksworked (morekids = twoboys twogirls) age, robust first
```

```
First-stage regressions
```

```
-----
```

```

Number of obs   =    254654
F(   3, 254650) =   1342.66
Prob > F        =    0.0000
R-squared       =    0.0151
Adj R-squared   =    0.0151
Root MSE       =    0.4819

```

```

-----+-----
      |               Robust
morekids |               Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
      |
      age |   .0143633   .0002769   51.88  0.000   .0138206   .014906
      twoboys |   .0578227   .0023012   25.13  0.000   .0533124   .0623329
      twogirls |   .0789634   .0023967   32.95  0.000   .0742659   .0836608
      _cons |  -.0902769   .0084654  -10.66  0.000  -.1068689  -.073685
-----+-----

```

```
Instrumental variables (2SLS) regression
```

```

Number of obs   =    254654
Wald chi2(2)    =   3518.15
Prob > chi2     =    0.0000
R-squared       =    0.0295
Root MSE       =   21.542

```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
weeksworked						
morekids	-5.58984	1.229054	-4.55	0.000	-7.998741	-3.180938
age	.7977256	.0213784	37.31	0.000	.7558246	.8396265
_cons	-3.099863	.373689	-8.30	0.000	-3.83228	-2.367446

Instrumented: morekids
 Instruments: age twoboys twogirls

(ii) How many instruments do we have? How many endogenous regressors? Is the regression overidentified, underidentified, or exactly identified?

We have two instruments and one endogenous regressor, so we are overidentified.

(d) Using Stata, test whether twoboys and twogirls are “strong” instruments.

We can carry out the first stage regression, and test the hypothesis that the coefficients on twoboys and twogirls are jointly zero.

```
. reg morekids twoboys twogirls age, robust
```

```
Linear regression                               Number of obs = 254654
                                                F( 3,254650) = 1342.66
                                                Prob > F      = 0.0000
                                                R-squared    = 0.0151
                                                Root MSE    = .48185
```

	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
morekids						
twoboys	.0578227	.0023012	25.13	0.000	.0533124	.0623329
twogirls	.0789634	.0023967	32.95	0.000	.0742659	.0836608
age	.0143633	.0002769	51.88	0.000	.0138206	.014906
_cons	-.0902769	.0084654	-10.66	0.000	-.1068689	-.073685

```
. test (twoboys=0) (twogirls=0)
```

```
( 1) twoboys = 0
( 2) twogirls = 0
```

```
F( 2,254650) = 659.77
Prob > F = 0.0000
```

The F -stat we obtain is 659.77, which is greater than 10, so we conclude that we have “strong” instruments.

(e) Using Stata, implement the over-identifying restrictions test. Specifically:

- (i) Calculate the J -statistic.
- (ii) Assuming a 5% significance level, what is the relevant critical value?
- (iii) What does the test conclude?

The commands/steps are as follows:

- Step 1: Estimate IV 2SLS regression: `ivregress 2sls weeksworked (morekids = twoboys twogirls) age, robust`
- Step 2: Obtain residuals: `predict uhat, resid`
- Step 3: Regress residuals on Z 's and W 's using *homoskedastic* errors: `reg uhat twoboys twogirls age`
- Step 4: Test the hypothesis that the coefficients on Z 's are jointly zero and obtain *homoskedastic* F -stat: `test (twoboys = 0) (twogirls = 0)`. Doing this in Stata (see output below), we obtain the F -stat = 1.50.
- Step 5: Calculate J -stat = $m \cdot F$ where m is the number of instruments. In this case, $m = 2$ so our J -stat = $2 * 1.50 = 3.00$.
- Step 6: Find the critical value from the Chi-squared distribution with $m - k$ degrees of freedom (where m is again the number of instruments Z , and k is the number of endogenous variables X). In this example, $m - k = 2 - 1 = 1$. Using the Chi-squared table at the back of the textbook and 5% significance, we find that the critical value is 3.84.

Instead of using the Chi-squared table to find this critical value, we also could have used the Stata command `di invchi2(1,0.95)` where `di` means display, the first number inside the parenthesis is the degrees of freedom, i.e., $m - k = 1$, and the second number 0.95 is because we are using $\alpha = 0.05$, so that $1 - \alpha = 0.95$.

- Step 7: Conclusion: If J -stat > critical value, we reject H_0 , which is evidence that at least one of the Z 's is not exogenous. If J -stat < critical value, we fail to reject H_0 , which is evidence that the Z 's are exogenous. In our example, we fail to reject H_0 because $3.00 < 3.84$.

```
. ivregress 2sls weeksworked (morekids = twoboys twogirls) age, robust
```

```
Instrumental variables (2SLS) regression                Number of obs = 254654
                                                       Wald chi2(2)   = 3518.15
                                                       Prob > chi2    = 0.0000
                                                       R-squared      = 0.0295
                                                       Root MSE      = 21.542
```

```
-----
```

		Robust				
weeksworked	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
morekids	-5.58984	1.229054	-4.55	0.000	-7.998741	-3.180938
age	.7977256	.0213784	37.31	0.000	.7558246	.8396265
._cons	-3.099863	.373689	-8.30	0.000	-3.83228	-2.367446

```
-----
```

```
Instrumented:  morekids
Instruments:   age twoboys twogirls
```

```
. predict uhat, resid
```

```
. reg uhat twoboys twogirls age
```

```
Source |          SS          df          MS          Number of obs = 254654
```

Model		1390.24171	3	463.413903	F(3,254650) =	1.00
Residual		118169993254650	464.048665		Prob > F	= 0.3923
					R-squared	= 0.0000
					Adj R-squared	= -0.0000
Total		118171383254653	464.048658		Root MSE	= 21.542

uhat		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
twoboys		-.1281283	.1026161	-1.25	0.212	-.3292531 .0729965
twogirls		.0750825	.1063013	0.71	0.480	-.1332652 .2834302
age		.000073	.0126058	0.01	0.995	-.0246339 .02478
_cons		.0139247	.3880438	0.04	0.971	-.7466309 .7744803

. test (twoboys = 0) (twogirls = 0)

- (1) twoboys = 0
- (2) twogirls = 0

F(2,254650) = 1.50
 Prob > F = 0.2236

. di invchi2(1,0.95)
 3.8414588

3 Final Exam Review: Spring 2014 Final, Question 4

In honor of Mother's Day last weekend, this question concerns a dataset consisting of more than a quarter of a million moms between the ages of 21 and 35 drawn from the 1980 Census. We focus on five variables in that dataset:

Variable	Description
weeksworked	number of weeks worked by the mom in 1979
morekids	= 1 if mom had more than 2 children
samesex	= 1 if 2 or more children and first two are same sex
hispan	= 1 if mom is Hispanic
age	age of mom in the 1980 census

A labor economist is interested in answering how the number of children affects mothers' labor supply decisions, and specifically whether there is a causal effect of the dummy indicator `morekids` on `weeksworked`. The researcher's population regression is: $weeksworked_i = \beta_0 + \beta_1 morekids_i + \beta_2 age_i + u_i$. Below you will find the log of a series of Stata program commands executed on this dataset by the researcher, and below that is a list of questions to be answered using this output.

```
. summ weeksworked morekids samesex hispan age

-----+-----
Variable |      Obs      Mean   Std. Dev.   Min        Max
-----+-----
weeksworked | 254654   19.01833   21.86728         0         52
morekids | 254654   .3805634   .4855263         0         1
samesex | 254654   .5055683   .49997         0         1
hispan | 254654   .0742066   .2621073         0         1
age | 254654   30.39327   3.386447        21         35

. correlate weeksworked morekids samesex hispan age
(obs=254654)
-----+-----
| weeksw-d morekids  samesex  hispan  age
-----+-----
weeksworked | 1.0000
morekids | -0.1196  1.0000
samesex | -0.0097  0.0695  1.0000
hispan | -0.0104  0.0777 -0.0002  1.0000
age | 0.1111  0.0999 -0.0031 -0.0657  1.0000

. regress weeksworked morekids age , robust

Linear regression                               Number of obs = 254654
                                                F( 2,254651) = 4252.27
                                                Prob > F      = 0.0000
                                                R-squared     = 0.0296
                                                Root MSE     = 21.541

-----+-----
|              Robust
weeksworked |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
morekids | -5.946385   .0865748   -68.68  0.000   -6.11607   -5.776701
age | .8028323   .0121562   66.04  0.000   .7790064   .8266582
_cons | -3.119385   .3674612   -8.49  0.000   -3.839599  -2.399171
-----+-----

. regress morekids samesex hispan , robust

Linear regression                               Number of obs = 254654
                                                F( 2,254651) = 1368.81
                                                Prob > F      = 0.0000
                                                R-squared     = 0.0109
                                                Root MSE     = .48288

-----+-----
|              Robust
morekids |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
samesex | .0675405   .0019132   35.30  0.000   .0637907   .0712902
hispan | .1439261   .0037629   38.25  0.000   .136551   .1513013
_cons | .3357368   .0013596  246.93  0.000   .333072   .3384017
-----+-----

. test samesex hispan

( 1)  samesex = 0
( 2)  hispan = 0

F( 2,254651) = 1368.81
```

Prob > F = 0.0000

. ivregress 2sls weeksworked (morekids = samesex) age , robust

Instrumental variables (2SLS) regression	Number of obs = 254654
	Wald chi2(2) = 3520.60
	Prob > chi2 = 0.0000
	R-squared = 0.0296
	Root MSE = 21.541

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
weeksworked						
morekids	-6.06062	1.258803	-4.81	0.000	-8.527828	-3.593413
age	.8044685	.0217279	37.02	0.000	.7618825	.8470544
_cons	-3.125639	.3739658	-8.36	0.000	-3.858599	-2.39268

Instrumented: morekids
Instruments: age samesex

. ivregress 2sls weeksworked (morekids = hispan) age , robust

Instrumental variables (2SLS) regression	Number of obs = 254654
	Wald chi2(2) = 3469.70
	Prob > chi2 = 0.0000
	R-squared = 0.0205
	Root MSE = 21.642

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
weeksworked						
morekids	-1.63085	1.037662	-1.57	0.116	-3.66463	.4029299
age	.7410219	.0192269	38.54	0.000	.7033379	.7787059
_cons	-2.8831	.3736879	-7.72	0.000	-3.615515	-2.150685

Instrumented: morekids
Instruments: age hispan

. ivregress 2sls weeksworked (morekids = samesex hispan) age , robust

Instrumental variables (2SLS) regression	Number of obs = 254654
	Wald chi2(2) = 3506.36
	Prob > chi2 = 0.0000
	R-squared = 0.0265
	Root MSE = 21.575

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
weeksworked						
morekids	-3.430894	.7992682	-4.29	0.000	-4.997431	-1.864357
age	.7668035	.0166953	45.93	0.000	.7340813	.7995257
_cons	-2.981656	.3707483	-8.04	0.000	-3.70831	-2.255003

Instrumented: morekids
Instruments: age samesex hispan

. predict u2slshat , resid

. regress u2slshat samesex hispan age

Source	SS	df	MS	Number of obs = 254654
Model	3410.54014	3	1136.84671	F(3,254650) = 2.44
Residual	118536486254650	465	487869	Prob > F = 0.0622
Total	118539896254653	465	495778	R-squared = 0.0000
				Adj R-squared = 0.0000
				Root MSE = 21.575

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
u2slshat						
samesex	-.1783145	.0855142	-2.09	0.037	-.34592	-.0107091
hispan	.2819951	.1634709	1.73	0.085	-.0384035	.6023937
age	.0013517	.0126525	0.11	0.915	-.0234469	.0261504
_cons	.0281409	.3904391	0.07	0.943	-.7371093	.7933912

. test samesex hispan

- (1) samesex = 0
- (2) hispan = 0

F(2,254650) = 3.66
Prob > F = 0.0256

- a) [5] Why did the researcher include the age of the mother in the OLS regression? What would you expect would happen to the coefficient on `morekids` if `age` was excluded?

Answer: The age of the mother should correlate with her employability within the 21-35 range, but also with the number of kids she has had. Leaving it out will cause omitted variable bias. The OLSE would be biased upward assuming that older moms tend to have more kids than younger ones.

- b) [4] Give one reason why you would suspect that the coefficient on `morekids` would not be unbiased and consistent estimate of the population coefficient.

Answer: One reason is that there is simultaneous causality occurring, with more weeks worked resulting in more income which, in turn, makes more kids affordable for the mother and her family. There could be omitted variable that affects both amount of work and number of kids. For example, mother's education level is omitted from the regression. Education level likely is positively related to employment opportunities and it is also known that the number of children is inversely related to education of the mother.

- c) [4] What is the interpretation of the coefficient on `morekids` in the OLS regression. Is it economically significant? Is it statistically significant?

Answer: A mother who has more than 2 kids works 5.95 fewer weeks on average compared to a mother who has just 1 or 2 kids. The average number of weeks worked is 19 so a change of 6 is quite economically significant. A t stat of 68 ensures it is strongly different from zero.

From the above output, the researcher thinks that two of the other variables in the dataset, `samesex` and `hispan`, are potential instruments for `morekids`. The variable `samesex` takes the value of 1 when the mother has had 2 or more children *and* the first two were both boys *or* both girls, and 0 otherwise.

- d) [5] Why might the variable `samesex` be a "relevant" instrument for the endogenous regressor `morekids`? Explain why there is empirical evidence in the Stata output supporting the relevance of both `samesex` and `hispan`.

Answer: `samesex` is relevant if it is statistically correlated with `morekids`. If, after having two children of the same sex, mothers and parents wish to have a third child in the hopes they will have a child of the other gender, then `morekids` and `samesex` will be positively correlated. The correlation matrix offers a bit of evidence that this is so. Also the test of whether both instruments are jointly zero in the first stage regression yields an $F = 1,368$ far in excess of the rule of thumb of 10.

- e) [4] Assuming that both candidates are valid instruments, why does the researcher have a case of "over identification"? What evidence do you see in the Stata output that confirms that there is an issue with overidentification?

Answer: There are two instruments but only one endogenous regressor: $m - k = 2 - 1 = 1$ degree of over identification. Inspection of the two TSLS regression results using the two candidates separately yields quite different coefficient estimates: -6.06 and -1.63 using `samesex` and `hispan` as instruments, respectively. These estimates are sufficiently different from one another to call into question exogeneity of one or both instruments.

- f) [4] For these variables, what does it mean for the candidate instruments to meet the second criterion of a valid instrument, i.e., to be "exogenous"? What is in the Stata output that should make you suspicious that the one or the other or both of these candidate instruments are not exogenous?

Answer: An instrument is "exogenous" if it is uncorrelated with the error term from the population regression. As in part (e) the TSLS estimates of coefficient on `morekids` differs depending on the instrument chose: -6.06 vs. -1.63.

- g) [7] How did the researcher attempt to determine whether the candidate instruments are exogenous? What should the researcher conclude?

Answer: The researcher has estimated the J statistic to perform a test of over identifying restrictions. First she performed TSLS, then found the residuals evaluated at the original values of the regressor (not first stage fitted values), then performed homoskedasticity-only regression of those residuals on the two instruments and the control. The $J = mF$ where F is the F stat testing whether coefficients on the two instruments are jointly zero. The J stat is asymptotically Chi Squared with 1 degree of freedom. Here $F = 3.66$ so $J = 7.32$, and so is much larger than the critical value of 3.84 at the 5% significance level that is found in the first row of Table 3.

TABLE 3 Critical Values for the χ^2 Distribution

Degrees of Freedom	Significance Level		
	10%	5%	1%
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.41	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89

This table contains the 90th, 95th, and 99th percentiles of the χ^2 distribution. These serve as critical values for tests with significance levels of 10%, 5%, and 1%.