ØAMET2200 · Fall 2019
Worksheet 1

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Exercise 1 State whether the following are discrete or continuous variables.

(a) Let X be the number obtained from rolling a fair die. Is X discrete or continuous?

(b) Let Y be the total number of heads obtained from two tosses of a fair coin. Is Y discrete or continuous?

(c) Let X be represent the weight of a 195-gram bag of Sørland potato chips. Is X discrete or continuous?

(d) Let Y be a household's electricity consumption in kWh (kilowatt-hour). Is Y discrete or continuous?

Exercise 2 Consider again the random variables in Exercise 1.

(a) What is the PMF of the random variable X in Exercise 1(a)?

(b) What is the PMF of the random variable Y in Exercise 1(b)?

Exercise 3 Let X_1, X_2, and X_3 be random variables representing the numbers of small, medium, and large pizzas, respectively, sold during the day at a pizza parlor. Suppose $E(X_1) = 25$, $E(X_2) = 57$, and $E(X_3) = 40$. The prices of small, medium, and large pizzas are \$5.50, \$7.60, and \$9.15. What is the expected revenue from pizza sales on a given day?

Exercise 4 Customers at MAX Burgers buy both burgers and drinks. The following is the joint distribution of the number of burgers (X) and drinks (Y) bought by customers.

		X	
		1 burger	2 burgers
	1 drink	0.40	0.20
Y	2 drinks	0.10	0.25
	3 drinks	0.00	0.05

(a) What does $P(X = 1, Y = 2)$ mean in words? What is its value?

(b) Are X and Y independent?

(c) What is $E(X)$? $Var(X)$? $SD(X)$?

(d) What is $E(Y)$? $Var(Y)$? $SD(Y)$?

(e) What is $cov(X, Y)$?

(f) What is $corr(X, Y)$?

(g) Interpret the size of the correlation for the manager of the MAX Burgers.

(h) If the profit earned from selling a burger is kr. 15 and from a drink is kr. 10, what is the expected value and standard deviation of the profit made from each customer?

Exercise 5 Let Y be a random variable that equals 1 if an individual has a bachelor degree, and 0 if she/he does not. Let X be another random variable that captures the individual's quartile in the income distribution. Specifically, $X = 1$ if the individual is in the first (lowest) income quartile, and the values 2, 3 and 4 indicate the next 3 quartiles. The table below shows the joint probability distribution of X and Y.

	$X = 1$	$X = 2$	$X = 3$	$X = 4$
$Y = 0$	6/30	8/30	4/30	2/30
$Y = 1$	1/30	2/30	4/30	3/30

(a) What is the probability that a randomly selected individual went to college and is in the highest income quartile?

(b) What is the probability that a randomly selected individual did not go to college and is in the 2nd income quartile?

(c) Are X and Y independent or dependent? Provide an intuitive explanation for the result you find.

Exercise 6 Let X,Y and Z be random variables with:

$E(X) = 2$	$Var(X) = 4$	$Cov(Y, Z) = -3$
$E(Y) = 5$	$Var(Y) = 9$	X and Y independent
$E(Z) = 3$	$Var(Z) = 1$	X and Z independent

Calculate the following:

(a) $E(8 + 3X - 2Y + 9Z)$

(b) $Var(2X + 3Y)$

(c) $Var(2Y - 3Z + 5)$

(d) $Cov(3 + 6Y, 5 - 7Z)$

(e) $Corr(3 + 6Y, 5 - 7Z)$

Exercise 7 Find the given probabilities for the following normally distributed random variables.

(a) Suppose that $X \sim \mathcal{N}(3, 4)$. What is the probability that $X \leq 1$?

(b) Suppose that $X \sim \mathcal{N}(4, 9)$. What is the $P(2 \leq X < 6)$?

Exercise 8 Find the value of z that makes the following probabilities hold.

(a) $P(Z < z) = 0.20$

(b) $P(Z \leq z) = 0.50$

(c) $P(-z \leq Z \leq z) = 0.50$

(d) $P(|Z| > z) = 0.01$

(e) $P(|Z| < z) = 0.90$

Exercise 9 (Stine and Foster, Chapter 12, Question 46) A tire manufacturer warranties its tires to last at least 20,000 miles or "you get a new set of tires." In its experience, a set of these tires lasts on average 26,000 miles with SD 5,000 miles. Assume that wear is normally distributed. The manufacturer profits \$200 on each set sold, and replacing a set costs the manufacturer \$400.

(a) What is the probability that a set of tires wears out before 20,000 miles?

(b) What is the probability that the manufacturer turns a profit on selling a set to one customer?

Exercise 10 A local Peppe's Pizza branch has just introduced a money-back guarantee: if they don't deliver a pizza to a customer within 30 minutes after the order is placed, the pizza is free. The branch manager is trying to decide whether to adjust that guarantee. After the pizza is made, the door-to-door delivery time (denoted T_d) has a normal distribution with a mean of 15 minutes and a standard dedvition of 5 minutes.

(a) What is the probability that a randomly selected door-to-door delivery will take more than 25 minutes?

(b) Once an order is received, the preparation time (denoted T_p)—which is the time it takes to cook and package the pizzas—has a normal distribution with a mean of 10 minutes and standard deviation of 3 minutes. Suppose that the door-to-door delivery time and preparation time are independent. What is the mean and standard deviation fo the total customer wait time to receive pizza after an order is placed ($T_p + T_d$)?

(c) The branch manager wants to set a guaranteed total customer wait time such that only 1% of orders will not arrive in time to satisfy the guarantee. What guaranteed time t^* should the manager set?

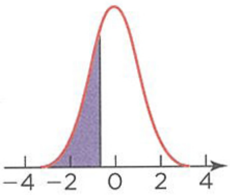
(d) So far, we have assumed that door-to-door delivery time and preparation time are independent. Now, let's consider what happens to the answer in part (c) when they are *not* independent. In particular, suppose that door-to-door delivery time and preparation time are *positively correlated*. Which of the following statements are true?

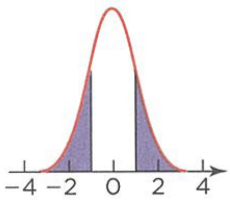
   (i) t^* will be larger than what we found in part (c).

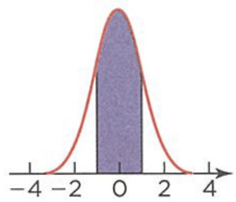
   (ii) t^* will be smaller than what we found in part (c).

   (iii) t^* does not depend on the correlation between T_d and T_p.

   (iv) There is not enough information to know how t^* will be affected.







z	$P(Z \le -z)$	$P(Z \le z)$	$P(\lvert Z\rvert > z)$	$P(\lvert Z\rvert \le z)$
0	0.50	0.50	1	0
0.0502	0.48	0.52	0.96	0.04
0.1004	0.46	0.54	0.92	0.08
0.1510	0.44	0.56	0.88	0.12
0.2019	0.42	0.58	0.84	0.16
0.2533	0.40	0.60	0.80	0.20
0.3055	0.38	0.62	0.76	0.24
0.3585	0.36	0.64	0.72	0.28
0.4125	0.34	0.66	0.68	0.32
0.4677	0.32	0.68	0.64	0.36
0.4959	0.31	0.69	0.62	0.38
0.5244	0.30	0.70	0.60	0.40
0.5828	0.28	0.72	0.56	0.44
0.6433	0.26	0.74	0.52	0.48
0.6745	0.25	0.75	0.50	0.50
0.7063	0.24	0.76	0.48	0.52
0.7388	0.23	0.77	0.46	0.54
0.7722	0.22	0.78	0.44	0.56
0.8064	0.21	0.79	0.42	0.58
0.8416	0.20	0.80	0.40	0.60
0.8779	0.19	0.81	0.38	0.62
0.9154	0.18	0.82	0.36	0.64
0.9542	0.17	0.83	0.34	0.66
0.9945	0.16	0.84	0.32	0.68
1.0364	0.15	0.85	0.30	0.70
1.0803	0.14	0.86	0.28	0.72
1.1264	0.13	0.87	0.26	0.74
1.1750	0.12	0.88	0.24	0.76
1.2265	0.11	0.89	0.22	0.78
1.2816	0.10	0.90	0.20	0.80
1.3408	0.09	0.91	0.18	0.82
1.4051	0.08	0.92	0.16	0.84
1.4758	0.07	0.93	0.14	0.86
1.5548	0.06	0.94	0.12	0.88
1.6449	0.05	0.95	0.10	0.90
1.7507	0.04	0.96	0.08	0.92
1.8808	0.03	0.97	0.06	0.94
1.9600	0.025	0.975	0.05	0.95
2.0537	0.02	0.98	0.04	0.96
2.3263	0.01	0.99	0.02	0.98
2.5758	0.005	0.995	0.01	0.99
2.8070	0.0025	0.9975	0.005	0.995
3.0902	0.001	0.999	0.002	0.998
3.2905	0.0005	0.9995	0.001	0.999
3.7190	0.0001	0.9999	0.0002	0.9998
3.8906	0.00005	0.99995	0.0001	0.9999
4.2649	0.00001	0.99999	0.00002	0.99998
4.4172	0.000005	0.999995	0.00001	0.99999