Numerical Simulation of Traffic Congestion in 1-D

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This study models traffic congestion with a system of differential equations which describe vehicles moving on a one-dimensional ring. Numerical integration using the fourth-order Runge-Kutta method was performed on the model. We found that the model develop shockwaves which propagate in the opposite direction of traffic, and also exhibits numerous spectral modes. Time evolution of the density and velocity profile were analyzed.

I. INTRODUCTION

The emergence of congestion in steadily flowing traffic is a well-studied problem in numerical modeling. Previous studies, as well as an actual test involving subjects driving cars around a circular single-lane track show that traffic jams can emerge in uniform traffic from small perturbations[1, 2]. One method of modeling this phenomenon is assigning to each vehicle an equation of motion which is determined by its relation to neighboring cars. This study investigates a particular class of accident-avoiding equations of motion which does not allow a car to surpass its neighbor.

II. DYNAMICAL MODEL

The dynamical model used here is described in detail in a previous study[1]. The equation of motion of car n (n = 1,2,...,N) is modeled by

$$\ddot{x}_n = a(V(\Delta x_n) - \dot{x}_n) \tag{1}$$

where

$$\Delta x_n = x_{n+1} - x_n \tag{2}$$

Since $V(\Delta x_n)$ is the effective speed limit, the equation describes an acceleration proportional to the difference between the speed limit and the current speed of a car. Periodic boundary condition is imposed, such that $x_{n+1} = x_1$. Many choices for $V(\Delta x_n)$ can be made, including linear relations such as the three-second following distance rule, $V(\Delta x_n) \propto \Delta x_n$, and non-linear relations such as $V(\Delta x_n) \propto tanh(\Delta x_n)$. These choices both allow a steady-state solution of

$$x_n = bn + ct \tag{3}$$

for constants $b = L/N$ and $c = V(b)$, where L is the circumference and N is the total number of cars. In addition, the stability of the solution is governed by parameters a and b. If the condition $V'(b) < \frac{a}{2}$ is met, the modes induced by a perturbation decay and the solution reduces to steady flow. However, if $V'(b) > \frac{a}{2}$, the modes are amplified [1]. In the cases where $V(\Delta x_n) \propto \Delta x_n$ or $\propto tanh(\Delta x_n)$, the modes eventually amplify until a car tagging behind catches up with a car in front, causing an "accident." In order to eliminate these "accidents," one can choose $V(\Delta x_n)$ as follows

$$V(\Delta x_n) = tanh(\Delta x - 2) + tanh(2) \tag{4}$$

III. NUMERICAL SIMULATION

In this study, we choose $N = 40$, $L = 60$, $a = 1$, which yields a steady flow velocity $v_0 = 0.5$. These parameters fulfill the requirement $V'(b)$ $0.786 > \frac{a}{2} = 0.5$ for oscillation modes to occur. Simulations with parameters outside this range were found to exhibit the steady flow solution. With initial conditions $\dot{x}_n = v_0$, we introduce a 10% extra velocity in the first car. The solution is then integrated using the RK4 algorithm.

The positions of all cars are plotted as a function of time in Fig. 1. The solution exhibits shockwaves, or "traffic jams", propagating in the reverse direction of the traffic, which emerge from the small perturbation in the initial condition. The shockwaves are rendered more visible when plotted as traffic density in Fig. 2.

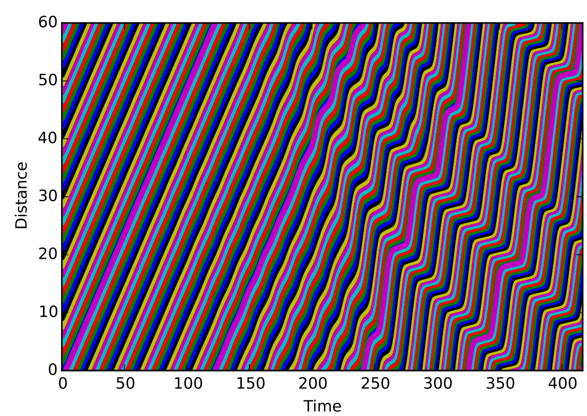

FIG. 1: Positions of all cars with time.

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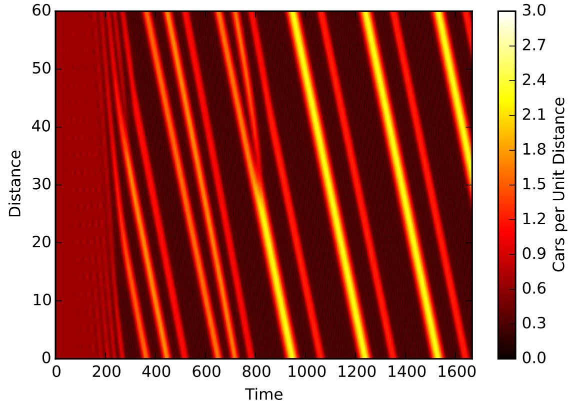

FIG. 2: Density of the traffic as a function of position and time. The density was calculated with a running average over 1/10 of the circumference. Traffic flows in the positive distance direction.

The temporal Fourier Transform reveals that numerous modes exist in the density of the traffic (Fig. 3). In particular, there is a harmonic with a frequency of ~0.5 throughout the spectrum. However, the dominant features of the spectrum are mostly in the low frequency range, where a harmonic of frequency spacing ~0.003 appears (Fig. 4). One can also observe that the velocity of the shockwaves tend to decrease with time, with one shockwave merging into another, forming a conglomerate with lower velocity. Although the shockwaves propagate with a fraction of the velocity of the cars, they can gradually slow down to two orders of magnitude of the steady flow velocity. This contributes to the spectral dominance of low frequency modes.

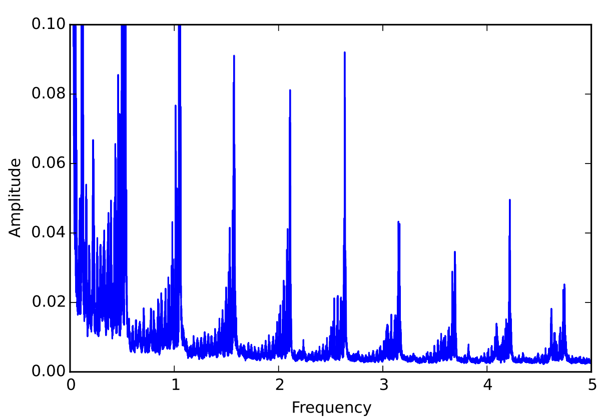

FIG. 3: The temporal Fourier Transform of the traffic.A harmonic with frequency spacing 0.5 is evident in the spectrum.

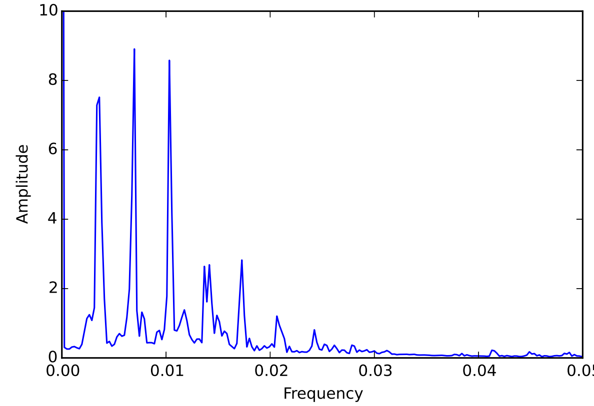

FIG. 4: The temporal Fourier Transform of the traffic at low frequencies. A harmonic with frequency spacing 0.003 appears in the low frequency spectrum.

The velocity profile of the traffic also exhibits changes, as expected from the traffic jam (Fig. 5). The cars which originally start out with the steady flow velocity gradually fall into two groups–one group with lower velocity (the congested group) and another with higher velocity transitioning between shockwaves.

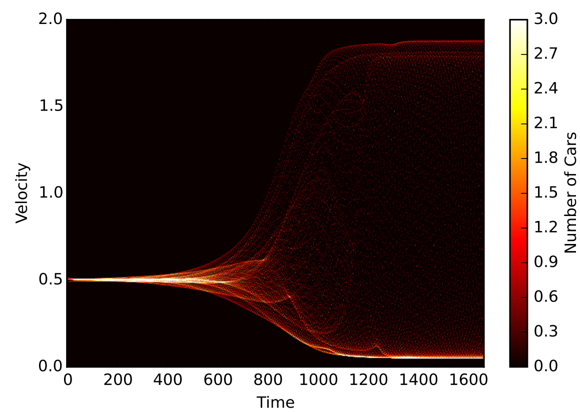

FIG. 5: The velocity profile as a function of time. As time passes, cars originally at the steady flow velocity gradually fall into two bunches.

IV. DISCUSSION AND CONCLUSION

The equations of motion, eq. 1, together with a choice of a speed limit function such as eq. 4 are likely to be a sufficient description of traffic jams. The main advantage is that these equations do not allow a car to overtake another car, thus avoiding "accidents." While this choice of the speed limit function cannot accommodate modeling of the delayed reaction time of drivers, it is nevertheless useful for observing the dynamics of a traffic jam.

In this study, we found that given sufficient vehicle density, traffic jams can grow from small perturbations. We also found that the shockwaves propagate in the opposite direction of traffic. In addition to previous findings that a common mode in traffic jams propagate at 20 km/hr in 40 km/hr traffic[2], this simulation demonstrated that evolution through time can slow traffic to much lower speeds. Lastly, the velocity profile clearly show that the vehicles evolve into two groups, one stuck in the traffic jam and the other transitioning between traffic jams.

V. ACKNOWLEDGEMENT

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[2] Y. Sugiyama, M. Fukui, M. Kikuchi, K. Hasebe, A. Nakayama, K. Nishinari, S. ichi Tadaki, and S. Yukawa, New Journal of Physics **10**, 033001 (2008).