Determining the Electron to Proton Mass Ratio through Isotope shift of Hydrogen and Deuterium Emission Spectra

Franklin Liou[1]
[1] Department of Physics, Stanford University, Stanford CA

E-mail: fliou@stanford.edu

Abstract. We determine the mass ratio of the electron to the proton by measuring the isotope shift in the Balmer alpha line of hydrogen and deuterium. We measure this ratio to be $(5.46 \pm 0.34) \times 10^{-4}$, within error of the known value 5.45×10^{-4}. We observe the Balmer alpha line of a gas discharge tube containing hydrogen and deuterium through a grating spectrometer, which passes light of a selected wavelength to a photomultiplier and ammeter.

I. Introduction

We determine the electron to proton mass ratio by observing the difference in atomic emission spectra of hydrogen and deuterium. This can be achieved because the difference in nucleus mass between hydrogen and deuterium atoms causes a shift in emission spectra of the two species. To obtain the emission spectra theoretically, the wavelengths λ_n of the Balmer lines of hydrogen are given by the formula

$$\frac{1}{\lambda_n} = R_\infty \left(\frac{1}{2^2} - \frac{1}{n^2}\right) \tag{1}$$

R_∞ is the Rydberg constant and n is the any integer greater than 2. However, since the nucleus is not infinitely heavier than the electron, we need to correct for reduced mass of the total electron-nucleus system. After the correction, the predicted wavelengths of hydrogen and deuterium emission lines $\lambda_n{}^H$ and $\lambda_n{}^D$ become

$$\lambda_n{}^H = \left(1 + \frac{m_e}{m_p}\right)\lambda_n \tag{2}$$

$$\lambda_n{}^D = (1 + \frac{m_e}{m_D})\lambda_n \tag{3}$$

The shift between these two wavelengths can be used to calculate the electron to proton mass ratio

$$\frac{\Delta\lambda_n}{\lambda_n} = \frac{1}{2}\frac{m_e}{m_p} \tag{4}$$

The theoretical emission spectra of hydrogen and deuterium are listed in Table 1.

Table 1: Predicted Emission Spectra for Hydrogen and Deuterium

	Alpha	Beta	Gamma	Delta
Hydrogen	6564.7	4862.7	4341.7	4102.9
Deuterium	6562.9	4861.4	4340.6	4101.8
Difference	1.8	1.3	1.2	1.1

II. Experimental Methods

We use high voltage to accelerate electrons across a gas discharge tube containing hydrogen and deuterium. When the accelerated electrons collide with a hydrogen or deuterium atom, it excites the atomic electrons, which migrate to a higher energy level. As these electrons decay back to the ground state, they emit visible light at discrete wavelengths. We pass the light from the tube through a grating spectrometer, which selects a particular wavelength, into a photomultiplier tube, which counts the number of arriving photons and amplifies the photocurrent to the microamp scale. We then use a digital ammeter to measure and record this current. We scan through wavelengths of interest, namely, the wavelengths of the Balmer alpha, beta, gamma and delta lines. To observe the alpha line, we adjust the slits of the spectrometer to 200 micrometers. For the beta and gamma lines, we adjust it to 250 micrometers. The lens used to focus the light from the lamp has a focus of 145 mm. We take three sets of spectra for each line.

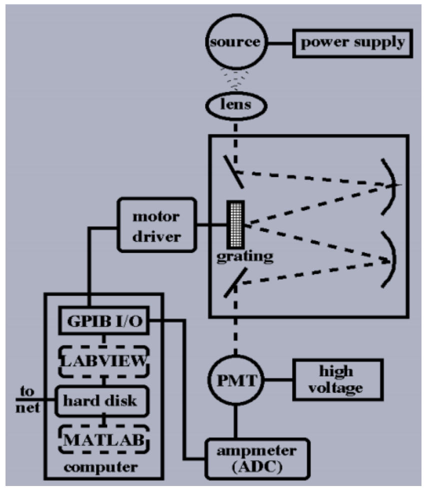


Figure 1 : Experimental Setup
Light from the hydrogen/deuterium lamp is passed through a grating spectrometer,
which selects a certain wavelength, then passes it to a photomultiplier tube (PMT). The current is amplified by the photomultiplier tube and recorded by a digital ammeter.

III. Results

We observe double peaks clearly near each predicted emission line, except at the Balmer delta line. The wavelengths of the peaks are shown in Fig 2. For each Balmer line, we have three measurements, as exemplified in Fig 3. We fit two Gaussian peaks to each spectrum, as shown in Fig 4. We take a total of three measurements of each peak and weight the position of each peak by its corresponding standard deviation to determine its position and an error estimate.

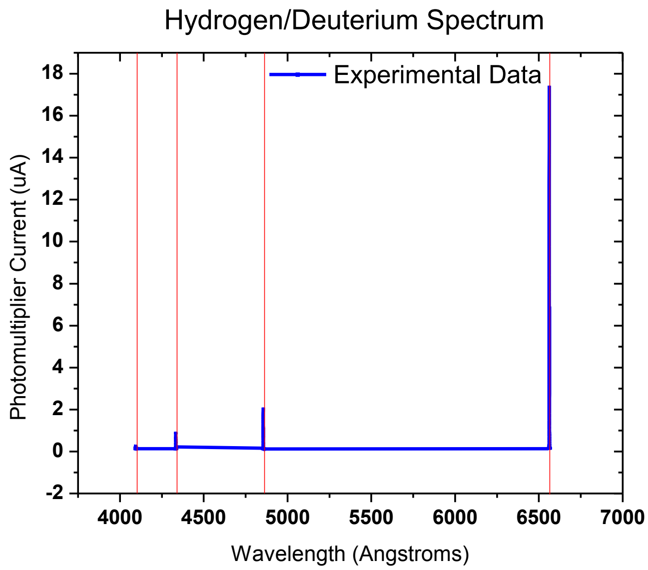


Figure 2 :Full Hydrogen/Deuterium Spectrum
The red lines indicate the theoretically predicted wavelengths

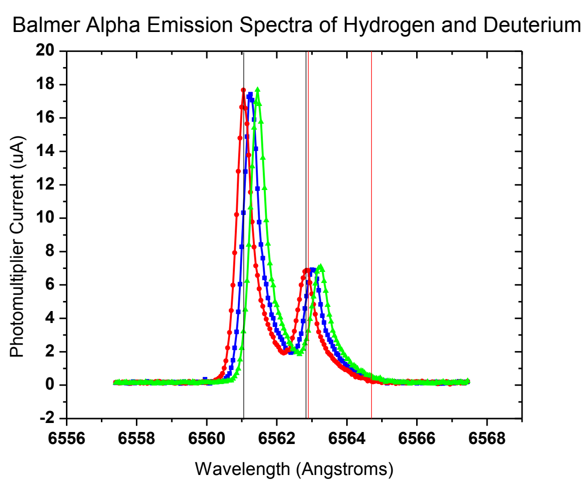


Figure 3 :Balmer Alpha Hydrogen/Deuterium Spectrum
The red vertical lines indicate the theoretically predicted wavelengths.

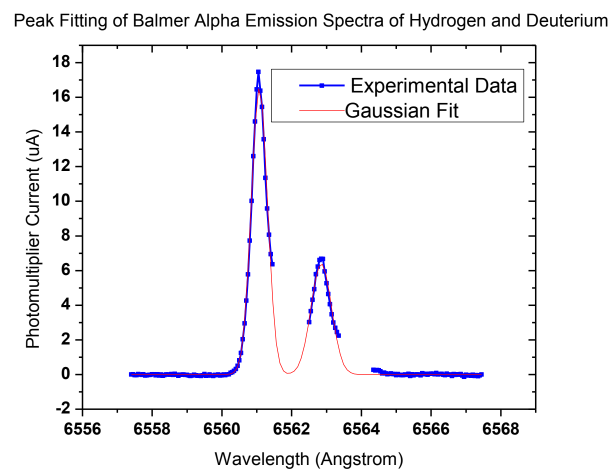
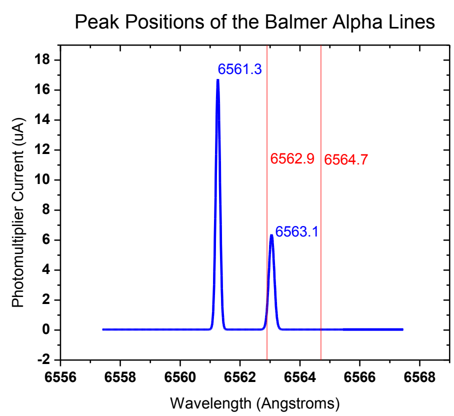


Figure 4 :Gaussian Fit for Balmer Alpha Hydrogen/Deuterium Spectrum

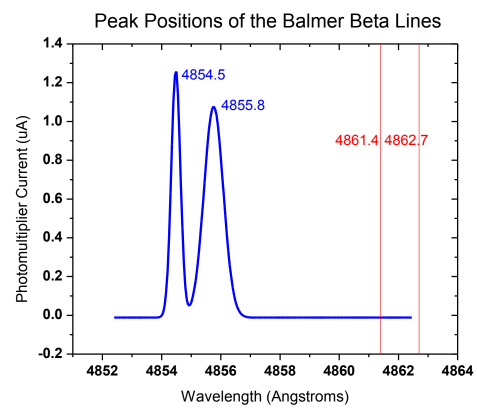
The wavelengths of the spectral lines are shown in Fig 5. We notice that the experimental peaks are shifted to lower wavelength than predicted, but the difference in wavelength between the hydrogen and deuterium lines remains close to the predicted value. Based on this peak shift, we calculate the electron to proton mass ratio in Table 2.

Table 2: Measured Isotope Shift and Derived Electron to Proton Mass Ratio

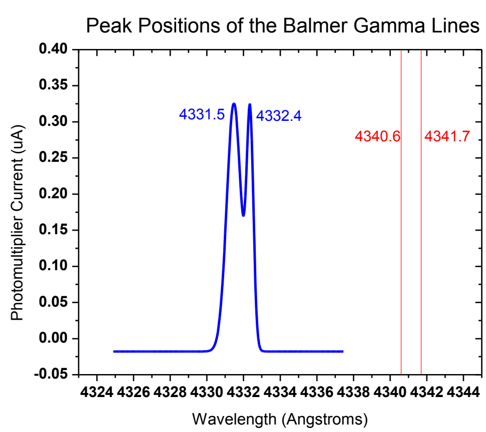
	Alpha	Beta	Gamma
Predicted Shift (A)	1.8	1.3	1.2
Observed Shift (A)	1.79 ± 0.23	1.28 ± 0.72	0.88 ± 0.86
Electron to Proton Mass Ratio	$(5.46 \pm 0.34) \times 10^{-4}$	$(5.36 \pm 1.48) \times 10^{-4}$	$(4.1 \pm 2.0) \times 10^{-4}$



(a)



(b)



(c)

Figure 5 :Fitted Gaussians for Balmer (a) Alpha, (b) Beta, (c) Gamma Lines. The theoretically predicted wavelengths are marked by red vertical lines.

IV. Discussion

The electron to proton mass ratio determined from the Balmer alpha line was determined to be $(5.46 \pm 0.34) \times 10^{-4}$, which is close and within error of the known value of 5.45×10^{-4}. However, the predicted wavelengths of the Balmer lines fall well out of range of the observed peaks, indicating systematic error. We may attribute this systematic error to non-linear mechanical offset of the stepper motor in the spectrometer, or to impurities in the hydrogen/deuterium mix which may absorb and emit photons from the gas. We also find that the electron to proton mass ratio decreases with increasing order of alpha, beta and gamma lines. Since the lines of decreasing wavelength also have higher statistical error in wavelength, the uncertainty in the electron to proton mass ratio also has higher uncertainty with decreasing wavelength.

We characterize the statistical error of our photomultiplier tube by finding the standard deviation of our noise around each Balmer line. Our signal to noise ratio for the alpha, beta and gamma lines are 490, 57, and 12 respectively.

V. Conclusion

We have measured the electron to proton mass ratio to be $(5.46 \pm 0.34) \times 10^{-4}$ through observing the difference in wavelength of hydrogen and deuterium Balmer alpha lines. Although this result is within range of the know value, we also observed that the position of the Balmer lines were shifted down in wavelength significantly. We postulate this systematic effect to be due to the non-linearity of the stepper motor inside our spectrometer, which may affect the reading of the selected wavelength, or impurities in the gas discharge lamp, which may absorb and re-emit photons at another frequency. We hope to recalibrate our instrument or use other gas discharge tube samples in the future to investigate this systematic error and to verify our results.

References

[1] Schleier-Smith, Monika and Pam, Rick. Stanford Physics 107 Lab1 Handout. (2014)