# Agenda Setting in Multiple Dimensions

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**Abstract**

We consider take-it-or-leave-it bargaining between a proposer and a veto player in a multi-dimensional policy space with Euclidean preferences. We compare two bargaining protocols: issue-by-issue and multidimensional bargaining. For any ideal points and status quo policy, the agenda setter always weakly (sometimes strictly) prefers multidimensional bargaining, and the veto player always weakly (sometimes strictly) prefers issue-by-issue bargaining. The reason is that multidimensional bargaining allows the agenda setter to force more change from the status quo than the veto player prefers on some dimensions, as a "price" for movement on dimensions where the veto player strongly dislikes the status quo. Issue-by-issue bargaining does not allow this because it requires Pareto improvements for movement in any dimension in equilibrium. Despite this, the equilibrium for issue by issue bargaining is not generally Pareto efficient, while it always is for multidimensional bargaining.

## 1 Models of Multidimensional Agenda Setting

Though 1-dimensional models predominate in the spatial agenda setting literature due to their tractability, $N$-dimensional spatial models are equally simple provided that a decisive proposer and decisive voter can be identified. There are many substantively important contexts approximated by this assumption, including policy bargaining between legislatures and committees (Shepsle 1979; Shepsle and Weingast 1981; Crombez et al. 2006); policy bargaining between a legislature and a chief executive (Carter and Schap 1990; Brown 2012); nation states with unitary negotiators in a multifaceted bargain (Mattes 2018); litigants in a complex dispute who may wish to bifurcate (i.e., independently resolve) some issues (Landes 1993); etc.

There are multiple methods by which the players can bargain over issues at play. In this paper we consider and compare the equilibrium properties of two natural methods in a complete information spatial model with take-it-or-leave-it offers: multidimensional agenda setting, in which multiple issues are resolved simultaneously; and issue-by-issue agenda setting, in which each issue is considered and settled one at a time. We give a full characterization of each player's preferences over these bargaining protocols, which perhaps surprisingly is not (to our knowledge) present in the literature. In particular, we show that with separable preferences, the veto player always weakly and sometimes strictly prefers issue-by-issue bargaining; the proposer always weakly and sometimes strictly prefers multidimensional bargaining.

**Notation.** To formalize this, consider a policy space $X \subset \mathbb{R}^N$ that is convex with arbitrary dimension denoted $i \in \mathcal{N} \equiv \{1, \ldots, N\}$. There is an exogenous status quo $q \in X$. There are two players, a proposer $P$ and a veto player $V$. The players have ideal points $x^P$ and $x^V$ in $X$ and each player $k \in \{V, P\}$ is assumed to have preferences over policy $x \in X$ represented by a complete, transitive, and continuous binary relation, $\succeq_k \in X^2$, that is *separable and spatial* in that, for all pairs of policies $(x, y) \in X^2$:

$$x \succeq_k y \Leftrightarrow \|x - x^k\| \leqslant \|y - x^k\|,$$

where $\|\cdot\|$ is the $N$-dimensional Euclidean distance metric.[1] The ideal point of player $k$ on dimension $i$ is denoted $x_i^k$. It is convenient to assume, without loss of generality, $x_i^P \leqslant x_i^V$ for all dimensions. All of this is common knowledge among the players.

Let $CC(x^P, x^V) \subset X$ denote the line segment connecting $x^P$ and $x^V$. This is the *contract curve*, or the set of Pareto efficient policies: there are no other policies $x \in X$ that make one player better off without making the other player strictly worse off.

Finally, define:

- $W^V(x) = \{y \in \mathbb{R}^N : \|y - x^V\| \leqslant \|x - x^V\|\}$, the policies that $V$ weakly prefers to $x$, and its dimension-$i$ analogue $W_i^V(x_i) = \{y \in \mathbb{R}^N : |y_i - x_i^V| \leqslant |x_i - x_i^V|\}$,
- $S^V(x) = \{y \in \mathbb{R}^N : \|y - x^V\| < \|x - x^V\|\}$, the policies that $V$ strictly prefers to $x$, and its dimension-$i$ analogue $S_i^V(x_i) = \{y \in \mathbb{R}^N : |y_i - x_i^V| < |x_i - x_i^V|\}$,
- $E^V(x) = \{y \in \mathbb{R}^N : \|y - x^V\| = \|x - x^V\|\}$, the policies that $V$ considers exactly as good as $x$, and its dimension-$i$ analogue $E_i^V(x_i) = \{y \in \mathbb{R}^N : |y_i - x_i^V| = |x_i - x_i^V|\}$,

Given separable preferences, $E_i^V$ and $S_i^V$ are unaffected by $x_j$ for any $j \neq i$.

With that in hand, we consider two distinct agenda setting games.

- In the **multidimensional agenda game (MDM)**, $P$ makes a single take it or leave it offer $y \in X$, which $V$ accepts or rejects. If $V$ accepts, then the policy outcome is $x^M = y$; if $V$ rejects, then $x^M = q$.
- In the **issue by issue agenda game (IBI)**, each dimension is resolved in sequence. First $P$ proposes a policy $y_1 \in X_1$ for the first dimension,[2] and $V$ accepts or rejects this proposal. Second, $P$ proposes a policy $y_2 \in X_2$ for the second dimension, and $V$ accepts or rejects this proposal; and so on for other dimensions. The policy outcome is $x^I$, where $x_i^I = y_i$ if $V$ accepted the proposal on dimension $i$, and $x_i^I = q_i$ if $V$ rejected the proposal on dimension $i$.

[1] These assumptions ensure that each player's preferences can be represented by a continuous utility function $u^k : X \to \mathbb{R}$, for example, $u^k(x) = -\sum_i (x_i - x_i^k)^2$ or $u^k(x) = -\|x - x^k\|$. Our results rely only on the spatial and separability properties of the ordinal preferences, not any specific utility representation.

[2] It will be shown that, due to preference separability, the subgame perfect equilibrium of the issue by issue game is not affected by the order in which dimensions are considered.

Both are extensive form games of complete information, for which the natural solution concept to ensure credibility of threats is subgame perfect Nash equilibrium (SPNE).[3] MDM has been used in the literature to study agenda setting and gatekeeping (Crombez et al. 2006). Special cases of IBI have been used to study the line item veto (Carter and Schap 1990; Brown 2012), though a characterization of its SPNE or properties such as efficiency have not been offered.

## 2 Multidimensional Agenda Game

The MDM game is a straightforward extension of the Romer and Rosenthal (1978) one-dimensional agenda setter model, which we review for reference.

In any SPNE, $V$ will accept any proposal $y \in X$ that is weakly preferred to the status quo, $q$ (*i.e.*, any policy $y \in W^V(q)$). Accordingly, in any SPNE, $P$ can do no better than to propose the policy closest to $x^P$ that satisfies $V$'s acceptance constraint, which $V$ accepts.[4] Thus, if $x^P \in W^V(q)$ (*i.e.*, the status quo is farther than $P$'s ideal point is from $V$'s ideal point), then $x^M = x^P$. Otherwise, $P$ proposes the point where $CC(x^P, x^V)$ and $E^V(q)$ intersect. This policy is accepted by $V$ as $x^M$. Each of these cases are summarized in the following lemma.

**Lemma 1 (Equilibrium and Efficiency of MDM.)** *In any SPNE of the multidimensional agenda game,*

1. *If $q \in S^V(x^P)$, the policy outcome is $x^M = E^V(q) \cap CC(x^P, x^V)$.*
2. *If $q \notin S^V(x^P)$, the policy outcome is $x^M = x^P$.*

*Proof*: Proofs of all numbered results are contained in the appendix. ∎

Lemma is illustrated in Figure 1 for the case of $\mathbb{R}^2$, where the status quo $q$ maps to $x^M(q)$ in equilibrium. Note $x^M(q)$ is on the contract curve. Before moving to the IBI game note that, because $x^M$ always lies on the contract curve, the SPNE of the MDM game is always Pareto efficient.

## 3 Issue by Issue Agenda Game

The SPNE of the issue by issue game, $x^I$, also has a very simple structure: $x^I$ is composed of the one-dimension Romser-Rosenthal SPNE outcomes from each dimension in isolation. For formal characterization, recall that $W_i^V(x_i^P) = [x_i^P, 2x_i^V - x_i^P]$ is the set of policies that $V$ weakly prefers to $x_i^P$ on dimension $i$.

[3] Because we are considering bargaining over a continuum of policies and players have continuous payoff functions, equilibrium behavior necessarily involves $V$ accepting with certainty any proposal $x \neq q$ such that $x \succeq_V q$.

[4] If $q$ is the point in $W^V(q)$ that is closest to $x^P$, then there are multiple SPNE of the game, but they are payoff equivalent and all yield the same final policy: $x^M = q$.

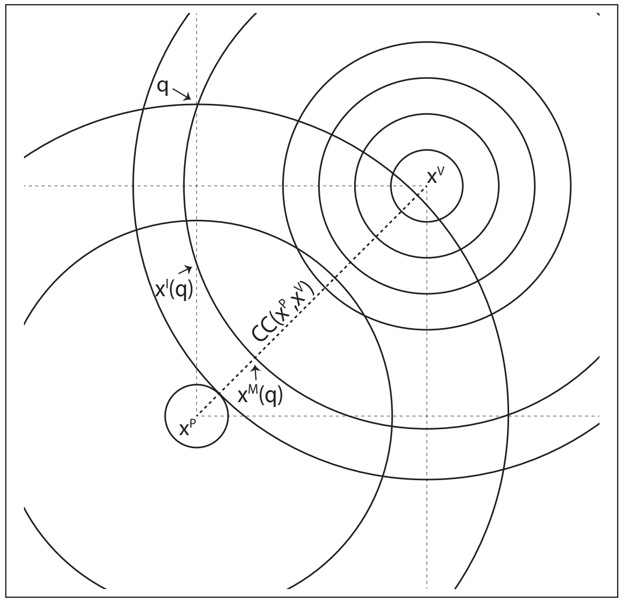


Figure 1. Equilibrium under MDM and IBI.
Status quo $q$ maps to $x^M(q)$ under MDM, $x^I(q)$ under IBI.

**Lemma 2 (Equilibrium of IBI.)** *In any SPNE of the issue by issue game, the policy outcome, $x^I$, is determined as follows. For each dimension $i \in \{1, \ldots, N\}$,*

1. *If $q_i \in S_i^V(x_i^P)$, then $x_i^I \in E_i^V(q_i)$.*

2. *If $q_i \notin S_i^V(x_i^P)$, then $x_i^I = x_i^P$.*

That is, on every dimension $i$ in equilibrium under IBI, the policy is either $P$'s ideal point on $i$, or is exactly as good for $V$ as $q_i$. This is also illustrated in Figure 1, where $q$ maps to $x^I(q)$.

Given separable preferences, the SPNE outcome is the same regardless of the order in which dimensions are considered. To see this, consider dimension $j$, and let $x_i^I$ be the policy already decided on dimension $i < j$. Since preferences are separable across dimensions, each player's ordinal rankings of policies on $j$ are unaffected by $x_i^I$ or by policies to be adopted on dimensions $k$ yet to be considered. Then the determination of $x_j^I$ follows the standard Romer-Rosenthal logic. Further, when dimension $i$ is considered, all players know that $x_j^I$ will result on dimension $j$, regardless of $x_i^I$. Therefore, determination of $x_1^I$ also follows a standard Romer-Rosenthal logic. Since this reasoning also holds if the dimension labels are reversed, the order in which dimensions are considered does not affect the outcome.

However, $x^I$ is not generally Pareto efficient. For instance, in Figure 1, $x^I(q)$ is not on the contract curve. To characterize the efficiency of IBI, let $D$ denote the set of all subsets of $\{1, \ldots, N\}$, $m$ an element of $D$, and $\widehat{CC}(x^P, x^V)$ as the set of line segments connecting $x^V$ to $(\{x_i^P\}_{i \in m}, \{2x_j^V - x_j^P\}_{j \notin m})_{m \in D}$. In words, $(\{x_i^P\}_{i \in m}, \{2x_j^V - x_j^P\}_{j \notin m})_{m \in D}$ are the vertices of the hyper-rectangle containing those points which $V$ prefers to $x_i^P$ on each dimension, and

$\widehat{CC}(x^P, x^V)$ contains the line segments emanating from the centroid of this hyper-rectangle ($x^V$) to the vertices. Figure 2 gives an example in $\mathbb{R}^2$: $\widehat{CC}(x^P, x^V)$ consists of the crossing, solid diagonal line segments. Letting $\delta(x^P, x^V) \in \{0, 1, \ldots, N\}$ denote the number of dimensions on which $x^P$ and $x^V$ differ, there are $2^{\delta(x^P, x^V)}$ segments in $\widehat{CC}(x^P, x^V)$. Note that $CC(x^P, x^V) \in \widehat{CC}(x^P, x^V) \subset W^V(x^P)$.

**Lemma 3 (Inefficiency of IBI.)** *The SPNE policy $x^I$ is Pareto efficient if and only if (i) there is no dimension for which $q_i \in S_i^V(x_i^P)$; or (ii) $q \in \widehat{CC}(x^P, x^V)$.*

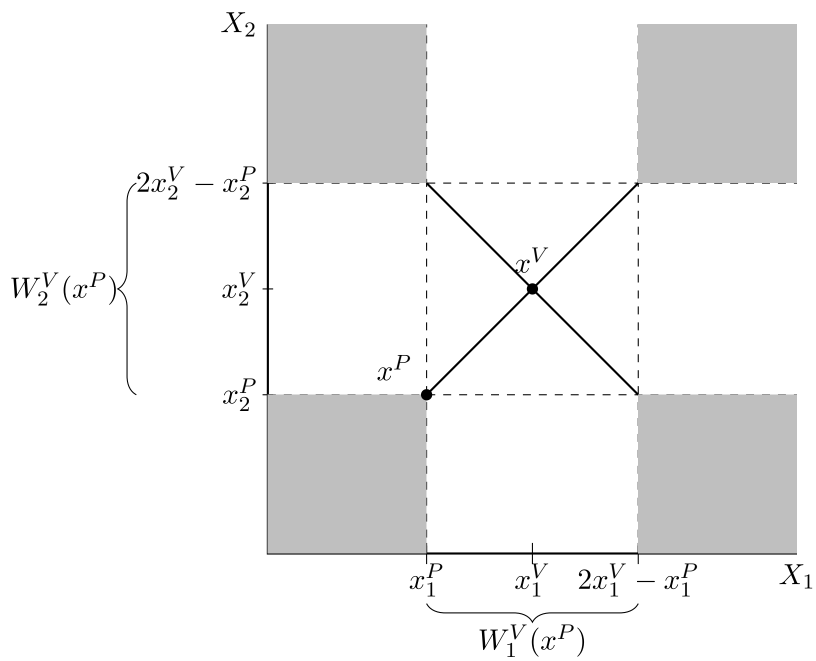


Figure 2. IBI is Pareto efficient for status quos in the shaded regions, or on the diagonal lines from $x^V$, $\widehat{CC}(x^P, x^V)$.

## 4 Comparing Agenda Institutions

The key results are that the veto player always weakly, and sometimes strictly, prefers the issue by issue process to the multidimensional process. But the agenda setter always weakly, and sometimes strictly, prefers the multidimensional process to the issue by issue process.

**Proposition 1 (Veto player's preference.)** *For any status quo $q \in X \subset \mathbb{R}^N$, if $q_i \in S_i^V(x_i^P)$ for some but not all dimensions, then the veto player strictly prefers the issue by issue game to the multidimensional game. If $q_i \in S_i^V(x_i^P)$ either for all dimensions or no dimensions, the veto player is indifferent between the two games.*

The logic behind Proposition 1 is depicted in Figure 3 for the case of $\mathbb{R}^2$. Considering $q$, note $q_i \notin W_i^V(x^P)$ for either dimension. Accordingly, $x^M = x^I = x^P$. On the other hand, considering $q'$, note $q'_i \in W_i^V(x^P)$ for dimension $X_1$ but not $X_2$. Here $x^M = x^P$ but $x^I = x'$.

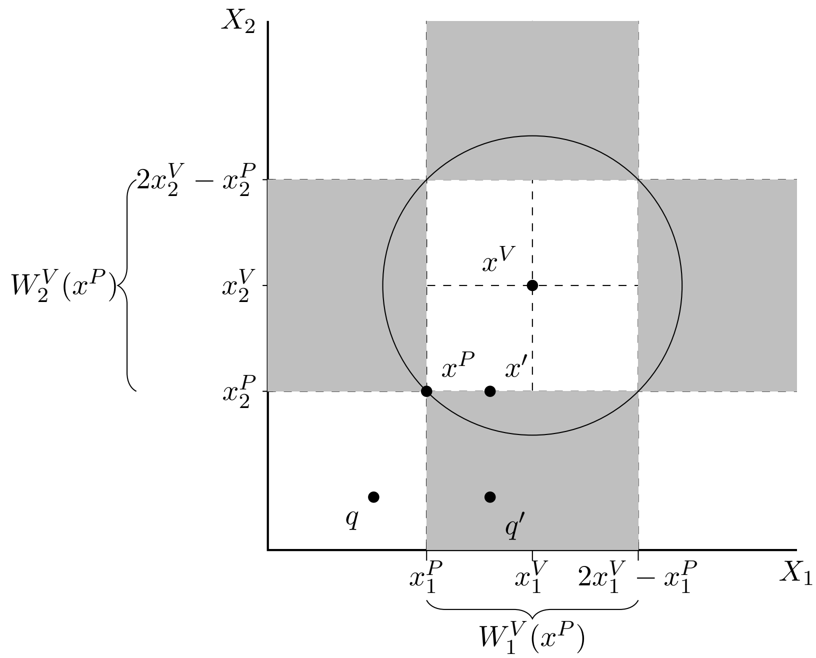


Figure 3. Unshaded: $V$ indifferent over game forms.
Gray shaded: $V$ strictly prefers issue by issue agenda setting.

Intuitively, in the multidimensional game, the proposer can bundle a change in one dimension that the veto player would rather avoid (but that the proposer desires), with a change in the other dimension that is very important to the veto player (and that the proposer also desires). In order to obtain the beneficial change in one dimension, the veto player must accept the costly change in the other. But in the issue by issue game, there is no bundling. The movement from $q$ on each dimension cannot make the veto player worse off.

**Proposition 2 (Proposer's preference.)** *For any status quo $q \in X \subset \mathbb{R}^N$, if the issue by issue equilibrium $x^I(q)$ is Pareto inefficient, then the proposer strictly prefers the multidimensional game to the issue by issue game. If $x^I(q)$ is Pareto efficient, the proposer is indifferent between the two games.*

Under the MDM bargaining protocol, $P$ finds her most preferred policy within $W^V(q)$, whereas under IBI, $P$ finds her most preferred policy within a strict subset of $W^V(q)$. For status quos $q$ under which the outcome of these two optimizations are different, the outcome must be Pareto inefficient and $P$ strictly prefers the MDM game form.

## 5 Implications and Extensions

Taken together, Propositions 1 and 2 identify exactly when $P$ and $V$ will disagree about the bargaining protocol. For instance, as the number of policy dimensions increases, the conditions for $V$ to strictly prefer IBI, and for $P$ to strictly prefer MDM, become easier to satisfy. Adoption of a bargaining protocol, such as a line item veto, by mutual agreement in a high dimensional context suggests that considerations besides the spatial policy implications alone are at play.

More generally, disagreement over bargaining protocols occurs when there are bargains to be struck on some, but not all, issues. In these situations, $P$ would prefer to leverage the gains available on some of the issues to extract concessions from $V$ on the other issues; conversely, $V$ prefers the IBI protocol because it allows $V$ to protect his or her interests on those other issues.

Another implication relates to the structure of committee jurisdictions in a legislature. Notions such as structure-induced equilibrium (Shepsle (1979)) indicate how carving a multidimensional policy space into lower-dimension jurisdictions can mitigate collective choice challenges such as majority rule "cycling" (*e.g.*, McKelvey (1976), Schofield (1978)). Proposition 1 complements this logic by showing that, in addition, a legislature is subject to more rent extraction by committees with more issues under their jurisdiction.

**The Scope of the Bargaining Problem: Adding Dimensions.** It is sometimes held that adding dimensions to a bargain raises the possibility of useful trades across dimensions that can sustain agreement, which is good for both players; thus, they should expand the dimensions under consideration when at an impasse (Bazerman et al. 2002). Suppose $P$ and $V$ follow a a fixed bargaining protocol, either MDM or IBI, with $N$ dimensions. They become aware of another dimension they could add to the problem, provided both agree to do it. If either disagrees, the status quo remains in place for dimension $N+1$. Then they play the pre-specified game.[5] Should each player agree to add the new dimension? Propositions 1 and 2 also shed light on this.

Indeed, $P$ is never worse off from adding another dimension under either MDM or IBI, and may be strictly better off. Under MDM, this occurs when $q_i \in W_i^V(x_i^P)$ for dimensions 1 through $N$, but $q_{N+1} \notin W_{N+1}^V(x_{N+1}^P)$. Adding dimension $N+1$ allows $P$ to obtain his or her ideal point on each dimension including $N+1$, while not adding it maintains $q_{N+1}$. Under IBI, $P$ strictly prefers to add dimension $N+1$ if $q_{N+1} \notin [x_{N+1}^P, x_{N+1}^V]$.

Under the issue by issue protocol, $V$ is also never worse off from adding another dimension. In this protocol, $V$ never does worse on any dimension than $q_i$. $V$ strictly prefers to add dimension $N+1$ under IBI if $q_{N+1} \notin W_{N+1}^V(x_{N+1}^P)$, because then $x_{N+1}^I = x_{N+1}^P$ which is strictly better than $q_{N+1}$.

However, under the multidimensional protocol, adding an additional dimension can make $V$ worse off than not adding that dimension. This is true even if $q_{N+1}$ is inefficient relative to

[5] That is, each player has de facto gatekeeping rights over dimension $N+1$.

dimension $N+1$ (i.e. $q_{N+1} \notin [x^P_{N+1}, x^V_{N+1}]$), and if $q$ in the $N+1$-dimensional problem is worse for $V$ than $x^P$. In particular, if $V$ prefers $x^P_i$ to $q_i$ on at least one dimension 1 through $N$, and $q_{N+1} \in W^V_{N+1}(x^P_{N+1})$, then adding dimension $N+1$ makes $V$ worse off on that dimension without improving the outcome on any other dimension.

Since the ability to preclude consideration of an issue is itself a form of issue by issue bargaining, these preferences for adding dimensions are not surprising in light of the previous results, but they do emphasize the effect of a bargaining protocol on the ability to resolve issues on which parties may disagree. If the bargaining protocol is fixed and dimensions can only be added by mutual agreement, then issue by issue bargaining is better than multidimensional bargaining at resolving the largest number of issues. While MDM bargaining is always Pareto efficient for a given set of issues and IBI is not, IBI may actually be mutually preferable if the set of issues considered is endogenous.

# 6 Conclusion

This paper shows that, in take-it-or-leave-it policy bargaining with a fixed set of issues, the veto player is always better off (sometimes strictly) by taking issues one at a time, while the proposer is always better off (sometimes strictly) by taking issues in multidimensional bundles. The reason is that multidimensional bargaining allows the proposer to extract rents on dimensions where the voter may not particularly dislike the status quo, as the price of obtaining movement on dimensions where the voter strongly dislikes the status quo. Since the voter's only credible commitment in multidimensional bargaining is to reject a proposal that is overall worse than the status quo, the proposer is able to take advantage of issue bundling power in this way.

Issue-by-issue bargaining does not allow this, since the proposal on each issue is taken one at a time rather than bundled with others. Under this protocol the voter can credibly commit to reject proposals that are worse than the status quo on any dimension, not just overall. This is a stronger commitment power than the multidimensional protocol allows, and the voter uses it to prevent more excessive rent extraction by the proposer.

# References


Bazerman, M., J. Baron, and K. Shonk (2002). *You can't enlarge the pie: Six barriers to effective government*. New York: Basic Books.

Brown, A. R. (2012). The item vetos sting. *State Politics & Policy Quarterly 12*(2), 183–203.

Carter, J. R. and D. Schap (1990). Line-item veto: where is thy sting? *Journal of Economic Perspectives 4*(2), 103–118.

Crombez, C., T. Groseclose, and K. Krehbiel (2006). Gatekeeping. *The Journal of Politics 68*(2), 322–334.

Landes, W. M. (1993). Sequential versus unitary trials: An economic analysis. *The Journal of Legal Studies 22*(1), 99–134.

Mattes, M. (2018). Chipping away at the issues: Piecemeal dispute resolution and territorial conflict. *Journal of Conflict Resolution 62*(1), 94–118.

McKelvey, R. D. (1976). Intransitivities in multidimensional voting models and some implications for agenda control. *Journal of Economic theory 12*(3), 472–482.

Romer, T. and H. Rosenthal (1978). Political resource allocation, controlled agendas, and the status quo. *Public choice 33*(4), 27–43.

Schofield, N. (1978). Instability of simple dynamic games. *The Review of Economic Studies 45*(3), 575–594.

Shepsle, K. A. (1979). Institutional arrangements and equilibrium in multidimensional voting models. *American Journal of Political Science*, 27–59.

Shepsle, K. A. and B. R. Weingast (1981). Structure-induced equilibrium and legislative choice. *Public choice 37*(3), 503–519.

# A Online Appendix: Proofs

**Lemma 1** *In any SPNE of the multidimensional agenda game,*

1. *If* $q \in S^V(x^P)$*, the policy outcome is* $x^M = E^V(q) \cap CC(x^P, x^V)$*.*
2. *If* $q \notin S^V(x^P)$*, the policy outcome is* $x^M = x^P$*.*

*Proof*: Given $V$'s behavior, the equilibrium policy is $x^M = \arg\min_y \|y - x^P\|$ s.t. $\|y - x^V\| \leqslant \|q - x^V\|$.

1. Suppose $q \in S^V(x^P)$ but $x^M \notin CC(x^P, x^V)$. Then $\exists x \in CC(x^P, x^V)$ such that $\|x - x^V\| = \|x^M - x^V\| \leqslant \|q - x^V\|$ but $\|x - x^P\| < \|x^M - x^P\|$, i.e. $x^M$ cannot be an equilibrium. Next suppose $q \in S^V(x^P)$ and $x^M \in CC(x^P, x^V)$ but $x^M \notin E^V(q)$. Given $V$'s best response this implies $\|x^M - x^V\| < \|q - x^V\|$. But then there is another $x \in CC(x^P, x^V)$ such that $\|x - x^V\| = \|q - x^V\|$ and therefore $\|x - x^P\| < \|x^M - x^P\|$, i.e. $x^M$ cannot be an equilibrium.
2. Suppose $q \notin S^V(x^P)$; then $\|x^P - x^V\| \leqslant \|q - x^V\|$ and $x^P$ solves $P$'s problem. ∎

**Lemma 2** *In any SPNE of the issue by issue game, the policy outcome,* $x^I$*, is determined as follows. For each dimension* $i \in \{1, \ldots, N\}$*,*

1. *If* $q_i \in S_i^V(x_i^P)$*, then* $x_i^I \in E_i^V(q_i)$*.*
2. *If* $q_i \notin S_i^V(x_i^P)$*, then* $x_i^I = x_i^P$*.*

*Proof*: For each dimension $i$, $x_i^I = \arg\min_{y_i} |y_i - x_i^P|$ s.t. $|y_i - x_i^V| \leqslant |q_i - x_i^V|$. Taking the cases in turn,

1. If $q_i \in S_i^V(x_i^P)$, then $q_i \in (x^P, x_i^V - x_i^P)$. If $q_i \in (x_i^P, x_i^V]$, then $x_i^I = q_i$. If $q_i \in [x_i^V, 2x_i^V - x_i^P)$, then $x_i^I = 2x_i^V - q_i$. In either case, $x_i^I \in E_i^V(q)$.
2. If $q_i \notin S_i^V(x_i^P)$, then $|x_i^P - x_i^V| \leqslant |q_i - x_i^V|$, so $P$ proposes $x_i^P$ and $V$ accepts. ∎

**Lemma 3** *The SPNE policy* $x^I$ *is Pareto efficient if and only if* $(i)$ *there is no dimension for which* $q_i \in S_i^V(x_i^P)$*; or* $(ii)$ $q \in \widehat{CC}(x^P, x^V)$*.*

*Proof*: We first prove sufficiency of the two conditions and then the necessity of their union.
SUFFICIENCY (*i.e.*, $(i)$ and $(ii)$ each imply that $x^I \in CC(x^P, x^V)$).

$(i)$ (Condition $(i)$ is sufficient.) If there is no dimension $i \in \mathcal{N}$ such that $q_i \in S_i^V(x_i^P)$, then Lemma 2 implies that $x_i^I = x_i^P$ $\forall i$, i.e., $x^I = x^P$, which is Pareto efficient.

(*ii*) (Condition (*ii*) is sufficient.) Suppose that $q \in \widehat{CC}(x^P, x^V)$. For each dimension $i \in \{1, \ldots, N\}$, it is the case that $q_i \in [x_i^P, 2x_i^V - x_i^P]$ If $q_i \in [x_i^P, x_i^V]$, then Lemma 2 implies $x_i^I = q_i \in CC_i(x^P, x^V)$. Similarly, if $q_i \in [x_i^V, 2x_i^V - x_i^P]$, then Lemma 2 implies $x_i^I = 2x_i^V - q_i \in CC_i(x^P, x^V)$. Thus $x_i^I \in CC_i(x^P, x^V)$ for each dimension $i \in \{1, \ldots, N\}$, implying that $x^I \in CC(x^P, x^V)$ and $x^I$ is Pareto efficient.

Necessity (*i.e.*, $x^I \in CC(x^P, x^V)$ implies that either (*i*) or (*ii*) are true). Suppose that $q_i \in S_i^V(x_i^P)$ for at least one dimension $i \in \mathcal{N}$ and $q \notin \widehat{CC}(x^P, x^V)$ (*i.e.*, $q$ is such that neither (*i*) nor (*ii*) hold). For all $i \in \mathcal{N}$ such that $q_i \notin S_i^V(x_i^P)$, Lemma 2 implies that $x_i^I = x_i^P$. Thus, if $\{i \in \mathcal{N} : q_i \notin S_i^V(x_i^P)\} \neq \varnothing$ and $x^I \in CC(x^P, x^V)$, then it must be the case that $x^I = x^P$, implying that $q_j \notin S_j^V(x_j^P)$ for all $j \in \{1, \ldots, N\}$, contradicting the presumption that $q_i \in S_i^V(x_i^P)$ for at least one dimension $i \in \mathcal{N}$.

Thus, suppose that $q_i \in S_i^V(x_i^P)$ for all $i \in \mathcal{N}$[6] and define the following partition of $\mathcal{N}$:

$$\begin{aligned} G(q, x^P, x^V) &= \{i \in \mathcal{N} : x_i^P < q_i \leqslant v_i^V\}, \\ B(q, x^P, x^V) &= \{i \in \mathcal{N} : x_i^P < v_i^V < q_i\}. \end{aligned}$$

Then $x_i^I = q_i$ for all $i \in G(q, x^P, x^V)$ and $x_i^I = 2x_i^V - q_i$ for all $i \in B(q, x^P, x^V)$. If $x^I \in CC(x^P, x^V)$, then $q_i$ is located on the line segment connecting $x^V$ to $(\{x_i^P\}_{i \in G(q,x^P,x^V)}, \{2x_j^V - x_j^P\}_{j \in B(q,x^P,x^V)})$, implying that $q \in \widehat{CC}(x^P, x^V)$.

Accordingly, we have shown that if $q_i \notin S_i^V(x_i^P)$ for some $i \in \mathcal{N}$ and $x^I \in CC(x^P, x^V)$, then it follows that there is no dimension for which $q_i \in S_i^V(x_i^P)$ (*i.e.*, condition (*i*) holds). On the other hand, if $q_i \in S_i^V(x_i^P)$ for all $i \in \mathcal{N}$ and $x^I \in CC(x^P, x^V)$, then it must be the case that $q \in \widehat{CC}(x^P, x^V)$ (*i.e.*, condition (*ii*) holds). Thus, satisfaction of either condition (*i*) or condition (*ii*) is necessary and sufficient for the policy $x^I$ to be Pareto efficient, as was to be shown. ∎

**Proposition 1** *For any status quo $q \in X \subset \mathbb{R}^N$, if $q_i \in S_i^V(x_i^P)$ for some but not all dimensions, then the veto player strictly prefers the issue by issue game to the multidimensional game. If $q_i \in S_i^V(x_i^P)$ either for all dimensions or no dimensions, the veto player is indifferent between the two games.*

*Proof*: If $q_i \in S_i^V(x_i^P)$ for no dimensions, then $q \notin S^V(x^P)$, so Lemma 1 implies that $x^M = x^P$. Similarly, Lemma 2 implies that $x^I = x^P$. If $q_i \in S_i^V(x_i^P)$ for all dimensions $i \in \mathcal{N}$, then $q \in S^V(x^P)$, and Lemma 1 implies that $x^M \in E^V(q)$. Similarly, Lemma 2 implies that $x_i^I \in E_i^V(q_i)$ for each dimension $i \in \mathcal{N}$, which by separability of $\succeq_V$ implies that $x^I \in E^V(q)$, implying that $V$ is indifferent between $q$, $x^M$, and $x^I$. In both cases, $V$ is indifferent between the equilibrium policy outcomes of the game forms.

If $q$ is such that $\{i \in \mathcal{N} : q_i \in S_i^V(x_i^P)\} \notin \{\varnothing, \mathcal{N}\}$, there are two subcases to consider.

1. If $q \notin S^V(x^P)$, then $x^M = x^P$ and $x^I \in S^V(x^M)$ by the separability of $V$'s payoffs, because

[6] Note that this is stronger than $q \in S^V(x^P)$, a point that returns in Proposition 1, below.

   (a) $x^I_j \in S^V_j(x^P_j)$ for each dimension $j \in \{i \in \mathcal{N} : q_i \in S^V_i(x^P_i)\}$ and
   (b) $x^I_k \in E^V_k(x^P_k)$ for each dimension $k \in \{i \in \mathcal{N} : q_i \notin S^V_i(x^P_i)\}$.

2. If $q \in S^V(x^P)$, then $x^M \in E^V(x^P)$ and $x^I \in S^V(x^M)$ by the separability of $V$'s payoffs, because

   (a) $x^I_j \in S^V_j(x^P_j)$ for each dimension $j \in \{i \in \mathcal{N} : q_i \in S^V_i(x^P_i)\}$ and
   (b) $x^I_k \in E^V_k(x^P_k)$ for each dimension $k \in \{i \in \mathcal{N} : q_i \notin S^V_i(x^P_i)\}$. ∎

**Proposition 2** *For any status quo $q \in X \subset \mathbb{R}^N$, if the issue by issue equilibrium $x^I(q)$ is Pareto inefficient, then the proposer strictly prefers the multidimensional game to the issue by issue game. If $x^I(q)$ is Pareto efficient, the proposer is indifferent between the two games.*

*Proof*: If $x^I(q)$ is Pareto efficient, Lemma 3 implies that either

1. $q \in \widehat{CC}(x^P, x^V)$, which implies $x^M \in E^V(q) \cap CC(x^P, x^V)$ and $x^I \in E^V(q) \cap CC(x^P, x^V)$, i.e. $x^M = x^I$; or

2. $\nexists\ i$ s.t. $q_i \in S^V_i(x^P_i)$, which implies $x^I = x^P$ by Lemma 2; and further implies $q \notin S^V(x^P)$ so $x^M = x^P$ by Lemma 1.

If $x^I(q)$ is Pareto inefficient, then either:

1. $q \in S^V(x^P)$ but $q \notin \widehat{CC}(x^P, x^V)$. Then $(i)$ $x^M \in CC(x^P, x^V)$ and $x^M \in E^V(q)$ by Lemma 1; $(ii)$ $x^I \geq_V q$ by Lemma 2; $(iii)$ $x^I \notin CC(x^P, x^V)$ by Lemma 3. Facts $(i)$ and $(iii)$ imply $\|x^M - x^P\| + \|x^M - x^V\| < \|x^I - x^P\| + \|x^I - x^V\|$; facts $(i)$ and $(ii)$ imply $\|x^I - x^V\| \leqslant \|x^M - x^V\|$. But then $\|x^I - x^M\| < \|x^I - x^P\|$.

2. $q \notin W^V(x^P)$ and $\exists i$ s. t. $q_i \in W^V_i(x^P_i)$. Then $x^M = x^P$ by Lemma 1; but $x^I \neq x^P$ by Lemma 2, so $P$ strictly prefers MDM to IBI. ∎