The Unraveling of Prolonged Stability: The Fall of the Old Kingdom in Ancient Egypt

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January 22, 2009

Abstract

I model the relationship between a central government and its tax-collecting proxies. The Old Kingdom was the first “great age” of Ancient Egypt, witnessing the construction of such wonders as the pyramids of Giza and the Sphinx. Its downfall was the culmination of a long process, whereby the balance of power gradually shifted from the royal court to an emerging provincial elite. I maintain that the process was born of the state’s policy. Incapable of detecting the dynamics governing the division of resources between itself and its local proxies, the royal court set the economy on a divergent path that empowered the provincial elite at the state’s expense. The underlying strategic interaction between the royal court and its proxies was present long before a provincial elite ever emerged. Careful attention is paid to the question of which decisions should be modeled by optimizing behavior and which should not. Finally, the paper fills a gap in the existing literature by addressing why, in the short run, the royal court could not decrease the flow of state resources to the provincial elite.

1 Introduction

The Old Kingdom was the first “great age” of Ancient Egypt. Together with the preceding Early Dynastic Period it spanned most of the third millennium BCE and saw the construction of such grand monumental architecture as the Great Pyramids of Giza and the Sphinx.¹ The period was one of extraordinary stability and uniformity, not just politically but also in terms of cultural, social and economic development. Remarkably little change took place over this immense stretch of time, compared with other historical periods. In the absence of any

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¹The two periods jointly span the years 3000 to 2181 BCE. The distinction between them is essentially a scholarly one, without any real discontinuity. See Malek (2000) page 89.
marked external events affecting Egypt, the Old Kingdom’s downfall was the outcome of an internal process that unfolded over centuries, whereby the balance of power gradually shifted from the royal court to an emerging provincial elite.

I maintain that the process was born of the Egyptian state’s policy. The emergence of the provincial elite is closely related to the relationship between the royal court (i.e. the state) and its local tax-collecting proxies. Local administrators fall among the proxies and so do the emerging provincial elite, either directly or indirectly through its affiliation with the former. In order to efficiently guarantee the proxies’ compliance, the court would have needed to permit them to keep a certain share of the taxes they collected. The size of this share would simultaneously depend on the existing balance of power and affect the future balance of power, yielding a dynamic system. Viewing the balance of power in terms of the court’s and the proxies’ resources implies that the state could control the balance of power by carefully managing its expenditure.

Despite the simplicity with which such a system can be modeled, the chain of causality would have been virtually impossible for the royal court to detect. Consequently, I maintain that the royal court continuously used more resources than required to keep the balance of power stable, setting the economy on a divergent path that empowered the provincial elite at the state’s expense. The underlying strategic interaction between the royal court and its local proxies was present from the onset of the Early Dynastic Period, when the former first levied taxes from outside its immediate vicinity, and many centuries before a provincial elite ever emerged. Moreover, while my analysis in what follows is specific to Ancient Egypt and to the era in question, the model that I have implicitly referred to is more widely applicable.

The gradual nature of the Old Kingdom’s downfall and its roots in political developments are well described in the literature. The authority of the royal court is said to have declined in the later years of the Old Kingdom as local administrators and the associated provincial elite gained power and increasingly rivaled the state. This trend occurred because an increasing amount of resources was channeled to these parties by the state, leading to its relative impoverishment and hence its weakening. Additional flows of resources, earmarked for the upkeep of the temples and tombs which are the hallmark of Ancient Egyptian religion only increased the strain on the state’s resources.

From such descriptions it appears that once the shift in the balance of power had progressed sufficiently, it was quite apparent for a very long period of time. How is it possible then, that not a single Pharaoh attempted to divert the flow of resources back to the state? Surely an absolute king (and a deity, no less!) could issue an immediate decree on the matter?

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2The link between the balance of power and the ratio of the court’s and the proxies’ resources is made formally in the model that follows by way of a *fighting function*, which postulates a positive relationship between the two.

3This statement is developed in more depth in section 5.


5Ibid.

6The flow of resources away from the royal court was accompanied by immaterial flows of power, too. Egypt was divided into local districts called *nomes* that were run by state-appointed administrators called
Old Kingdom, they ought to provide a convincing answer to this question, which I address via the model implicitly outlined above.\textsuperscript{7}

In the model, the state incentivizes proxies to act as its loyal agents by allowing them to maintain a share of the tax revenue they collect. It sets the level of this share such that the proxies are indifferent between complying with the state and confronting it militarily. Thus, the proxies’ compliance is guaranteed and with it the royal court’s income, and military conflict between the state and its proxies is kept off of the equilibrium path. In this scenario, the royal court cannot, at least in the short-run, decrease the share of resources it affords the local proxies.

In the long run, however, the royal court can influence the proxy’s share via its policy. In particular, it can control the state’s expenses so as to alter the ratio of resources held by the royal court and its proxies, which implies the balance of power.

Nevertheless, it is highly unlikely that members of the royal court were historically aware of the dynamic system governing the balance of power, and therefore I pay careful attention to distinguishing which decisions can be modeled by optimizing behavior and which cannot. As discussed in more detail in section 5, the Pharaohs and their aides were almost certainly incapable of detecting the long-term dynamic relationship between state expenditure and the subsequent share of resources afforded to proxies. Due to this inability, I argue that the state failed to control its expenses so as to maintain a stable balance of power between itself and its proxies, and that the Egyptian economy consequently followed a divergent path, whereby the state was impoverished as it afforded the provincial elite an ever-increasing share of its tax revenue. Finally, I argue that the ratio of resources was such that the state could no longer credibly threaten to confront the proxies militarily, and it was effectively rendered powerless.

The period following the Old Kingdom, known as the First Intermediate Period, lasted for almost two centuries and is often perceived as a dark age. During this period the state dissipated into several polities and the country was struck by strife and bouts of famine. Very little is known with any certainty about the nature of the transition into the First Intermediate Period.\textsuperscript{8}

Throughout the periods involved, agricultural output in Egypt depended heavily on local administrators, who were responsible for developing and maintaining local irrigation systems 

\textsuperscript{nomarchs.} Grimal (1992) (page 92) and Seidlmayer (2000) (pages 120-121) mention that during the 5th and 6th Dynasties (the last two Dynasties of the Old Kingdom) hereditary transfer of the nomarch position gradually replaced the previous method of appointment by the state, allowing nomarchs to establish a stronger hold over their districts. Eradicating this norm seems like an easy enough task for a Pharaoh, so why was it not abolished?\textsuperscript{7}

Absolute religious ideology may explain the reticence to pull resources away from some uses, such as temples and funerary cults, but it can hardly explain the whole picture. Moreover, from a cynic’s point of view it is hard to believe that a powerful Pharaoh would voluntarily permit religious ideology to develop in a way that sheds him of power, let alone entire Dynasties of Pharaohs surrounded by intelligent, savvy advisors.

\textsuperscript{8}Despite the certainty with which Bell (1971) states that Egypt fell into anarchy with seeming suddenness, it is uncertain whether the deterioration of the state was gradual or if abrupt events occurred. See Grimal (1992) pages 137-140, and Seidlmayer (2000), pages 119-122.
composed of floodwater dams and canals. The state, on the other hand, managed the storage of grain. The main determinant of agricultural output was the level of annual Nile floods that served to irrigate the land, and which was inherently volatile. By storing grain the state could smooth the food supply over time so as to cushion the hazardous consequences of this volatility, and it could do the same over space by transferring food from areas of plenty to areas of shortage. The state also provided Egypt with external security.

In the absence of the state the food supply could not be smoothed over space, and the capacity to smooth it over time was at best on par with what it had been earlier. Thus, it is no mystery how the collapse of the Old Kingdom may have led to famine, while internal strife may have preceded the famine, followed it, or both.

Section 2 provides a set of stylized facts that portray the era in question. A model introduced in section 3 is then used in section 4 to analyze the stylized facts. Section 5 provides justification for the way various agents’ decisions are modeled and section 6 concludes.

2 Stylized Facts

The facts:

1) *Beginning with the formation of the state, the region from which taxes were levied gradually expanded and the tax collection system solidified. The process came to an end roughly during the 3rd Dynasty when the tax system encompassed all of Egypt. Most of the growth in the state’s income occurred up until the 3rd Dynasty with more modest growth, if any, taking place afterwards.*

2) *The peak of state expenditure occurred during the 4th Dynasty, and is marked by the construction of the Great Pyramids at Giza. During the 5th and 6th Dynasties state expenditure was substantially reduced.*

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10 Although its origin is likely to be from a later period (see Knohl (2008)), the biblical story of Joseph and Pharaoh’s dreams is a perfect example of the state preventing famine this way. In the story, Joseph oversees the royal bureaucracy as it stocks grain over seven years of plenty and then cushions the blow of the seven lean years that followed (Genesis, cap. 41). The state’s role in inter-temporally smoothing the food supply is commonly acknowledged.


12 An alternative theory explaining the fall of the Old Kingdom is that of Bell (1971), who hypothesizes that collapse occurred due to a catastrophic sequence of unusually low Nile floods, which could not be mitigated by grain storage (For a thorough description of Nile floodplain agriculture and the ramifications of too low or too high a flood, see Butzer (1976), pages 39-56). Implicitly, the low Nile floods induced famine, which in turn brought about strife. However, the only relevant evidence of unusually low Nile floods around the time of the collapse is comprised of inferences from contemporary texts describing famine and strife (there is much physical evidence that Nile floods were lower during the later centuries of the Old Kingdom. See Butzer (1976), pages 30-33). While there is no ruling out the climatic theory of Bell (1971), I agree with others who argue that it is redundant in explaining the fall of the Old Kingdom. See Butzer (1984), pages 101 and 106, and Seidlmayer (2000), page 129.
3) The provincial elite was negligible in size and wealth before the 5th Dynasty. During the 5th and 6th Dynasties it grew substantially larger and wealthier, at the expense of the state.

4) From the end of the 6th Dynasty the state lost de-facto authority over the country, although not necessarily by way of any military conflict.

Basis for the facts:

1) Beginning with the formation of the state, the region from which taxes were levied gradually expanded and the tax collection system solidified. The process came to an end roughly during the 3rd Dynasty when the tax system encompassed all of Egypt. Most of the growth in the state’s income occurred up until the 3rd Dynasty with more modest growth, if any, taking place afterwards.

The unification took place roughly during the first century of the third millennium BCE, marking the beginning of the Early Dynastic Period. The Old Kingdom refers to the period from 2686 BCE to circa 2150 BCE, and is formally distinguished from the Early Dynastic Period by the reign of King Djoser (3rd Dynasty), whose construction projects foreshadowed the subsequent wonders of the 4th Dynasty. Referring to the “enormous control exercised by the Crown” with respect to the step-pyramid complex of King Djoser, Bard (2000) writes the following:

“...such power must have been developed incrementally throughout the 1st and 2nd Dynasties, following the unification of the large territorial state... The Early Dynastic Period was a time of consolidation of the enormous gains of unification... when a state bureaucracy was successfully organized and expanded to bring the entire country under royal control. This was done through taxation, to support the Crown and its projects on a grand scale...”

The “enormous gains of unification” to which Bard (2000) refers are the focus of Allen (1997), who argues the following:

“Successful states in the ancient world depended on the ability of elites to extract a surplus from farmers and other producers. This ability was greatest when the population was immobile. The success of the Pharaohs was due to the geography of Egypt - the deserts bordering the Nile meant that habitation was confined to the valley. Farmers could flee tax or rent collectors only along the river. The population control problem was, thus, simpler than elsewhere and was the reason a unified state was created and lasted for millennia.”

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13 King Djoser is known for constructing the first step-pyramid which provides the link between the later, true pyramids, and the preceding mastaba tombs. The given classification into periods was imposed by 19th century historians and was not accompanied by discontinuous changes on the ground - see Malek (2000), page 89.


During the formative period of the tax system that preceded the 3rd Dynasty the state’s income grew both on the extensive margin, due to expansion of the taxed region, and on the intensive margin due to more thorough extraction. From the 3rd Dynasty onwards, increasing the state’s income was possible only on the intensive margin. Therefore, and in light of the quotes above, it is reasonable to assume that most of the growth in the state’s income occurred up until the 3rd Dynasty, with more only more modest growth, if any, taking place afterwards.

2) The peak of state expenditure occurred during the 4th Dynasty, and is marked by the construction of the Great Pyramids at Giza. During the 5th and 6th Dynasties state expenditure was substantially reduced.

This stylized fact is inferred from viewing the construction of monumental architecture, and in particular of royal tombs, as a proxy for state expenditure. There are no marked periods of increased warfare during the Early Dynastic Period or the Old Kingdom, nor do there appear to be any other grounds to suspect that the share of monumental construction in the total state expenditure fluctuated significantly during the period, so at least as a first pass this seems to be a valid proxy.

The step-pyramid of King Djoser, in the 3rd Dynasty, overshadowed all royal construction that preceded it and was itself overshadowed by the Great Pyramids built by the 4th Dynasty kings. The later pyramids of the 5th and 6th Dynasties are significantly smaller. Referring to the reign of the King Pepi II (6th Dynasty), the last of the Old Kingdom’s kings, Kemp (1983) states that “the ability of the court to build on a truly monumental scale seems to have gone altogether”.

A potential objection to this argument would be that the size of a king’s pyramid is a function of how long he reigned. Kings Khufu, Khefren and Menkaura (4th Dynasty), builders of the three great pyramids at Giza, ruled for 23, 26 and 18 years, respectively. While some of the 5th and 6th dynasty kings were not so lucky, two of the 5th Dynasty kings ruled for over 30 years and four of them ruled for over 20 years. During the 6th dynasty three kings ruled in excess of 30 years.

Another potential objection is that construction technology improved due to learning over time, so that a larger tomb at a later date could imply a smaller expenditure than a smaller one built earlier. While this argument may support the claim that state expenditure was greater at the time of King Djoser than it was at the height of the 4th Dynasty, it only supports further the claim that state expenditure was lower during the 5th and 6th Dynasties than it was at the height of the 4th.

3) The provincial elite was negligible in size and wealth before the 5th Dynasty. During the 5th and 6th Dynasties it grew substantially larger and wealthier, at the expense of the state.

According to Seidlmayer (2000), “[u]ntil well into the 5th Dynasty, nothing of the cultural expansion of the Egyptian state is beyond the scope of this paper.

For a sequential list of pyramid base sizes by king and dynasty, see Grimal (1992) pages 116-119.

See Kemp (1983), page 112.
achievements that attest to the grandeur of the Old Kingdom was to be seen outside the Memphite region” (Memphis being the capital of the Old Kingdom). He continues:19

“However, a profound change in the system began to appear in the 5th Dynasty and was completely in place by the end of the 6th... Originally, economic resources were concentrated at the royal residence and redistributed to the beneficiaries by the central administration. Now, however, the nobles residing in the provinces were able to gain direct access to the products of the country... The provincial aristocracy was eager to ensure that its way of life was on a par with the style of the royal court. This is evident in the decorated monumental tombs that began to appear in the cemeteries of the regional centres throughout the country... These tombs, however, are only the tip of the iceberg; in fact, the various provincial elites and their staff acted as separate centres within the political organization, sustaining specialist professionals and keeping a growing amount of local produce for use within the provinces themselves (rather than allowing it to be exploited by the royal court)...”

The process whereby the provincial elite gained power was a very gradual one, and was a result of the state’s financial policy. Malek (2000) describes how state officials were typically remunerated in the Old Kingdom:20

These officials were remunerated for their services in several different ways, but the most significant was an ex officio lease of state (royal) land, usually estates settled with their cultivators. Such estates produced practically all that their personnel needed... and the ex officio remuneration was their surplus produce. This land reverted, at least in theory, to the king after the official’s term of office expired and so could be assigned as remuneration of another official. In an economic system that did not know money it was a very effective way of paying salaries of officials, but it also represented a significant erosion of the king’s resources.”

The remuneration of officials was not limited to the realm of material compensation. Grimal (1992) describes a great number of new official titles appearing during the 5th and 6th Dynasties that bore no real meaning and were clearly granted to satisfy political needs.21

An even greater drain on the royal treasury than ex-officio remunerations were pious foundations. The latter were funds, usually established by a donation of property. Their role was to guarantee perpetual maintenance of religious sites and the cults that accompanied them, such as temples, the tombs of royalty and the tombs of growing numbers of other wealthy individuals.22 Referring to the growing number of funerary endowments of provincial officials for which the state was responsible, Grimal (1992) states that:23

19Seidlmayer (2000), pages 120-121. See also Kemp (2006), page 309. For a reflection of the growing wealth of the provincial elite on the relative importance of local deities, see Malek (2000), page 111.
21See Grimal (1992), page 90, as well as Kemp (1983), page 80.
22See Kemp (1983), page 85.
23See Grimal (1992), pages 92-93.
“...this principle... contained the seeds of the state’s destruction, in that it favored the dissemination of wealth and the gradual - and ultimately irreversible - impoverishment of the king. The profits enjoyed by the recipients of these concessions acted as a drain on the economy, since they effectively lay outside the redistribution network provided by the state. But even this was not the most important effect. The most serious problem was the social mechanism that was created by these concessions: ...the recipients attempted to acquire not only wealth but also the prerogatives associated with royal property.”

The historians’ quotations presented clearly identify the symptoms of the process by which the state’s power diminished in terms of economic resources. The contribution of this paper is in analyzing the mechanism causing these symptoms.

4) From the end of the 6th Dynasty the state lost de-facto authority over the country, although not necessarily by way of any military conflict.

Describing the end of the Old Kingdom following the 6th Dynasty, Malek (2000) is very straightforward: “Centralized government all but ceased to exist, and the advantages of a unified state were lost.”24

There is no material or textual evidence that the end of the Old Kingdom came about through military conflict. Rather, it may have been the conclusion of a gradual process in which the royal court’s power diminished and it fell into political irrelevance for much of the country. There appears to be no archaeological evidence for a social revolution or a civil war at the end of the Old Kingdom.25

Whether or not the remnants of the Old Kingdom at the end of the 6th Dynasty were destroyed in armed conflict is inconsequential. Both possibilities will prove to be consistent with the theory presented below.

The following section presents a model with which these stylized facts can be analyzed.

3 Model

Consider the king, the royal court and the state to be interchangeable terms describing one player, whose marginal utility from an additional unit of the single existing type of good, grain, is always positive.

The royal court extracts surplus product from the economy by way of local proxies. The proxies may be nomarchs (administrators of local districts called nomes), mayors, temple priests, combinations of these, or perhaps bearers of different posts or informal roles.

Consider then a proxy for the state, $i$, who presides over a domain with product $Y_i$ and extracts its surplus. He transfers to the state all but a share of the extracted surplus, which he retains for himself. This share is referred to as the proxy’s share and is labeled $M_i$. His

24See Malek (2000), page 117.
transfer combines with that of other proxies to comprise the state’s tax revenue.  

The proxy’s share is retained under the auspices of the state, either formally or informally by way of the court turning a blind eye. The product of ex-officio remunerations and pious foundations falls under the former category, whereas embezzlement of tax revenue by local officials falls under the latter.

The proxy’s share is an essential incentive for him to fulfill his duty. If he complies with his role by transferring all but his share of surplus to the state he retains:

\[ M_i \cdot \tau Y_i \tag{1} \]

where \( \tau \) is the share of product that is extracted as surplus, i.e. the tax rate. According to Brewer and Teeter (2007) it appears that approximately a tithe of the total harvest was appropriated as tax. For simplicity, assume that tax is either levied at a fixed rate, \( \tau \), or not levied at all. Consequently, the state’s tax revenue is:

\[ \sum_{j \in J} (1 - M_j) \cdot \tau Y_j \tag{2} \]

where \( J \) is the set of all proxies.

Alternatively, if the proxy chooses to revolt by not transferring tax revenue he may retain all, some or none of the extracted surplus from his domain. The amount which he retains is a random variable that depends on whether or not the state chooses to confront him and on the outcome and the cost of such a confrontation if it takes place. Designate \( P_i \in [0, \infty) \) to be the share of extracted surplus retained by the proxy if he revolts, so that in a particular realization he retains:

\[ P_i \cdot \tau Y_i \]

and in expectancy he retains:

\[ E(P_i) \cdot \tau Y_i \tag{3} \]

\( P_i \) can be greater than one, because a military confrontation may potentially result in grand victory for the proxy, where he is left with more resources than he extracted from his domain to begin with. On the other hand, it is also possible that \( P_i = 0 \) if the state crushes the proxy.

Regardless of whether a military confrontation is realized, assume that \( E(P_i) \) is a positive function of the proxy’s potential military ability versus the royal court, and refer to it as a fighting function. Assume further that the ratio of resources available to the parties at the

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26 The royal court may extract some surplus directly from its vicinity. This is inconsequential to the analysis.
28 The cost of conflict is in terms of the grain used to hire military force.
29 To be more precise: let \( x \) be a parameter positively describing the proxy’s military ability versus the royal court such that \( P_i(x) \in [0, 1] \) is a random variable with cdf \( F(P_i|x) \). The assumption can be re-stated as \( F(P_i|x') \leq F(P_i|x'') \ \forall x' \geq x'' \) (first-order stochastic dominance), which implies \( \frac{\partial}{\partial x} E(P_i(x)) \geq 0 \).
time of confrontation is a valid proxy for their relative, potential military ability. Doing so is especially appealing in the context of ancient Egypt, in which military force was generally hired and even the state itself never maintained a standing army.\textsuperscript{30}

Should the proxy choose to revolt, the resources available to him are those he extracts from his domain in the current period, $\tau Y_i$, and whatever remains of his share from previous periods, which I shall assume to be zero for simplicity.\textsuperscript{31} The royal court’s resources are the current period’s tax revenue save that of the revolting proxy, and what remains of the previous period’s tax revenue. The length of time periods is normalized so that resources can be stored for no more than one period.\textsuperscript{32} Thus:

$$E(P_{i,t}) = E(P'(\frac{X_{i,t}}{X_{c,t}}; Z_t)) \text{ s.t. } E(P'(Z)) > 0 \quad (4)$$

where $Z_t$ is a vector of all exogenous parameters affecting the outcome at time $t$, such as the location and characteristics of potential sites of battle, the parties’ military and political strategies, the players’ temperaments, talents and so forth. $X_{i,t}$ and $X_{c,t}$ are the resources of the proxy and the court in period $t$, respectively, conditional on the proxy choosing to revolt:

$$X_{i,t} = \tau Y_i \quad (5)$$

$$X_{c,t} = \sum_{j \in J} [(1 - M_{j,t}) + (1 - \alpha_{t-1})(1 - M_{j,t-1})] \tau Y_j - (1 - M_{i,t}) \tau Y_i \quad (6)$$

where $(1 - \alpha_{t-1}) \in (0, 1)$ is the proportion of resources that remain stored from the previous period and is controlled by the state in period $t - 1$.

As will become evident, the results of the model depend crucially on the dynamic link between the royal court’s present resources, $X_{c,t}$, and the proxy’s share in the previous period, $M_{i,t-1}$. This link is intact as long as the court’s resources are partly comprised of the previous period’s tax revenue that has been stored by the state. Therefore, I shall assume $(1 - \alpha_t) > 0$ for all $t$ and express $X_{c,t}$ as a function of the lagged proxy’s share, $M_{i,t-1}$:

$$X_{c,t}(M_{i,t-1}) = K - (1 - \alpha_{t-1}) M_{i,t-1} \tau Y_i$$

where $K$ follows from (6).

\textsuperscript{30}See Brewer and Teeter (2007), page 74. Also, according to Seidlmayer (2000) much of Egypt’s external warfare was conducted by local administrators who hired temporary force. See page 130.

\textsuperscript{31}Without this simplifying assumption, the proxy’s resources would depend positively on the proxy’s share in the previous period, leading further support to the results developed below.

\textsuperscript{32}This choice of normalization makes the model more tractable, and in particular more amenable to graphic presentation. Note that a normalized period may be longer than the intervals at which tax revenue is transferred to the royal court (these were probably annual in ancient Egypt). If this is the case then the resources actually in the hands of a revolting proxy are only the part of the resources extracted from his domain in the latest tax transfer interval, which he has not yet transferred to the court and can be expressed as $\frac{1}{\gamma} \tau Y_i$, where $\gamma$ is the number of tax transfer intervals in one normalized period (the storage lifetime of grain). Carrying the coefficient $\frac{1}{\gamma}$ through all of what follows is inconsequential to the analysis, so I assume $\gamma = 1$ throughout even though this is unrealistic.
Note that this assumption is far from arbitrary and is most likely to have held in reality. To see this, suppose that the royal court’s resources depended entirely on the present period’s tax revenue. If this were the case, then any coalition of proxies which comprised a sufficient proportion of the total could have revolted with reasonable odds of success at any moment. This would have been an inherently unstable situation and the state could not have persisted over any length of time under these circumstances, let alone for the many centuries that it did (during which it relied heavily on proxies).33

For the royal court to assure the proxy’s compliance, it must set the proxy’s share so that by complying he will dynamically maximize his utility. To eliminate unnecessary complexity, I make two simplifying assumptions:

1) The proxy is assumed to be risk-neutral. To justify this, note that if the proxy is risk-averse \( P_i(\cdot) \) can be replaced with some function \( \tilde{P}_i(\cdot) \in [0, \infty) \) that is also strictly increasing and which is referred to as the risk-adjusted fighting function. A formal proof is provided in the appendix.34

2) The proxy is assumed to be myopic. In addition, I assume that changes over time in the proxy’s share are very gradual if he does not revolt, and are therefore negligible in his foreseeable horizon. It is rather tedious to show that the models with myopic and non-myopic proxies are equivalent, so I relegate this to the appendix, too.35

Given these assumptions, the royal court’s task is to maintain the following condition, which states that the proxy’s present utility is at least as great as his present expected utility from revolting.

\[
u_i|\text{comply} \geq E[u_i|\text{revolt}]
\]

\( u(\cdot) \) is a utility function representing the proxy’s preferences over resources in the current period, and is assumed to exist and to be “well behaved”.36

Because the state always has positive marginal utility from resources, it will ensure that this condition binds. Normalizing \( u(0) \) to zero, this condition yields:

\[
u_i(M_{i,t}\tau Y_i) = u_i(E(P_{i,t})\tau Y_i)
\]

\[
\Rightarrow \quad M_{i,t} \equiv E(P_{i,t})
\]

(7)

If the proxy’s share, \( M_{i,t} \), were smaller he would choose to revolt and if it were larger the royal court would reduce it, keeping more for itself. (7) is an equivalence because \( M_{i,t} \) is constantly adjusted by the state so as to maintain it.

33This line of thought also sheds light on the historical role of granaries, where resources were stored in grain. Prior to the introduction of currency and perhaps even thereafter, the state’s motivation for maintaining granaries went well beyond their role in buffering the food supply from shocks. Granaries were a crucial vehicle for the power and stability of the state. In their absence, the state could not have existed for any length of time on a large geographic scale (one that required reliance on proxies for tax collection).

34See appendix section 1

35See appendix section 2.

36That is: \( \frac{\partial u(x)}{\partial x} > 0 \) and \( \frac{\partial^2 u(x)}{\partial x^2} < 0 \) \quad \forall x.
Substituting (4) into (7) we obtain the dynamic path of $M_i$:

$$M_{i,t+1} = E(P\left(\frac{\tau Y_i}{K - (1 - \alpha_t)M_{i,t}\tau Y_i}; Z_{t+1}\right)) \quad (8)$$

Shortly, we shall proceed to examine the dynamics of the proxy’s share graphically, however there are some further issues that must be addressed before doing so.

First, notice that until now I have implicitly modeled the state’s strategy as an automatic response to that of the proxy’s: it provides him with his proxy’s share if he complies and transfers tax revenue to the state and confronts him otherwise. This strategy, however, is not necessarily applied. When confronting a revolting proxy, the state takes upon itself a risk of loss in the battlefield that may leave it with fewer resources than if it were to acquiesce to the proxy. Formally, if the state chooses to acquiesce it obtains:

$$X_{c,t}\mid\text{acquiesce} = \sum_{j \neq i} (1 - M_{j,t})\tau Y_j \quad (9)$$

in the current period, whereas if it confronts the proxy then in expectancy it maintains:

$$E(X_{c,t}\mid\text{confront}) = E(Q_{c,t} \cdot \sum_{j \in J} (1 - M_{j,t})\tau Y_j) \quad (10)$$

where $Q_{c,t}$ is a fighting function viewed from the royal court’s perspective, i.e. whose argument is the inverse resource ratio, $\frac{X_{c,t}}{X_{i,t}}$. Thus, given a rival with sufficient resources, the state prefers to acquiesce to a revolt rather than confront it. Equating (9) and (10) yields a unique proxy share, $\bar{M}_i$, beneath which the state confronts a revolting proxy and above which it acquiesces.

If the proxy’s share exceeds $\bar{M}_i$ then he ceases to transfer tax revenue to the state, knowing that no military confrontation will be required of him. This effectively means that the proxy’s domain becomes independent of the state.

Second, so far I have considered only a single proxy, $i$, however there is no barring the cooperation of several proxies in revolting against the state. Clearly, several proxies would stand a better chance of success if they revolted simultaneously. This raises the question whether the state determines each proxy’s share based on his potential to rebel single-handedly, or whether it takes into account his potential role in a larger coalition. In order to avoid this matter, let us assume that the royal court can correctly identify coalitions that are most likely to revolt and predict their internal division of resources, and that it determines each proxy’s share accordingly. This amounts to giving the members of the royal court credit as savvy political players, who know the intricate human material of their proxies and can

---

The fighting function $Q_{c,t}$ differs from $1 - P_{i,t}$ in two respects. First, if either the state, the proxy or both are not risk-neutral then their risk attitudes are incorporated and the two are distinct. Second, $P_{i,t}$ is “truncated” at zero for instances where the proxy is absolutely defeated (supposing some latent, underlying fighting function). Consequently, $1 - P_{i,t}$ may take on negative values and never exceeds 1, whereas $Q_{i,t} \in [0, \infty)$, similar to $P_{i,t}$. 

12
establish the correct payoffs to efficiently guarantee their compliance. Moreover, historically it must have been the case that the royal court was proficient at determining proxy shares that successfully guaranteed compliance, or else it would not have lasted the extensive period that it did. Accordingly, let subscript $i$ denote any relevant set of proxies acting as a single player, rather than just any single proxy, and let us continue referring to it as the proxy’s share.

Let us proceed to examine the dynamics of the proxy’s share by graphing $M_{i,t}$ as a function of $M_{i,t-1}$. Recalling the assumption that the fighting function is strictly increasing, the dynamic curve to the left of $M_i$ in figure 1 must be strictly increasing, too, because the state’s resources in period $t+1$ diminish when $M_{i,t}$ increases. The linear presentation of the dynamic curve to the left of $M_i$ is merely a simplification, and there may in fact be multiple equilibria along the depicted stretch. We shall assume, however, that there exists at least one stable equilibrium with $M < M_i$, on the grounds that the Old Kingdom state held on to power for more than a brief moment.

Thus, there are at least two stable equilibria in the model: one or more low equilibria to the left of $M_i$ and a high equilibrium where $M = 1$. When a low equilibrium prevails tax revenue is transferred to the state by the proxies and it maintains its hegemony over the country. In contrast, when the high equilibrium prevails no tax revenue is transferred to the state, and it is in fact no longer a state. Rather, it is a royal court stripped of its power. The high equilibrium is an apt description of the state of affairs during the First Intermediate Period, while any low equilibria describes the state of affairs during the Old Kingdom.

Finally, let us address one last aspect of the model: the state’s ability to influence the outcome. Even given the state’s choice of the proxy’s share and its decision whether to acquiesce or confront a revolting proxy, it still has the ability to maneuver between the model’s multiple equilibria. The state can alter the position of the dynamic curve via its policy, thereby affecting both the location of stable equilibria (and in some instances their number). There are two categories of policy it can apply: first, it can inadvertently affect the amount of resources it will have in the future by spending more today, as modeled by the parameter $\alpha_t$. Increased (decreased) spending in period $t$ is represented by a higher (lower) level of $\alpha_t$, leaving the state with fewer (more) resources to confront a potential revolt in period $t+1$, and thereby shifting upwards (downwards) the section of the dynamic curve to the left of $M_i$. To see this, recall equations (6) and (7), and note that $M_{i,t+1}$ can be expressed as follows:

$$M_{i,t+1} = P\left(\frac{\tau Y_i}{K' + (1 - \alpha_t)K''}; Z_{t+1}\right) \quad \forall t \quad (11)$$

where $K'$ and $K''$ follow from equation (6).

Secondly, the state can take measures that affect the fighting function via the parameter vector $Z$, and which may or may not be accompanied by the use of resources and a consequently higher level of $\alpha_t$. Maintaining a fortress overlooking a proxy’s abode would qualify into this category, as would mounting obstacles to the construction of a rebel coalition. I will assume, however, that the effect of changes to $Z$ is limited when they are not accompanied by the use of any resources.
4 Analysis

The hypothesis brought forth in this paper is that the state’s policy, reflected by the control variable $\alpha$, was such that the equilibrium proxy’s share increased gradually until it eventually reached the level $\bar{M}$. At that point the state ceased to confront revolting proxies and there occurred a shift to the high equilibrium, marking the end of the Old Kingdom and the onset of the First Intermediate Period.

In what follows I present this hypothesis more carefully and in greater detail, incorporating each of the stylized facts presented earlier into the framework of the model. To do so, I track the likely value over time of four variables, from which the path of $\alpha$ over time can be reconstructed:

1) $GI$: the state’s gross income.
2) $M$: the proxy’s share (recall the issue of proxy coalitions, addressed above).
3) $NI$: the state’s net income, which refers to the income remaining after deducting the proxy’s share and is equal to $GI - M$.
4) $E$: the state’s expenditure of present-period income. To understand the qualifier “present-period”, recall from equation (6) that $\alpha_t$ refers to the share of the tax revenue received in period $t$ that is spent in the same period, leaving behind a share $1 - \alpha_t$ for use in period $t + 1$. Thus, $E = \alpha \cdot NI$.\(^{39}\)

\[^{38}\]In what follows: $TI = \sum_{j \in J} \tau Y_j$; $M = \sum_{j \in J} M_j \tau Y_j$; $NI = \sum_{j \in J} (1 - M_j) \tau Y_j$.

\[^{39}\]An implicit assumption made here is that each period the state first spent resources remaining from the previous period, before they went to waste, and only then turned to using resources collected in the present
Having described the paths of these variables, it is then straightforward to reconstruct $\alpha$ as:

$$\alpha = \frac{E}{NI} = \frac{E}{GI - M}$$ (12)

Figure 2 is useful in illustrating the analysis that follows.

1) Beginning with the formation of the state, the region from which taxes were levied gradually expanded and the tax collection system solidified. The process came to an end roughly during the 3rd Dynasty when the tax system encompassed all of Egypt. Most of the growth in the state’s income occurred up until the 3rd Dynasty with more modest growth, if any, taking place afterwards.

First of all, note that a new proxy introduced in period $t$ has a previous-period proxy share of $M_{i,t-1} = 0$. Therefore, with all else equal, his share necessarily converges to the lowest existing equilibrium, as indicated by figure 1.

Next, consider the dynamics of the proxy’s share when the amount of surplus extracted by the state is increasing. Equation (7) is key so it is repeated here:

$$M_{i,t+1} = E(P\left(\frac{X_{i,t}}{X_{c,t}}; Z_{t+1}\right))$$

This stylized fact is essentially an observation on $GI$. As long as the growth in surplus extraction was more or less evenly distributed among proxies the resource ratio for each individual proxy, $X_{i,t}/X_{c,t}$, is likely to have been constant.\(^{40}\) Inasmuch as the increase in surplus extraction involved the introduction of new proxies, it would have yielded reductions in individual proxies’ resource ratios. In figure 1, any increase in $X_{c,t}$ given an existing level of $M_{i,t}$ induces a downward shift of the dynamic curve, leading to a drop in the levels of all low equilibria. Thus, it is likely that while the state’s extraction of surplus expanded from its inception through the 3rd Dynasty, the proxy’s share did not grow and probably even decreased.

The state’s gross income, $GI$, increased during this period. The lack of increase in the proxy’s share, $M$, established above implies that the state’s net income, $NI$, followed gross income very closely. While the state’s present-period expenditure, $E$, was rising, it is unclear whether or not it was rising relative to net income, so the path of $\alpha$ while the state’s surplus extraction system was expanding is unclear (recall $\alpha = E/NI$).\(^{41}\)

\(^{40}\)The Egyptian state was subdivided into administrative units of roughly similar size called nomes, of which there were finally several dozen. Thus, momentarily equating proxies with nomarchs (administrators of local districts called nomes), even if the expansion of surplus extraction was concentrated in the domains of a small number of nomarchs during some period, it is unlikely that this was the case throughout any length of time.

\(^{41}\)Of course, one could assert the path of $\alpha$ more accurately by making educated assumptions about the state’s spending behavior. Also, given that the state continuously wielded power throughout this period, a sharp increase in $\alpha$ of the kind that would induce a shift to the high equilibrium can be ruled out.
Figure 2: The development of GI, NI, E and $\alpha$ during the Early Dynastic Period and the Old Kingdom. Note that from the 4th Dynasty onwards, the level of $\alpha$ steadily increases as it converges to 1.
2) The peak of state expenditure occurred during the 4th Dynasty, and is marked by the construction of the Great Pyramids at Giza. During the 5th and 6th Dynasties state expenditure was substantially reduced.

This stylized fact is essentially an observation on $E$. The state’s present-period expenditure peaked during the 4th Dynasty and dropped thereafter. In combination with the next stylized fact, it is possible to map out the path of alpha through the end of the 6th Dynasty, so let us proceed.

3) The provincial elite was negligible in size and wealth before the 5th Dynasty. During the 5th and 6th Dynasties it grew substantially larger and wealthier, at the expense of the state.

This stylized fact, on the other hand, is essentially an observation on $M$. The proxy’s share was very small until roughly the 5th Dynasty, after which it steadily grew larger. For this to have been the case, it is necessary for $\alpha$ to have been increasing, thereby raising the dynamic curve in figure 1 and shifting any low equilibrium to a higher level of $M$.\footnote{It is inconsequential whether the increase in $M$ occurred through a gradual shift of a realized low equilibrium or through a jump to some higher low equilibrium in a set of multiple equilibria.}

But is such an increase in $\alpha$ plausible? The proxy’s share was negligible until the 3rd Dynasty (see above), so $\alpha = E/NI \approx E/GI$. For $\alpha$ to have increased, it must have been the case that the state’s present-period expenditure, $E$, increased more quickly than the state’s income, $GI$. Recall from the first stylized fact that the increase in the state’s gross income, $GI$, slowed substantially after the 3rd Dynasty and possibly even stopped. The emergence of a provincial elite was therefore the result of state expenditure, $E$, whose growth after the 3rd Dynasty did not slow as quickly as gross income.

The mutually unsustainable paths of state expenditure and income eventually induced the state to curb its expenditure, albeit not enough to curb the growth of $\alpha$.\footnote{Income here refers to both $GI$ and $NI$, which diverged now that the proxy’s share, $M$, was no longer negligible.} The curbing of state expenditure brought the path of $E$ to a peak during the 4th Dynasty. The growth of $\alpha$, however, was only slowed but not stopped and so the proxy’s share continued to expand so that it became necessary to curb $E$ even further. This occurred continuously, sending state expenditure on a decreasing path well below the 4th Dynasty peak and raising the proxy’s share to unprecedented levels. Eventually, it hit the critical level $\bar{M}$ at which the state ceased to confront revolting proxies. The result was stylized fact number four.

4) From the end of the 6th Dynasty the state lost de-facto authority over the country, although not necessarily by way of any military conflict.

Earlier, I stated that whether or not the remnants of the Old Kingdom at the end of the 6th Dynasty were destroyed in armed conflict is inconsequential, and both possibilities are consistent with the analysis. To clarify this point, note that a risk-averse proxy may choose to comply with the state even when the proxy’s share exceeds the critical level $\bar{M}$, for fear that the state will confront him despite the odds.

Formally, the interaction between the state and the proxy can be modeled as an extensive form game where the proxy moves first and can either comply with the state or revolt. In the
event that the proxy revolts, the state can either confront the proxy militarily or acquiesce. When the proxy’s share exceeds the critical level, $M$, and the proxy is sufficiently risk-averse with respect to the outcome, $(\text{revolt}, \text{acquiesce})$ may not qualify as a trembling-hand perfect equilibrium, even if it is a perfect Bayesian equilibrium.

5 What Is Optimized And What Is Not:

Before proceeding, note that all of the decisions in the model are made by way of optimization with one exception: the state’s expenditure is not optimally controlled. The contrast is sharpest with respect to the state’s optimal determination of the proxy’s share. This matter is worth dwelling upon.

The grounds for modeling the state’s choice of the proxy’s share as an optimizing one is that its results were apparent to the state almost immediately. A royal court member setting an insufficient proxy’s share would quickly be confronted - possibly in person - either by a proxy withholding tax revenue (i.e. revolting), by a threat to do so, or at least by an expression of discontent on the proxy’s part that served as a warning. This type of interaction can be viewed as implicit bargaining, and perhaps the state even bargained with the proxies explicitly. I broadly assume that this feedback allowed the state to gauge the proxies’ perception of their relative power as well as their risk attitude, and consequently to set near-optimal proxy’s shares. The process was likely one of learning through trial and error on the state’s part, which yielded political intuition and successfully guaranteed the proxies’ compliance. Therefore, optimal determination of the proxy’s share appears to be a reasonable approximation of the state’s behavior.

A feedback loop such as this, which was immediate in historical terms, did not exist with respect to the level of state expenditure. The meager feedback that did exist would have looked like this: suppose the state failed to cut back on its expenses in response to a reduction in its net income - for instance a reduction in net income due to an increase in a certain proxy’s share. Such a failure by the state to respond would not be unusual behavior even for a modern government today, let alone for an ancient one. In terms of the model this would constitute an increase in $\alpha$ for that period (recall once again that $\alpha = E/NI$). After the next harvest season, when tax revenue was transferred to the royal court, the state’s resources would have dwindled, placing it in an inferior position with respect to the proxies (if not after the following harvest then cumulatively, after a longer stretch of time). The proxies, with or without knowledge of the state’s exact fiscal condition, could then drive a harder bargain and subsequently up their proxy’s share.

It seems unlikely that any one person would make the connection between the state’s initial inaction and the eventual, implicitly harder bargain driven by proxies. In fact, picking up on such a causal link was practically impossible given the cumulative nature of the process, which occurred over several generations, and so I assume that it was never made. In light of this, it does not appear that optimal control over $\alpha$ incorporating perfect knowledge of the model would be a reasonable approximation of the state’s behavior.

When state expenditure was eventually and insufficiently curbed, it was likely in a general
attempt to economize and does not indicate any perception of the causal link between excess spending at one point in time and dwindled net income later on. Kemp (2006) addresses this matter:  

> “In revenue and expenditure terms the sum of [the pious foundations’] activities plus a general level of royal expenditure on court life, on large and thus long-term building programmes, and on the military, represented a general ‘budget’ or balance-sheet for the country. It was probably never seen in quite so abstract a way. But complaints from below of insufficient resources would have signalled to senior officials a degree of imbalance, which they could then have sought to correct.”

It is likely that signals of shortage and a call for economization arose in a very simple manner, when the stocks in the state’s granaries were reported.

Given the following quote, also from Kemp (2006), the inference of the causal link by the Egyptian state seems all the more unlikely:  

> “Within any one channel the procedures [of the Egyptian government] could be remarkably effective (though not efficient) in achieving a given target, such as quarry, transport, and erect a colossus of a particular size. This is where bureaucratic talents flourished. But we will look in vain for evidence of conscious integration of the individual parts into a general scheme of management.”

## 6 Conclusion

The prolonged stability of the Early Dynastic Period and the Old Kingdom came to an end when local administrators and the affiliated provincial elite grew too powerful for the royal court to credibly threaten them militarily, rendering the state powerless. This outcome was the culmination of a long and gradual process, rooted in the relationship between the royal court and its tax-collecting proxies, who directly or indirectly comprised the provincial elite. By making the minimal assumption of a positive relation between the two parties’ ratio of resources on the one hand and the outcome of a military conflict between them on the other, I characterize the dynamics of the balance of power. The formal model yields that in the short run the state could not deviate from its policy by reducing the flow of resources from its treasury to the provincial elite, because doing so would fail to guarantee the proxies’ compliance with their role. This result fills a gap in the existing literature, which does not provide a robust explanation why the Pharaohs did not exercise their power to immediately halt or even slow the flow of resources to the provincial elite.

In the long term, the state could potentially have prevented its fall by carefully manipulating its expenses, however the nature of the dynamics governing the balance of power

\[44\] See Kemp (2006), page 236.

\[45\] See Kemp (2006), page 235.
was essentially unobservable to the royal court. Failing to understand the link between state expenditure and the balance of power, the royal court failed to maintain a stable ratio of resources with its proxies. Thereby, an ever increasing flow of resources from the state to the provincial elite was required to guarantee the latter’s cooperation with the state, and eventually eroded the royal court’s power beyond the threshold of making a credible military threat. A critical issue here is distinguishing between policy decisions that can be modeled by optimizing choices and decisions that cannot.

The analysis presented in section 4 is specific to Ancient Egypt and to the era in question, however the model to which it applies is more general, and raises the possibility of wider applicability. The relationship between central government and tax-collecting proxies modeled here is a general one and may be widely applicable to ancient periods as well as to more recent ones.

References


Appendix

1 A risk-neutral proxy as a simplifying assumption

Suppose the proxy is strictly risk-averse, so his current-period utility function, \( u(\cdot) \), is continuous, strictly concave and strictly increasing. By (3), (4) and Jensen’s inequality:

\[
P_i(\cdot) \cdot u[\tau Y_i] \leq u[P_i(\cdot) \cdot \tau Y_i]
\]

Because \( u(\cdot) \) is strictly increasing, there exists a function \( \tilde{P}_i : \mathbb{R} \to [0, 1] \) referred to as the \textit{risk-adjusted fighting function}, s.t.:

\[
P_i(\cdot) \cdot u[\tau Y_i] \equiv u[\tilde{P}_i(\cdot) \cdot \tau Y_i]
\]

Note that when \( P(\cdot) = 1 \) then \( \tilde{P}(\cdot) = 1 \) and recall that \( u(0) = 0 \) by normalization, so that when \( P(\cdot) = 0 \) then \( \tilde{P}(\cdot) = 0 \). Otherwise \( \tilde{P}(\cdot) \in (0, P(\cdot)) \). Because equation (A-1) is an equivalence it will hold for the derivatives of both sides, yielding:

\[
\partial \frac{\partial}{\partial (\cdot)} [u(\tilde{P}_i(\cdot) \cdot Y_i)] \equiv \partial \frac{\partial}{\partial (\cdot)} [P_i(\cdot) \cdot u(Y_i)]
\]

\[
\Rightarrow \quad \tilde{P}_i'(\cdot) = P_i'(\cdot) \cdot \frac{u(Y_i)}{\tau Y_i \cdot u'(\tilde{P}_i(\cdot) \cdot Y_i)} > 0 \quad \forall (\cdot)
\]

where the strict inequality follows from (4) and from the characterization and normalization of \( u(\cdot) \). Thus, \( \tilde{P}_i(\cdot) \) is strictly increasing.

2 A myopic proxy as a simplifying assumption

Suppose the proxy is not myopic, but that all other assumptions made in the main text remain applicable. In particular, assume the proxy’s inter-temporal utility function is of the “standard”, time-consistent form:

\[
U_{i,t} = \sum_{s=0}^{\infty} \beta^s u_{i,t+s}; \quad \beta \in [0, 1)
\]

Every period the proxy maximizes \( E(U_{i,t}) \) over the policy set \( \{comply_t, revolt_t\} \). It is useful to express the proxy’s optimal policy in terms of a value function, \( V \):

\[
\max\{E(U_{i,t})\} \equiv V(M_{i,t}, Y_i) \equiv \max\{V_{comply_t}, V_{revolt_t}\}
\]

where:

\[
V_{comply_t} \equiv u(M_{i,t} \tau Y_i) + \beta V(M_{i,t+1} | comply_t, Y_i)
\]

\[
V_{revolt_t} \equiv E[u((P_{i,t} | M_{i,t-1}) \cdot \tau Y_i)] + \beta [P^- \cdot U + P^+ \cdot U]
\]

where:

\[
P^- \equiv \text{Prob}(P_{i,t} | M_{i,t-1} = 0) = \text{Prob}(\text{Revolt fails});
\]

\[
P^+ \equiv \text{Prob}(P_{i,t} | M_{i,t-1} > 0) = \text{Prob}(\text{Revolt succeeds});
\]

\[46\]Note that the additive form of \( V_{revolt_t} \) below does not indicate risk neutrality, because \( V(\cdot, \cdot) \) can be adapted to incorporate risk-aversion similarly to \( P(\cdot) \), as shown in appendix section 1.

22
\( U \) is the proxy’s long-run utility if he is crushed by the state and \( \bar{U} \) is his long-run utility if he successfully revolts. Naturally, I assume \( U < \bar{U} \). It is also reasonable to assume that \( \bar{U} \) depends positively on the amount of resources the proxy retains after revolting, seeing as he may need to confront the state militarily later on. The last assumption, however, is inconsequential and so I omit it in what follows.

In every period the state provides the proxy with the smallest share, \( M_{i,t} \), such that the proxy chooses to comply (assume that when indifferent, the proxy complies), as long as the proxy’s share is below the critical level, \( M \). Hence, in every such period \( M_{i,t} \) is set so that:

\[
V(M_{i,t}, Y_i) = V_{comply} = V_{revolt}
\]

Note from equation (A-3) that \( V_{revolt} \) does not depend on \( M_{i,t} \). This independence occurs because once the proxy chooses to revolt in period \( t \), his prospects thereafter are not affected by the counterfactual proxy’s share that he would have received had he chosen to comply.

Now, in order to simplify the situation, I assume that changes over time in the proxy’s share were very gradual if a proxy did not revolt, sufficiently so for them to be negligible within the proxy’s foreseeable horizon. This implies:

\[
M_{i,t+1|comply} \simeq M_{i,t}
\]

Equations (A-2) and (A-4) now yield:

\[
V_{comply} \simeq \frac{u(M_{i,t} \tau Y_i)}{1 - \beta}
\]

Substituting equation (A-5) into (A-2) and using these to re-write condition (A-4) gives:

\[
\frac{u(M_{i,t} \tau Y_i)}{1 - \beta} \simeq E[u((P_{i,t}|M_{i,t-1}) \cdot \tau Y_i)] + \beta \cdot [P^- \cdot \bar{U} + P^+ \cdot \bar{U}]
\]

Finally, applying the simplifying assumption of a risk-neutral proxy and re-arranging yields an equation for the dynamic path of \( M_{i,t} \) when proxies are non-myopic:

\[
M_{i,t} \simeq (1 - \beta) \cdot (P_{i,t}|M_{i,t-1}) + \beta \cdot \frac{(1 - \beta)}{u(\tau Y_i)} \cdot [P^- \cdot \bar{U} + P^+ \cdot \bar{U}]
\]

In light of equations (4) and (6), \( P_{i,t} \) is increasing in \( M_{i,t-1} \). Therefore, given the dynamic path, equation (A-6) and the definitions of \( P^- \) and \( P^+ \), \( M_{i,t+1} \) is unambiguously increasing in \( M_{i,t} \) when the two are below \( M \). This state of affairs is qualitatively similar to that when proxies are myopic.