Re-Interpreting Burdett and Mortensen: Equilibrium Wage Dispersion with Balanced Matching

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Abstract

A known shortcoming of Burdett and Mortensen’s celebrated result of pure equilibrium wage dispersion is that it hinges upon the assumption of random matching. This paper presents a modified version of their model, in which pure equilibrium wage dispersion arises under a form of balanced matching. Rather than modeling workers and firms, the model addresses workers and jobs. Matching is assumed to be random with respect to jobs, which amounts to balanced matching with respect to firms, when firm size is measured in terms of jobs. The bizarre implication of random matching with firms, whereby splitting a firm in two increases its recruitment rate, is eliminated. In addition to increasing employment, speeding up recruitment and reducing worker turnover, higher wages in the modified model also reduce the rate of job vacancy.

1 Introduction

The fundamental result in Burdett and Mortensen’s celebrated model is that even in the absence of worker or firm heterogeneity, search frictions alone give rise to equilibrium wage dispersion.1 In a nutshell, the mechanics of worker flows are such that firms posting higher wages recruit workers more frequently and are quit upon less often. This state of affairs results in high wage firms retaining a larger steady-state workforce, thereby giving rise to an iso-profit wage-workforce schedule along which firms can disperse.2 An immediate implication is that firms posting higher wages employ more labor. This consequence is especially important because it means that firms face upward sloping labor supply curves, i.e. they

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1Burdett and Mortensen (1998).

2That firms necessarily disperse along an iso-profit wage-workforce schedule in equilibrium is shown in the most memorable step of Burdett and Mortensen’s proof. Manning (2011) questions how accurately this step captures the real process leading to dispersion of the wage offer distribution, because it relies on workers switching jobs for the sake of infinitesimal gains.
have monopsony power. This insight theoretically underpins the competitive monopsony literature.\footnote{Ashenfelter et al. (2010) and Manning (2011) provide brief and extensive overviews of this literature, respectively, and Manning (2003) is the definitive volume on the topic.}

The devil, of course, is in the details. Burdett and Mortensen assume that workers randomly search among employers and assign each firm equal odds of being encountered, irrespective of firm size. This regime is known as random matching (with firms), and it generates some troubling implications. Burdett and Vishwanath (1988) point out that random matching with firms has the bizarre implication that by spitting itself in two, a firm can increase its recruitment rate. Less abstractly, Kuhn (2004) lets the reader judge random matching with firms based on the implication that “regardless of its size, every firm - from the local bakery to Microsoft - receives the same absolute number of job applications per period.”\footnote{Kuhn (2004), page 375.} At the opposite end of the spectrum is the alternative of balanced matching (with firms), in which a firm is encountered with probability proportional to its workforce. Burdett and Vishwanath (1988), who coined the terms for these two matching regimes, were also the first to show that, under balanced matching, equilibrium wage dispersion degenerates into a single mass point at the competitive wage.\footnote{The breakdown of equilibrium wage dispersion under balanced matching also arises in Mortensen and Vishwanath (1991), Manning (1993) and Mortensen and Vishwanath (1994). This result is reviewed in section 10.3 of Manning (2003).} Balanced matching with firms eliminates pure equilibrium wage dispersion.

The relative importance of random versus balanced matching with firms in actual labor markets is crucial in determining the amount of monopsony power firms can muster, and hence also in determining the empirical relevance of competitive monopsony. For this reason Manning (2003) discusses the matter carefully.\footnote{See sections 10.3 and 10.4 of Manning (2003).} He argues that some methods of recruitment, such as using existing employees’ word-of-mouth, are more suggestive of balanced matching with firms than are other methods, such as the use of public employment services or advertisement. He then provides empirical evidence that methods tending less strongly towards balanced matching with firms are quite commonly used, which is compelling because it suggests that at the very least a complete absence of monopsony power is unlikely. Nevertheless, it remains easy to argue that just about any recruitment method sends more job seekers to the Microsofts of the world than to local bakeries, raising doubt with respect to the extent of firms’ monopsony power. The ground on which competitive monopsony has been standing is shaky.

This paper attempts to secure competitive monopsony on a more solid foundation. A modified version of Burdett and Mortensen’s model is proposed, in which pure equilibrium wage dispersion arises under a form of balanced matching with firms. Instead of workers and firms seeking each other out, workers in the model seek jobs and firms seek workers to fill jobs. I assume random matching between workers and jobs (not firms!), with two consequences: first, doing so allows me to formulate a model that follows the lines of Burdett and Mortensen’s original very closely. In particular, the modified model retains the key result
of pure equilibrium wage dispersion and it continues to imply that employers face an upward sloping labor supply curve, indicating monopsony power. Second, random matching with jobs implies that the rate at which a firm is contacted is proportional to its measure of jobs, which amounts to a form of balanced matching with firms. Firms wield monopsony power even when matching is balanced.

Wording the statement that random matching with jobs amounts to a form of balanced matching marks a nuance in the way balanced matching is defined here. Burdett and Vishwanath (1988) define balanced matching as holding when the probability that a worker contacts a given firm, conditional on contacting some firm, equals the number employed by that firm divided by the total employed labor force. Random matching with jobs implies a form of balanced matching with firms that is closely related, but distinct. This form of balanced matching holds when the probability that a worker contacts a given firm, conditional on contacting some firm, equals the number of jobs at that firm divided by the total number of jobs in the labor market. Thus, I define balanced matching with firms in terms of the number of jobs at the firms, rather than the number of workers they employ.

Clearly, the these definitions of balanced matching refer to different statistics. It is also clear, however, that both definitions are consistent with the casual observation that larger firms receive greater numbers of applications per period than small firms do. In this respect both definitions are similarly apt. Unless one insists that the matching regime is balanced precisely with respect to the number of workers employed and not the number of jobs at a firm, the argument that Microsoft attracts more job seekers than the local bakery no longer poses a challenge. Moreover, when matching is balanced in terms of jobs then a firm’s recruitment rate is not affected by the act of splitting the firm, so the bizarre implication pointed out by Burdett and Vishwanath (1988) is eliminated.

The proposed model closely resembles Burdett and Mortensen’s original. The labor market comprises two continuums, one of workers and one of jobs. The mechanics of worker flows are such that jobs associated with higher posted wages recruit workers more frequently, and are quit upon less often. This state of affairs results in high wage jobs maintaining lower steady-state vacancy rates, thereby giving rise to an iso-profit wage-vacancy rate schedule along which firms can disperse their jobs.

Some appealing results carry over from the original model. Raising wages speeds up recruitment. By increasing the wage posted for a vacant job, an employer increases the probability that an encountered worker will forego his prior job in favor of its offer. Raising wages reduces worker turnover, too. By increasing the wage posted for an occupied job, an employer reduces the probability of the incumbent worker quitting in favor of a higher paying alternative.

Note that both results are stated in terms of jobs, not firms. A firm in the model is no more than an arbitrary grouping of jobs. That any two jobs belong to the same firm does not prevent that firm from posting different wages for them, because the model is essentially blind as to which firm a job belongs to.\footnote{The model makes no prediction with respect to within-firm wage distributions, but it is worth noting that wage-posting models in which it is assumed that firms post a single wage for all employees (such as Burdett} Whether a firm in the model encompasses one job
or seventeen remains arbitrary, too, and this matter is worth dwelling upon.

Most wage-posting models implicitly assume that in equilibrium, steady-state firm size is determined by labor supply alone, or equivalently that all firms have excess demand for labor.\(^8\) This implicit assumption stems from the simplifying assumption that (conditional on worker and firm characteristics) the marginal revenue product of labor is fixed. Because firms post wages below the marginal revenue product of labor, their labor demand at the posted wages is infinite, leaving finite labor supply to determine firm size. In this paper it is assumed that firm size, measured by the number of jobs available in a firm, is determined solely by labor demand. Specifically, it is assumed that the marginal revenue product of labor in each firm is fixed until an arbitrary firm-specific level of employment is reached, after which it drops to zero. That a firm’s labor demand is satisfied after employment hits a firm-specific cap reflects, in crude approximation fashion, the role of non-labor inputs as well as conditions in the firm’s output market, both of which are taken to be exogenous. That each firm’s measure of jobs is exogenous means that the model makes no prediction with respect to firm size (by either measure thereof - jobs or employment).

Even though firm size is undetermined in the model, firms still face an upward-sloping labor supply. To see this note that while fixing the number of jobs in the firm, raising wages reduces the firm’s steady-state vacancy rate. As employment is simply the measure of occupied or non-vacant jobs, raising wages must increase the firm’s steady-state number of employees, facing the firm with upward-sloping labor supply. This point is crucial because it is what tells us that even with balanced matching, firms wield monopsony power.

The model is presented in the next section, followed by a brief conclusion.

2 Model

2.1 Setup:

The labor market consists of a continuum of workers with measure \(m\) and a continuum of jobs with measure normalized to 1. Certain mutually exclusive sets of jobs comprise firms, and firms maximize the steady-state flow of profits from their constituent jobs. No single firm encompasses sufficient jobs to exceed measure 0, and without loss of generality it is assumed that each firm is comprised of a single job. Workers can be employed or unemployed and

\(^8\)This is the state of affairs, for example, in Burdett and Mortensen (1998), as well as Mortensen (1998), Mortensen (1999), Bontemps et al. (2000), Postel-Vinay and Robin (2002), Burdett and Coles (2003) and Cahuc et al. (2006), as well as in most of the models presented in Manning (2003). Exceptions to this rule are the wage-posting models containing some degree of balanced sampling: Burdett and Vishwanath (1988), Mortensen and Vishwanath (1991), Manning (1993) and Mortensen and Vishwanath (1994). In these papers decreasing marginal returns to labor are assumed and the size of some (or all) firms is determined by labor demand.
jobs can be occupied or vacant.\textsuperscript{9}

All jobs are identical outside of a fixed wage flow, \( w \), which is posted by the employer at the onset. The wage-offer distribution is denoted \( F \). The flow utility of an employed worker is his wage \( w \), and the flow utility of an unemployed worker is normalized to 0.\textsuperscript{10} All workers are identical, they discount the future exponentially at rate \( r \), and they yield their employer a revenue product flow of \( p \in (0, \infty) \) when employed in any available job.

Workers conduct on-the-job search, encountering jobs according to a Poisson arrival process with intensity \( \lambda \). I focus on the steady-state only, and for simplicity I assume that the arrival intensity is independent of a worker’s employment status.\textsuperscript{11} Upon encountering a job, workers discover its wage, which is a draw from \( F \). If the job is vacant, it is offered to the worker, who decides whether or not to accept it based on an acceptance rule, \( A : \mathbb{R}^2 \to \{0, 1\} \), whose arguments are the offered wage and the worker’s current wage. Job-worker matches separate exogenously at a positive rate, \( \delta \), and it is assumed that \( \lambda/\delta \in (0, \infty) \).

Whereas the wage-offer distribution \( F \) is the distribution of wages among jobs, the distinct distribution of wages among employed workers is denoted by \( G \). The relationship between these two distributions is simple: the number of workers employed at any wage is equal to the number of non-vacant jobs offering that wage. The following identity formally characterizes their relationship, and is the key assumption distinguishing the model from Burdett and Mortensen’s original.

\[
m(1 - u(t|A,F)) G(w,t) \equiv F(w,t) - \int_{-\infty}^{w} v(\tilde{w},t|A,F) dF(\tilde{w}) \quad \forall w.
\]  

Here, \( u(t|A,F) \) is the unemployment rate at time \( t \) (conditional on the behavior of workers and firms, as summarized by \( A \) and \( F \)), and the LHS is the measure at that time of all employed workers earning a wage less than or equal to \( w \). The RHS is the measure at time \( t \) of all non-vacant jobs offering a wage less than or equal to \( w \). In particular, \( F(w,t) \) is the measure of all jobs - vacant or not - offering wages less than or equal to \( w \) at time \( t \), and the integral on the RHS is the contemporary measure of all vacant jobs offering wages less than or equal to \( w \). \( v(w,t|A,F) \) denotes the share of vacant jobs among jobs offering wage \( w \) at time \( t \), given that workers follow an acceptance rule \( A \) and that firm post wages distributed \( F \).

“Non-vacant jobs” and “occupied jobs” are synonymous, reflected by

\[
o(w,t|A,F) \equiv 1 - v(w,t|A,F),
\]  

\textsuperscript{9}The notion of a vacant job may be intuitive, but it is elusive to define and its use in an empirical context poses a challenge - see e.g. the discussion in section 10.1 of Manning (2003). Nevertheless, vacancies are common currency in labor economics: they are a key ingredient in matching models along the lines of Mortensen and Pissarides (1994), and in light of Mortensen (1998) one could argue that they are implicit in Burdett and Mortensen (1998) as well.

\textsuperscript{10}Implicitly, the wage offered by a job is the pecuniary equivalent of the flow utility from all of the job’s characteristics, whether these characteristics are pecuniary or not. This point is inconsequential in theory, but requires careful attention in any empirical context.

\textsuperscript{11}The consequences of this simplification are addressed in a subsequent footnote, below.
where \( o(w,t|A,F) \) is the occupancy rate of jobs offering wage \( w \) at time \( t \), given \( A \) and \( F \). Substituting occupancy for non-vacancy in the identity (1) using (2) yields

\[
m(1 - u(t|A,F)) G(w,t) \equiv \int_{-\infty}^{w} o(\tilde{w},t|A,F) dF(\tilde{w}) \quad \forall w
\]

as an equivalent form.\(^{12}\) Plugging \( w \to \infty \) into (1) and (3) yields

\[
m(1 - u(t|A,F)) = 1 - v(t|A,F) \equiv o(t|A,F),
\]

where \( v(t|A,F) \equiv \int v(w,t|A,F) dF(w) \) and \( o(t|A,F) \equiv \int o(w,t|A,F) dF(w) \) are the vacancy and occupancy rates for all jobs.

The setup described comprises a wage-posting game. Each worker’s strategy space is the set of acceptance rules and his (instantaneous) payoff at every instant is his value flow. Each firm’s strategy space is \( \mathbb{R} \), from which it selects a wage, and its payoff is the steady-state flow of profits from its constituent job. An equilibrium of the game consists of an acceptance rule for each worker and a posted wage for each job. As in Burdett and Mortensen’s original model the game has a unique equilibrium, in which all workers apply a common simple acceptance rule and firms post wages along a continuous, non-degenerate distribution.

2.2 Preliminaries:

2.2.1 Acceptance Rule:

A worker’s steady-state value of employment at wage \( w \) is\(^{13}\)

\[
J(w) = \frac{1}{r + \lambda + \delta} \left[ w + \lambda \int v(\tilde{w}|A,F) \max\{J(w),J(\tilde{w})\} dF(\tilde{w}) + \delta J(0) \right],
\]

where the first bracketed term reflects the instantaneous utility flow from wage, the second the option value of switching to a higher paying job, and the third the possibility of exogenous separation. Note that encountering a higher paying job does not imply switching to one. When a worker encounters a non-vacant job he discovers its posted wage, but is only offered the job if it is vacant - hence the role of \( v(\tilde{w}|A,F) \) in the expression. As all workers are identical, firms have nothing to gain by replacing the incumbent worker with a fresh hire.

The steady-state value of unemployment, \( J^0 \), is

\(^{12}\)The occupancy rate of jobs offering wage \( w \) plays a similar role in this model to that of firm size (measured by employment) of firms offering wage \( w \) in Burdett and Mortensen’s original (\( l(w|R,F) \) therein, where workers’ reservation wage \( R \) comprises their acceptance rule).

\(^{13}\)To derive this equation, define \( J^0 \equiv E_T[e^{-rT} \int v(\tilde{w}|A,F) \max\{J^0,J^1(\tilde{w})\} dF(\tilde{w})] \), where \( T \) is the time elapsed until the next job offer arrives and is exponentially distributed with intensity \( \lambda \) (i.e. the density of \( T \) is \( \lambda e^{-\lambda T} \)) and \( \tilde{w} \) is the wage drawn from \( F \) when the first job offer arrives. \( E_T \) denotes an expectation with respect to the random variable \( T \). A useful primer on deriving Bellman equations of this type and on Poisson processes can be found in the appendices of Zenou (2009).
\[ J^0 \equiv J(0) = \frac{1}{r + \lambda} \int v(\tilde{w}|A,F) \max\{J(0), J(\tilde{w})\} dF(\tilde{w}). \tag{6} \]

Whereas the \( J(w) \) is strictly increasing in \( w \), \( J^0 \) is independent of \( w \), so 0 is the unique reservation wage such that \( J(w) > J^0 \) if and only if \( w > 0 \).\(^{14}\) The acceptance rule selected by all workers is thus

\[ A(\tilde{w}|w) = 1 \{ \tilde{w} > \max\{w,0\} \}, \]

i.e. “accept any job with a positive wage if unemployed, and accept any job associated with a wage increase otherwise.” For notational brevity, conditioning on workers’ acceptance rule \( A \) is omitted in what follows.\(^{15}\)

2.2.2 Steady-State Unemployment and Vacancy Rates:

The steady-state unemployment, vacancy and occupancy rates are fully determined by the exogenous parameters \( m, \lambda \) and \( \delta \). To see this, note that the steady-state flows in and out of unemployment are \( m(1-u) \cdot \delta \) and \( mu \cdot \lambda v \), respectively, where the conditioning of \( u \) and \( v \) on \( F \) has already been omitted. Equating the two and using \( m(1-u) = 1-v \) from (4) yields

\[ \frac{1-u}{u} = \frac{\lambda}{\delta} (1 - m(1-u)) \tag{7} \]

That the LHS is decreasing in \( u \) and the RHS is increasing in it implies the existence of a unique steady-state unemployment rate \( u \), which depends only on \( m, \lambda \) and \( \delta \), and (4) implies the same for the steady-state vacancy and occupancy rates, \( v \) and \( o \).

Note that while this argument renders \( v \) and \( o \) independent of \( F \), the steady-state vacancy and occupancy rates for jobs offering a specified wage \( (v(w|F) \) and \( o(w|F) \)) may still depend on \( F \).

2.3 Steady-State Job Occupancy:

Given an initial allocation of workers to firms and their derived acceptance rule, the number of employed workers receiving a wage no greater than \( w \) at time \( t \), \( m(1-u(t)) G(w,t) \), can

\(^{14}\)That \( J(w) \) is increasing in \( w \) is intuitive, as it raises the instantaneous flow utility without affecting the worker’s opportunities, and it is often taken as a standard result without proof. Showing it formally is not straightforward, but can be achieved - for example - by applying proposition 3 of Smith and McCardle (2002) to the property of monotonicity.

\(^{15}\)That workers’ reservation wage is not greater than 0 is driven purely by the simplifying assumption that employed and unemployed workers face the same job offer arrival rate. In contrast, Burdett and Mortensen (1998) find that the reservation wage, \( R \), is such that \( R > b \geq 0 \), where \( b \) is the non-negative flow value of unemployment. This discrepancy arises from their assumption that job opportunities arrive more frequently during unemployment whereas I have assumed a rate of arrival which is independent of state. In their model workers refuse an offer if the wage does not compensate for the reduced rate of the future flow of offers, whereas here accepting a job implies no such reduction, so any job offer with positive wage is accepted.
be calculated. Its time derivative can be written as
\[
\frac{d}{dt}(m(1-u(t))G(w, t)) = \lambda \int_0^w v(\hat{w}|F)dF(\hat{w})mu(t) - \left[\delta + \lambda \int_w^\infty v(\hat{w}|F)dF(\hat{w})\right]m(1-u(t))G(w, t).
\]

Consequently, the steady state distribution of wages earned by employed workers is
\[
G(w) = \frac{kv_0 w}{1 + kv_w} \cdot \frac{u}{1-u},
\]
(8)

where \( k \equiv \lambda/\delta \) and \( v_a^b \equiv \int_a^b v(\hat{w}|F)dF(\hat{w}) \) is a convenient abbreviated notation.

The steady state occupancy rate of jobs offering wage \( w \) can be expressed as
\[
o(w|F) = \lim_{\epsilon \rightarrow 0} \frac{G(w) - G(w - \epsilon)}{F(w) - F(w - \epsilon)} m(1-u).
\]
(9)

Following Burdett and Mortensen (1998), define the fraction, or mass, of jobs offering wage \( w \) as
\[
\nu(w) \equiv F(w) - F(w^-) = \lim_{\epsilon \rightarrow 0} F(w) - F(w - \epsilon),
\]
and note that together with \( u = (1 + kv)^{-1} \) from (4), (8) implies\(^{16}\)
\[
\lim_{\epsilon \rightarrow 0} G(w) - G(w - \epsilon) = \frac{uk}{1-u} \left( \frac{v_0 w}{1 + kv_w} - \frac{v_0 w^-}{1 + kv_w^-} \right) = \frac{k\nu(w)v(w|F)}{(1-u)(1+kv_w)(1+kv_w^-)}.
\]
(10)

Substituting the above into (9) and re-arranging yields
\[
o(w|F) = \frac{mk}{(1+kv_w)(1+kv_w^-) + mk} \in (0, 1),
\]
(11)

Combined with (10), equation (11) implies that \( G \) is continuous if and only \( F \) is continuous (i.e. \( \nu(w) = 0 \quad \forall w \)).\(^{17}\)

\( o(w|R, F) \) is the steady state occupancy rate of jobs offering wage \( w \), and (11) implies the following properties:

1. \( o(w|R, F) \) is increasing in \( w \).

\(^{16}\)The second equality is derived by obtaining a common denominator as follows \( \frac{v_0 w}{1+kv_w} - \frac{v_0 w^-}{1+kv_w^-} = \frac{v_0 (1+kv_w^-) - v_0 w^- (1+kv_w^-)}{(1+kv_w)(1+kv_w^-)} \) and applying the identity \( ab - cd = (a-c)b - c(d-b) \) to yield that \( v_0 w (1+kv_w^-) - v_0 w^- (1+kv_w^-) = (v_0 w - v_0 w^-)(1+kv_w^-) - v_0 w^- k(v_w^- - v_w^-) = \nu(w)v(w|F)(1+kv_w(1+kv_w^-)) = \nu(w)v(w|F)(1+kv), \) where \( v \equiv v_0 w $.\(^{17}\)To see this, note first that if \( F \) is continuous then \( \nu(w) = 0 \) for all \( w \), so by (10) \( G \) must be continuous. In the other direction, if \( G(w) \) is continuous then the RHS of (10) must equal zero, but because \( k \in (0, \infty) \), \( v(w|R, F) \equiv 1 - o(w|R, F) \in (0, 1) \) and the denominator is necessarily finite \( u \in (0, 1), k \in (0, \infty) \) and \( v_a^b \in (0, 1) \quad \forall a, b \) this implies that \( \nu(w) = 0 \), so \( F \) must be continuous.
2. \( o(w|R,F) \) is strictly increasing in \( w \) on the support of \( F \) and is a constant on any connected interval off the support of \( F \).

3. \( o(w|R,F) \) is continuous if and only if \( F \) is continuous (and hence \( G \), too).\(^{18}\)

### 2.4 Equilibrium Wage Dispersion:

For each job, firms post a wage so as to maximize their steady-state flow of profits from that job,

\[
\pi = \max_w (p - w) \cdot o(w|F). \tag{12}
\]

In equilibrium \( F \) must be such that

\[
(p - w) \cdot o(w|F) = \pi \text{ for all } w \text{ on support of } F
\]

\[
(p - w) \cdot o(w|F) \leq \pi \text{ otherwise.}
\]

Denote the infimum and supremum of the support of an equilibrium \( F \) (supposing that one exists) by \( w^- \) and \( w^+ \). Note that no employer will offer a wage \( w < 0 \) because he would have a permanent vacancy which is costly to maintain, so I consider only \( w \geq 0 \).

The following argument rules out continuous wage offer distributions, and is worded as closely as possible to Burdett and Mortensen’s original text. \( o(w|F) \) is discontinuous at \( w = \hat{w} \) if and only if \( \hat{w} \) is a mass point of \( F \) and \( \hat{w} \geq 0 \). This implies that any employer offering a wage slightly greater than \( \hat{w} \), a mass point where \( 0 \leq \hat{w} < p \), has a significantly larger steady state occupancy rate for the job and only a slightly smaller profit per unit of occupancy\(^{19}\) than an employer offering \( \hat{w} \), as \( (p - w) \) is continuous in \( w \).\(^{20}\) Hence, any wage just above \( \hat{w} \) yields a greater profit. If there were a mass of \( F \) at \( \hat{w} \geq p \), all jobs offering such a wage yield non-positive profit. However, any job offering a wage slightly lower than \( p \) yields a strictly positive profit as it maintains a positive steady state occupancy rate. In short, offering a wage equal to a mass point \( \hat{w} \) cannot be profit maximizing in the sense of (12).

As non-continuous offer distributions have been ruled out, (11) implies that for \( w \),

\[
o(w|F) = \frac{mk}{(1 + kv)^2 + mk} \in (0,1) \tag{14}
\]

where the equality follows from \( v_w^\infty = v \) by the definition of \( w \). Thus \( o(w|F) \) depends only on the parameters \( m, \lambda \) and \( \delta \) (as do \( u \) and \( v \)) and is independent of \( F \). Note that the

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\(^{18}\)The link between the continuity of \( v^0_w \) and that of \( F \) and \( G \) is provided by (1), which can be re-written in steady-state using the \( v^0_w \) notation as \( v^w_w = F(w) - m(1 - u)G(w) \).

\(^{19}\)For concreteness, “a unit of occupancy” can be measured, e.g., in terms of days that a job is non-vacant per year.

\(^{20}\)Algebraically, if \( \hat{w}^+ \equiv \lim_{\epsilon \to 0} \hat{w} + \epsilon \) then \( (p - \hat{w}^+) \cdot o(\hat{w}^+|F) - (p - \hat{w}) \cdot o(\hat{w}|F) = (p - \hat{w}) \cdot (o(\hat{w}^+|F) - o(\hat{w}^+|F)) - (\hat{w}^+ - \hat{w}) \cdot o(\hat{w}^+|F) \geq 0 \) for \( \hat{w} < p \), because \( o(\hat{w}^+|F) - o(\hat{w}^+|F) \geq 0 \) if \( \hat{w} \) is a mass point, whereas \( \hat{w}^+ - \hat{w} \to 0 \) and \( o(\hat{w}^+|F) \geq 0 \).
occupancy rate at wage }w = w_0\text{ is independent of }w_0, \text{ too, implying that the lowest paying job in the market will yields the maximum profit flow if and only if }w = 0. 

In equilibrium, every offer must yield the same steady state profit, which equals

$$\pi = p \cdot \frac{mk}{(1 + kv)^2 + mk} = (p - w) \cdot o(w|F)$$

(15)

for all }w\text{ on the support of }F, \text{ yielding }^{21}

$$o(w|F) = \frac{p}{p - w} \cdot \frac{mk}{(1 + kv)^2 + mk}. \tag{16}$$

Equating (11) with (16) and re-arranging yields

$$\frac{p}{p - w} = \frac{(1 + kv)^2 + k}{(1 + kv^\infty)^2 + k} \in (0, 1), \tag{17}$$

which, given the parameters }\lambda, \delta \text{ and }m, \text{ uniquely determines }v_\infty^w \equiv 1 - v_0^w \text{ for all }w\text{ on the support of }F \text{ (recall from section (2.2.2) that both }u \text{ and }v \text{ are fully determined by these three parameters, too). Substituting (8) into (1) yields

$$F(w) = v_0^w \frac{1 + k(mu + v_\infty)}{1 + kv_\infty^w}, \tag{18}$$

implying that }F\text{ is unique}. \text{ }^{22}

As }v_\infty^w \text{ must equal }0, \text{ it follows from (17) that

$$p - w = p \cdot \frac{1 + k}{(1 + kv)^2 + k} > 0. \tag{19}$$

As the occupancy rate and profits per unit of occupancy are both positive for a firm offering wage }w, \text{ and equilibrium profit flows are equal for all firms offering wage on the support of }F, \text{ it must be that }\pi > 0 \text{ for all firms}. \text{ }^{23}

Finally, to complete the proof that the acceptance rule obtained in section (2.2.1) coupled with the wage offer distribution }F\text{ constitute the unique equilibrium of the wage posting game, it must be shown that no wage off of the support of }F \text{ yields an employer a profit flow greater than }\pi. \text{ A job offering wage }w < 0 \text{ will have a vacancy rate of }1, \text{ and so yields zero profits. A job offering wage }w > \bar{w} \text{ has the same vacancy rate as a job offering }\bar{w}, \text{ because }v(w|F) \text{ is constant on any connected interval off the support of }F, \text{ but has a lower profit flow because }w > \bar{w}.

\text{ }^{21}(16) \text{ implies that }o(w|F) \equiv o(w) \text{ and }v(w|F) \equiv v(w) \text{, i.e. that the occupancy and vacancy rates at each wage level }w \text{ do not depend on }F, \text{ but only on the exogenous parameters }m, \lambda, \delta \text{ and }p. \text{ However, this does not imply that their integral }v_0^w \text{ (over strictly less than the complete support of }F) \text{ is independent of }F, \text{ because }v_0^w \equiv \int_0^w v(\tilde{w}|R)dF(\tilde{w}). \text{ }^{22}That }F(\bar{w}) = 1 \text{ and }v_\infty^\bar{w} = 0 \text{ implies that }v = (1 + mku)^{-1}. \text{ }^{23}In line with footnote 6 of Burdett and Mortensen (1998), at this stage one can endogenize the measure of firms by assuming the existence of a positive fixed cost }c > 0 \text{ and invoking free entry of firms, so that

$$\pi = p \cdot \frac{mk}{(1 + kv)^2 + mk} - c = 0.$$
3 Conclusion

A known shortcoming of Burdett and Mortensen’s result of pure equilibrium wage dispersion is that it hinges upon random matching, whereby workers are equally likely to encounter any firm, regardless of its size. The casual observation that larger employers attract more job applicants than small ones contrasts starkly with random matching. On the other hand it is well known that with balanced matching, whereby a worker’s probability of encountering a firm is proportional to its employment, equilibrium wage dispersion degenerates to a single mass at the competitive wage. The unrealistic nature of random matching with firms, coupled with the lack of wage dispersion under balanced matching, casts doubt on the empirical relevance of Burdett and Mortensen’s result. The stakes are raised further by the fact that Burdett and Mortensen’s result provides the theoretical underpinning for the competitive monopsony literature.

This paper presents a modification of Burdett and Mortensen’s model, in which the key result of pure equilibrium wage dispersion holds with balanced matching. The matching technology is balanced in the sense that workers encounter every firm with probability that is proportional to the number of jobs at the firm (but not with employment at the firm). The key insight is that instead of addressing the matching of workers and firms, a variation of the original model can be used to address the matching of workers and jobs, and that random matching with jobs amounts to a form of balanced matching with firms. In particular, random matching with jobs implies that a worker’s probability of encountering a firm is proportional to the number of jobs at that firm (which differs from the number of employees). The upshot, however, is that the casual observation that larger employers attract more job applicants than small ones is not at odds with balanced matching in terms of jobs, so this observation no longer challenges the empirical relevance of Burdett and Mortensen’s result. This paper removes an otherwise persistent source of doubt in the empirical relevance of pure equilibrium wage dispersion, and by doing so provides a more solid theoretical foundation for the competitive monopsony literature.

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