Frege's Critical Arguments for Axioms

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Abstract

Why does Frege claim that logical axioms are 'self-evident,' to be recognized as true 'independently of other truths,' and then offer *arguments* for those axioms? I argue that he thinks the arguments provide us with the *justification* that we need for accepting the axioms, and that this is compatible with his remarks about self-evidence. This compatibility depends on philosophical considerations connected with the 'critical method': an interesting approach to the justification of axioms endorsed by leading philosophers at the time.

1 The Puzzle about Frege's Arguments

There is something puzzling about Frege's introduction of his 'Basic Laws' or 'axioms' of logic. On the one hand, he thinks that 'it is part of the concept of an axiom that it can be recognized as true independently of other truths,'[1] and he offers a regress argument to show that there must be truths with that feature:

> The grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all.[2]

Axioms, whose justification rests on this 'something else,' are called 'self-evident' and 'unprovable.'[3]

On the other hand, in the *Basic Laws of Arithmetic*, Frege introduces his axioms by offering arguments for them. For example, Basic Law I is introduced as follows:

[1] Frege 1899–1906, 168.

[2] Frege 1879–1891a, 3.

[3] E.g., at Frege 1884b, §5.

> $\Gamma \to (\Delta \to \Gamma)$ would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; therefore $a \to (b \to a)$.[4]

What follows the 'therefore' is Basic Law I itself. To appreciate this argument apparently requires us to recognize other truths: a truth about the conditions under which $\Gamma \to (\Delta \to \Gamma)$ would be the False, and the truth that these conditions cannot obtain. It does not matter for now what these truths are, though we will consider the details later. What matters is that this argument invites us to accept the axiom on the basis of *some* other truths, while according to Frege's regress argument, our being justified in accepting anything at all depends on our being justified in accepting axioms independently of *any* other truths. For Basic Laws IIa, IIb, III, and IV, Frege also gives clearly articulated arguments with distinguishable steps and at least one 'therefore' on the way to the final statement of the law, while he simply states that Basic Law VI 'follows' from something already discussed and gives a similarly brief argument for Basic Law V.

The puzzle is: why does he offer these arguments? In the following section, I argue that he offers them to provide the justification he thinks we need to accept the axioms, and that this is consistent with the rest of what he says about axioms. In the third section, I argue that this will make good historical and philosophical sense if—and perhaps only if—Frege's arguments for axioms fit a particular pattern, centering in a certain way around *cognitive goals*. In the fourth section, I argue that Frege's arguments do fit the relevant pattern.

2 What are Frege's Arguments For?

A standard purpose for giving an argument is to provide justification for accepting its conclusion to those who accept the premises. But given Frege's regress argument and claims of self-evidence, commentators on Frege usually suppose that he must have some *alternative purpose* for giving his arguments for axioms, and attempted to find such a purpose.

Perhaps the most well-known of these attempts is that of Tyler Burge. Burge thinks that when Frege says that the axioms are 'self-evident,' this means that 'understanding the content of an axiom suffices to warrant one in believing it.'[5] Once we fully and

[4] Frege 1893a, §18, translating Frege's 'also' in Frege 1893b as the standard 'therefore' rather than the Ebert and Rossberg's 'accordingly.' See §§18, 20, and 25 for the other six arguments.

[5] Burge 1998, 338. Many other readers have hit upon similar ideas about Frege's talk of 'self-evidence'. (E.g.: Jeshion 2001, 944: 'clearly grasping them is a sufficient and compelling basis for recognizing their truth.' Weiner 2005, 341: 'All we need, in order to see that the primitive laws…are true, is to understand the *Begriffsschrift* statements of these laws.')

clearly understand the sentences that express the axioms, we are, just on the basis of that understanding, justified in accepting those axioms. This suggests a purpose that the arguments could usefully serve: 'the point of the arguments is to articulate the content of the axioms and to elicit a firm understanding of them.'[6] The arguments can do this because 'understanding a thought requires understanding its inferential connections to other thoughts.' Hence, 'Frege's arguments "for" his axioms elicit understanding of the axioms by bringing out these connections.'[7] On this reading, by showing how the axioms follow from other truths, the arguments can help us understand the axioms because part of understanding a truth is understanding such relations.

Let us grant that an argument could help someone to understand an axiom along these lines, and that Frege could offer the arguments for that purpose without violating his claims above about the justification of axioms.[8] We grant this in order to ask: why should we think that *is* the purpose of Frege's arguments? Burge does not identify any passages in which Frege *says* that this is their purpose. Instead, he points to a parallel between the *Basic Laws* and an earlier work of Frege's, in which a version of the argument above for the axiom $a \to (b \to a)$ is sandwiched between two translations of that axiom from symbols into natural language. According to this earlier passage, the axiom

> says "The case in which *a* is denied, *b* is affirmed, and *a* is affirmed is excluded." This is evident, since *a* cannot at the same time be denied and affirmed. We can also express the judgment in words thus, "If a proposition *a* holds, then it also holds in case an arbitrary proposition *b* holds."[9]

Having quoted this passage, Burge immediately concludes: 'Frege clearly regards his argument as an elaboration of what is contained as evident in the axiom itself. It is

[6]Burge 1998, 338. Again, others have had the same idea. (E.g. Ricketts 1998, 177: 'These arguments...are to bring an audience who is just learning *Begriffsschrift* to grasp and recognize as true the thoughts expressed by the axioms.' Weiner 2002, 168: 'Frege is...enabling his readers to understand what is meant by these expressions.' Sometimes this is put in technical Fregean terms: the arguments help us 'grasp the sense' ['Sinn'] of the expressions stating the laws. Thus Bar-Elli 2010 thinks the arguments are 'explicating the modes of presentation of...the things [the axioms] are about,' 'presenting [the axioms] as expressing features of the *Sinne*.')

[7]Burge 1998, 337. Burge also more tentatively suggests that some of Frege's arguments might have been intended to help us understand simply by stating the axioms in different ways: 'there is some circumstantial evidence for thinking that [it] might have been Frege's view of those arguments,' that they are not 'moving from one truth to another, but simply thinking through and expressing in different ways, via ordinary language, the character of the axiom which is the apparent conclusion of the reasoning.' (335).

[8]This is delicate. What is involved in 'understanding [a thought's] inferential connections to other thoughts'? If it involves *recognizing the truth* that a certain thought implies another, then it seems that our justification for accepting axioms will not be independent of our recognition of *any other truths*.

[9]Frege 1879a, §14. (Following the translation Burge uses.)

an elaboration of an understanding of the thought...He sees himself as articulating in argument form what is contained in the very content of the basic truth he is arguing for,' which makes it likely that in the *Basic Laws*, 'the basic procedure is the same.' That is Burge's reason for reading the *Basic Laws* passage the way he does.[10]

But what Burge says about the earlier passage looks like mere assertion. Why should we read the argument as serving this purpose in the *earlier* passage either? Again, Frege seems to appeal in support of his axiom to a truth: that *a* cannot be at the same time affirmed and denied. Again, this truth seems to be distinct from the axiom itself, which (among other things) has a *b* in it. Again, Frege does not *say* that he gives this argument to help us understand the axiom. Aside from the shift from Frege's late-period talk of 'the True' and 'the False' to his early-period terms 'affirmed' and 'denied,' the main difference between the *Basic Laws* passage and this earlier one is the presence of the translations of the axiom into ordinary language. I grant that the point of the translations is to help us understand the axiom, but I see no reason to think that the distinct claim which apparently supports the axiom serves the same purpose. What makes this point especially clear is that in the following pages, Frege sometimes accompanies the official *proof* of a theorem—which is certainly there to provide justification for accepting the conclusion—with two translations into words.[11]

At most, then, Burge's discussion identifies something useful that the arguments *could* do, consistent with Frege's regress argument and claims of self-evidence. Since there is a genuine puzzle about what the arguments' purpose is, identifying such a possibility is a helpful interpretive contribution. But if there are other possibilities that are also consistent with what Frege says, we would need some reason to prefer this one in particular. Other interpreters have suggested purposes for the argument that seem equally possible. (Richard Kimberly Heck, for example, suggests that when Frege gives an argument for an axiom, the purpose is 'to uncover the source of its truth, to demonstrate that it is indeed a law of logic': the argument shows that the truth of the law has the kind of 'source'

[10]Burge 1998, 328-329. Other authors sometimes claim that the unfamiliarity of Frege's artificial language makes it especially likely that he would employ this device to help readers understand the axioms. (See, for example, the Ricketts passage in note 6 above.) But it is worth recalling that at the point in the *Basic Laws* when the axioms appear, Frege's notation and connectives would no longer be unfamiliar. The *Begriffsschrift* conditional, for example, had been heavily employed on every one of the nineteen pages preceding the discussion of the axioms. Moreover, Frege's standard method for helping us understand unfamiliar or especially difficult formulae is the one he employs in the earlier passage: giving straightforward translations into natural language. Throughout §12 and §13, for example, he gives several translations 'in words' of *Begriffsschrift* sentences. (e.g.: 'In words: -1 is a square root of 1 and fourth root of 1.')

[11]For example, in §18 of Frege 1879b, the proof of proposition 36 from proposition 1 (an axiom) and proposition 34 is accompanied by the following two translations of proposition 36: 'The case in which *b* is denied, $\neg a$ is affirmed, and *a* is affirmed does not occur. We can express this as follows: "If *a* occurs, then one of the two, *a* or *b*, takes place".'

appropriate to a law of logic.)[12] In fact, Frege could have had more than one reason for offering these arguments, so these proposals need not even compete with one another.

I think, however, that there is a very general reason why seeking alternative purposes along any of these lines will not enable us to understand Frege's arguments. The reason is that Frege seems to *tell* us that the purpose of the arguments is to justify the axioms in a certain way. Here is how he distinguishes, later in the *Basic Laws*, what he himself has done (which he calls 'significant' arithmetic) from 'formal' arithmetic, which treats the modes of inference and axioms of an arithmetical system like the rules in a game:

> in Formal arithmetic we spare ourselves any account of why we lay down the rules in exactly this way...We do not extract these rules from the referents of the signs; rather we lay them down on our own absolute authority...acknowledging no necessity to justify these rules.[13]

Given the intended contrast, Frege's point is that in significant arithmetic, we *do* acknowledge a necessity to justify axioms and modes of inference by providing some kind of 'account' that 'extracts' them from the referents of the signs. Apparently, then, he thinks he has done that. But the only passages in the *Basic Laws* that could provide such an account for the axioms are the arguments with which they are introduced—which means that he thinks that the arguments justify us in accepting the axioms. If Frege's is to be a significant arithmetic, such justification is needed. We will not, then, understand Frege's arguments for axioms until we understand not just how they serve *some* important purpose, but how they provide *justification* along the lines just described.[14] Frege

[12]Heck 2012, section 2.1. Other readers similarly take the arguments to show or explain something *about* the axioms. Jason Stanley, for example, has suggested that only by providing arguments like Frege's 'can the arithmetician take himself as accounting for the application of his formal theory of arithmetic to the special sciences,' (Stanley 1996, 61.) and the chapter 'Metatheory' in Blanchette 2012 proceeds along similar lines. These authors sometimes read the claims that make up the arguments as meta-linguistic, which is ultimately not admissible, as Weiner 2005 §§2-4 points out. But the metalinguistic reading of the claims that make up the argument is inessential: even if what the arguments *say* is object-level, their *purpose* can be to make a meta-linguistic point. (The purpose of an object-level identity statement 'This is Bill' might be to show us something about the name 'Bill'.)

[13]Frege 1903a, §§89, 94. In discussing formal arithmetic, Frege refers to both axioms and modes of inference as 'rules;' axioms would be rules for what we may write down anytime. (See Heck 2012, 42 (fn36) for this point.)

[14]Recognizing the importance of this passage, Heck and Stanley call Frege's arguments for axioms 'justifications,' but remark that it is 'unclear what it would be to "justify" the laws of logic,' questioning what 'sense' of justification could be in play here. (E.g., Heck 2012 28-29, Stanley 1996, 60.) The alternative purposes for the arguments that they suggest might then qualify as an unusual sort of 'justification', which avoids conflict with Frege's regress argument. But there is no indication in the passage that Frege is introducing a new sense of 'justify'; the only reason to hypothesize one is if we do not see how Frege's claims

may, of course, think his arguments are useful in other ways, but one reason he has for giving them is to justify us in accepting the axioms, and this reason will be sufficient on its own to explain why he gives them. The real task, then, is to understand how justifying axioms by arguments is consistent with the rest of what Frege says.[15]

Let us begin by looking at the key move of Frege's regress argument:

> The grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes.[16]

Frege's claim is about what is required if we are to 'recognize' any truths at all. By this term—'erkennen' or 'anerkennen'—he must mean not just *accepting* things as true, but being *justified* in doing so—otherwise, how could a regress of justification threaten our ability to 'recognize' truths? (Frege often uses 'recognize' in this epistemically loaded way. For example, when he says that axioms 'can be recognized as true independently of other truths,' he does not mean merely that we can accept axioms as true, but that we can be justified in doing so.) This means that when Frege says that 'this' cannot be the only form that justification takes, he is talking about cases in which our justification for accepting a truth is found in other truths that we are already *justified* in accepting.

The conclusion of the regress argument, then, is that our justification for accepting axioms cannot depend on our recognition—justified acceptance—of other truths. This leaves open that the justification for accepting axioms could come from other truths that

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could otherwise be reasonable and consistent with the rest of the things he says about the justification of axioms.

[15]It is worth noting that if Frege meant to rule out justifying axioms by argument, this may put him in conflict with Kant's account of geometry. For Kant, axioms are claims that are 'immediately certain,' and 'indemonstrable,' as well as 'general.' (Kant 1781/1787, A164/B205.) The question is whether he thinks that 'immediately certain' and 'indemonstrable' claims are to be justified by argument, and it seems that he does: as Hintikka argues, for Kant, ' "immediate" and "indemonstrable" did not serve to distinguish immediate perception from an articulated argument, but to distinguish a certain sub-class of particularly straightforward arguments from other kinds of proofs.' (Hintikka 1967, Section 7.) These 'particularly straightforward' arguments are ones that involve every stage of a traditional Euclidean proof except for the *apodeixis*, in which appeal is made to previously established truths: axioms or theorems. Frege presumably does not mean to rule out Kant's view of geometrical axioms, since he endorses it or takes it seriously at Frege 1874 56-7, Frege 1884b §13 and §89, and Frege 1903b, 273—though he always refrains from discussing the justification of axioms in any detail. (E.g. Frege 1903b, 273: 'Here we shall not go into the question of what might justify our taking these axioms to be true.')

[16]The original reads: 'Die Gründe nun, welche die Anerkennung einer Wahrheit rechtfertigen, liegen oft in anderen schon anerkannten Wahrheiten. Wenn aber überhaupt Wahrheiten von uns erkannt werden, so kann dies nicht die einzige Art der Rechtfertigung sein. Es muss Urteile geben, deren Rechtfertigung auf etwas anderem beruht, wenn sie überhaupt einer solchen bedürfen.' (Frege 1879–1891b, 3.)

we accept *without justification.* The rest of what Frege says about the justification of axioms leaves this possibility open, too. His claim that axioms are 'unprovable,' for example, means that they cannot be justified by a *proof*, which he explains elsewhere to be an argument beginning with already 'known theorems, axioms, postulates, or definitions.'[17] This leaves open that axioms might be justified by an argument that does *not* begin from such things. Again, when he says that axioms 'can be recognized as true independently of other truths,' he must mean other *recognized* truths. (He cannot mean other truths simply *being* true, and the only word it would make sense to elide is the one already used: 'recognized.') The claim of 'self-evidence' in turn, presumably means that what makes axioms evident does not depend on the *evidence* that any other truths have. All of this leaves open that the justification for accepting axioms could come from other truths which are not yet known, recognized, or evident: truths we accept without justification.

Let us call an argument 'epistemically transmuting' if it justifies us in accepting its conclusion partly through our accepting claims—such as its premises—that we are not yet justified in accepting.[18] In that case, the premises have a different epistemic status than the conclusion; the status of the premises is not preserved, but changed by the argument. My claim is that Frege thinks that logical axioms are to be justified by epistemically transmuting arguments and that his own arguments are of this kind: that is why he gives them. My main reason for this claim is that it follows from everything Frege says about the justification of axioms, when understood as above. To review: given his remarks about the difference between formal and significant arithmetic, the arguments must justify the axioms; but given his regress argument and the rest of what he says about the justification of axioms, these arguments would have to justify us in accepting an axiom without our being justified in accepting their premises. This makes his arguments epistemically transmuting. This is the only reading that fits with everything Frege says about the axioms, read in the most straightforward way.

Of course, there could be some major problem that makes this reading, overall, unacceptable. I see two possibilities:

1. The first is substantive. Epistemic transmutation can seem a strange idea. Suppose that in fact, the idea makes no sense at all, or that it makes no sense that Frege's arguments for axioms should be epistemically transmuting. In that case, Frege's taking his arguments to be epistemically transmuting would be a rather bizarre error. Given ordinary constraints of interpretive charity, that would be a major problem for the reading. It will, then, be a strong objection to this reading

[17] See Frege 1914a, 204-206 for his technical use of 'proof.'

[18] The alchemical overtones here are deliberate. See Section 3 below, especially note 39.

if there is no good reason to think that Frege's arguments really are epistemically transmuting.

2. Even if there is good reason to think that Frege's arguments are epistemically transmuting, there could still be a serious historical objection to this reading. Suppose the reason to think that Frege's arguments are epistemically transmuting—or perhaps the whole idea of epistemically transmuting arguments—was unheard of in Frege's own time. In that case, for him to offer such arguments would be a major epistemological insight and a bold innovation. But then the fact that he does not describe what he is doing in any detail, while accompanying what he does with traditional language like 'self-evidence', becomes astonishing: it would virtually guarantee that no one in Frege's audience would understand him. It would, then, be a strong objection to this reading if epistemically transmuting arguments for axioms, the particular reason to think that Frege's arguments are epistemically transmuting, or the associated use of traditional language were unknown at the time. Indeed, there will be a good objection along these lines unless all of these things were *fairly well-known* at the time, since only then could Frege reasonably expect anyone to understand what he is doing without his explaining it.

The rest of this paper addresses these objections, in two steps. The following section argues that for Frege to suppose that axioms could be justified by epistemically transmuting argument would not be an innovation, because the foremost philosophers of Frege's time claimed, in popular works that Frege read, that a certain kind of epistemically transmuting argument was a viable approach—even the *only* viable approach—to the justification of logical axioms. They also employed traditional language to describe things justified by such arguments. Moreover, the key ideas behind this approach are reasonable ones. This means that to fully meet both objections, one must only show that Frege's arguments conform to this particular approach: if so, he is pursuing a reasonable method with which he could reasonably expect his readers to be familiar. In Section 4, I argue that they do so conform.

In addition to defending my reading from objections, this discussion actually produces an additional argument for it. Not only does my main claim follow from things Frege says when read in the most straightforward way, but it also is able to overcome the historical objection, and I expect no other reading can. We readers face the interpretive puzzle about Frege's arguments because Frege does not describe in any detail what the point of his arguments is or how his axioms are justified. It is not, then, in *itself* a strong objection to any reading that Frege does not say that he is doing what the reading says he is. But a historical objection arises for any reading which implies that Frege is doing something with which his audience would not be familiar, since in that case he could not

expect anyone to understand what he is doing unless he told them. And to my knowledge, the only idea about what arguments for axioms can do that was widespread at the time was that of an epistemically transmuting argument along the lines I describe.[19]

3 The Critical Method

Neo-Kantians on the Justification of Axioms

It is often observed that there are close philosophical connections between Frege and Neo-Kantian philosophers.[20] This is no surprise: as Frederick Beiser puts it, 'Neo-Kantianism was the predominant philosophical movement in Germany in the final decades of the 19th century.'[21] Philosophers in this movement endorsed an approach to the justification of logical axioms that came to be called the 'critical method.'

We find such an endorsement in Hermann Lotze's *Logic*, which had a tremendous influence in Germany and which Frege himself carefully read.[22] Lotze endorses two approaches to the 'justification'—also called the establishment of the 'validity'—of axioms, both of which lend axioms 'immediate certainty'.[23] The first is a version of a traditional

[19]This is also why other readings cannot embrace my main claim as any kind of friendly emendation to their own readings. As noted in note 14 above, Heck is interested in taking Frege's arguments to provide some kind of justification, and as noted in note 8, it is a delicate question whether Burge's reading can *avoid* an appeal to epistemically transmuting arguments. These authors might, then, hope to abandon their claims that these arguments do not justify (in the ordinary sense of 'justify'), and embrace my resolution of the puzzle in terms of epistemically transmuting arguments, while nonetheless insisting that the point of Frege's arguments is the one they originally identified: that it is because his arguments do *those* things that they are epistemically transmuting, not because they do what I describe. But these readings will be unable to overcome the historical objection described, and perhaps the substantial objection too. Even if Frege's arguments can do what Heck and Burge suggest, and even if those are important things to do, I see no reason why it would make these arguments epistemically transmuting—and even if it would, I see no way that Frege could reasonably expect readers to understand that, since it would connect with no tradition familiar to them. (Burge-style readings can be linked up with a forward-looking Wittgensteinian tradition, and Heck-style readings with a forward-looking Tarskian one—see, e.g., Weiner 2001, and Heck 2012, 27-28. But neither is a tradition of offering epistemically transmuting arguments, and Frege's audience in 1893 had not read Wittgenstein or Tarski.)

[20]See, e.g., Sluga 1984, Sullivan 1991, Gabriel 2002, Schlotter and Wehmeier 2013, and Glock 2015.

[21]Beiser 2014, 1.

[22]The discussion is mainly in §§64-65 of Lotze 1874, which was 'perhaps the most widely read logic text in Germany during Frege's early career,' (Heis 2013, 122.) Dummett 1981 was the first to show that Frege had read and taken notes on the book. Additional work by Sluga 1984 and Hovens 1997 shows that he was probably reading the 1880 edition in 1882. Frege also claims Lotze as an influence and attended his lectures in Göttingen. (See Gabriel 2002.)

[23]Lotze's terms are 'Berechtigung', 'Geltung'/'Gültigkeit', and 'unmittelbare Gewissheit.'

appeal to 'logical intuition', though like much of Lotze's book, it sounds alarmingly psychological: we are justified in accepting claims which 'we feel immediately to be necessary, and the opposite we feel with equal conviction to be impossible in thought.' Lotze thinks that the logical axiom of identity, '$A = A$', can be justified in this way. He identifies no role for arguing or reasoning in this approach. The second approach is quite different: when Lotze comes to consider a different axiom,[24] he gives an argument designed 'to show that an extension of our knowledge is possible *if* there is [such] a principle.' Lotze's argument thus establishes a connection between a purpose or goal of thought on the one hand—the extension of our knowledge—and the axiom which serves those purposes on the other. He thinks that the argument establishing this connection justifies the axiom by showing that it 'serves the purposes of thought.'[25] Lotze claims, then, that there are two sources of justification and immediate certainty for logical axioms, one of which involves giving an argument connecting the axiom to a cognitive purpose or goal.

In the increasingly strict anti-psychologistic purge that characterized Neo-Kantian philosophy in the last decades of the century and which we recognize in Frege's writings,[26] Lotze's goal-based approach to justification came to be seen by prominent figures as the *only* legitimate way to justify logical axioms. I focus here on the account provided by Lotze's student Wilhelm Windelband, who along with Hermann Cohen was one of the two central figures in the dominant Neo-Kantian movement when Frege was writing: as Beiser puts it, 'In the 1870s and 1880s, Cohen and Windelband seemed to be working together in shaping a new conception of epistemology...Windelband's and Cohen's views on these issues became the new orthodoxy of the 1880s and 1890s.'[27] There is good reason to think that Frege was familiar with Cohen's and Windelband's work in particular,[28]

[24]The axiom is a version of a principle of sufficient reason which he represents as '$A + B = C$': i.e., for every C, there is some A and B that together are the ground or reason for it. Lotze credits Herbart with 'having brought within the ken of formal logic' a principle 'so prominent in all scientific practice.'

[25]Lotze thinks it also matters that the axiom is confirmed 'by the concentrated impression of all experience,' but such inductive confirmation is not enough to justify the axiom without the connection to a purpose.

[26]Edgar 2008: 'In the 1870s and 1880s...anti-psychologism...went from the margins of German philosophy to become commonplace, and by the 1890s it had hardened into an orthodoxy.' (54)) Beiser 2014 describes the 'climate of thought' that starts to take hold in the 1870s: 'Psychologism...had now become passé.' Frege would have expected widespread agreement among mainstream Neo-Kantians when he claims that we must 'separate sharply the psychological from the logical' and prevent 'the ruinous incursion of psychology into logic.' (Frege 1884b, Introduction; Frege 1893a, Foreword.)

[27]Beiser 2014, 492.

[28]The connection to Cohen is very direct: Frege 1885b is a review of Cohen 1883. Evidence given by Sluga 1997 and Gabriel 2002 that Frege read or had close second-hand knowledge of Windelband includes the observation that Frege seems to have adopted from him the technical term 'truth-value.' ('*Wahrheitswert.*') Glock 2015, like Picardi 1987 183, points out that this latter could be a co-incidence—after all, Frege's logic

and in any case, he was surely aware of the dominant movement's orthodox position on the topics he wrote about.

In his most sustained treatment of axioms, published ten years before the first volume of Frege's *Basic Laws*, Windelband also identifies the issue through a regress argument: 'As our proving cannot go back infinitely, it must have an absolute beginning, and this must be sought in [that] which cannot be proved. Everything provable is mediately certain; the final premises of all proof are immediately certain.'[29] Like Frege, Windelband identifies the axioms of logic as among the absolute starting points, raising the question how we are to acquire immediate certainty for them. Crediting Lotze with insight into the problem, he argues that the 'purposes of thought' approach, which he dubs the 'critical method', is the *only* viable option: to justify axioms, we must 'show...that their validity must be recognized, if certain purposes are to be accomplished.'[30] Thus, 'the recognition of axioms is always conditioned through a goal, that as the ideal... must be presupposed.'[31] When it comes to logical axioms, the relevant purposes are purposes of *thought*, or *cognitive* goals. In connection with the cognitive goal of truth, for example, the critical method justifies an axiom like this: 'Logic can say to everyone: you want truth? Then remember, you must recognize the validity of these [axioms], if your desire is ever to be fulfilled.'[32]

On this view, the only way to justify an axiom is to show that there is a connection between that axiom and a cognitive goal. But the reasoning that shows the connection must proceed from *unjustified* claims, since the axioms themselves are the 'absolute beginning' of justification: whatever claims about goals lie behind them must be unjustified. That makes the argument by which the goal is connected with the axiom epistemically transmuting: it *yields* justification, though we are not already justified in accepting the

'was based on a generalization of mathematical function-theory, and the idea of a function is that of a mapping of arguments onto values.' But Frege surely does not call them 'truth-values' simply because they are the values of certain functions for certain arguments. For one thing, they are equally the *arguments* of certain functions that yield certain values, and he does not call them 'truth-arguments.' For another, *everything* is the value of certain functions for certain arguments, and he does not call everything a '-value.' I would add that there are textual echoes of Windelband in Frege's writings. For example: the (unpublished) Frege 1879–1891a (2) claims that the causes of our judgements take place in accordance with psychological laws, which 'are just as capable of leading to error as of leading to truth'; Windelband 1882, 47 had put the same point for the same reason in the same way, including the somewhat odd opposition between truth and error rather than the expected opposition between truth and falsity. ('kann...ebensosehr zum Irrtum wie zur Wahrheit führen' versus Frege 1879–1891b: 'können ebenso wohl zum Irrtum wie zur Wahrheit führen.')

[29]Windelband 1883, 322

[30]Windelband 1883, 328.

[31]Windelband 1883, 331

[32]Windelband 1883, 330.

claims we accept along the way. (Accordingly, Windelband never describes the statements about the goal as justified, certain, or valid; they are only 'presuppositions'. He refers to the kind of reasoning that connects axioms with goals as 'argument' which 'shows' something, but never 'proof' which 'proves' anything, claiming that 'it belongs to the concept of an axiom…to be unprovable'. Axioms justified by the critical method, he calls 'self-evident' and 'immediately evident.')[33]

Though Lotze and Windelband differ on some details, and though it admits of clarification and elaboration in many ways, the basic idea behind the Lotze-Windelband critical method for the justification of axioms is clear enough: if, starting from unjustified presuppositions about a goal, we can establish an inferential connection between an axiom (or our acceptance of it) and the achievability of the goal, that justifies us in accepting the axiom.[34] Moreover, they ascribe to axioms justified in this way the traditional status of 'self-evident' and 'immediately certain'. Since we argue for them, we clearly depend on other claims for the certainty, evidence, and justification of axioms; but we do not depend on the certainty, evidence, or justification of these other claims.

Is This a Reasonable Method?

A full discussion of the epistemological credentials of the critical method is beyond the scope of this paper, but it is important for our purposes that it is not an *obviously* unreasonable approach to the justification of logical axioms. (That will make it available as an interpretation of Frege, within the bounds set by charity.) We can see this by identifying the main claims which lie behind it, bringing out why they might strike us as obviously unreasonable and sketching available responses.

There are two basic ideas that lie behind the critical method:

1. Cognitive goals can be a source of justification.

[33]We engage in 'Argumentation' which can 'aufweisen' and 'zeigen'; it is never said to be 'Beweis' or able to 'beweisen.' Axioms are 'unbeweisbar' and 'selbstverständlich', possessing 'unmittelbare Evidenz.' (Windelband 1883, 326-328, and 330).

[34]There are important variations across Windelband's and Lotze's formulations. For one: Lotze seems to require us to show that our purposes can be reached if what the axiom says is *actually so*; Windelband, if we *recognize the axiom's validity.* Second, the inferential connection seems to go the other way around: while Lotze requires our arguments 'to show that [our purposes can be reached] *if* there is [such] a principle,' Windelband requires us to 'show...that [axioms'] validity must be recognized, if certain purposes are to be accomplished.' And finally: Lotze seems to require us to show only that the axiom 'serves the purposes of thought', perhaps *better* enabling us to reach an open-ended goal like the extension of knowledge; Windelband seems to require us to show that the axiom 'must be recognized' if the relevant purpose is to be achievable at all.

2. Claims can be justified by epistemically transmuting arguments (i.e. arguments that require us to accept things without justification.)

The obvious concern with the first claim is that unless something bears on what is true, it cannot be relevant to justification—but whether or not something is a cognitive goal of ours has nothing to do with what is actually true. Two responses are worth mentioning.

1. One might deny that only what bears on what is true can be a source of justification. Thus, in a recent defence of the idea that cognitive goals can be a source of justification, David Enoch and Joshua Schechter claim that 'epistemic justification should not be taken to be solely concerned with truth'; rather, it is 'closely related to the notions of epistemic responsibility and blameworthiness...to the question of whether [one] is being a responsible believer in holding the relevant belief.'[35] In particular, Enoch and Schechter claim that some cognitive goals are 'rationally required' and that this bears directly on responsible believing, making such goals relevant to justification.[36]

2. One might accept that sources of justification must bear on what is true, but insist that cognitive goals do. Picking up on Enoch and Schechter's notion of a 'rationally required' cognitive goal, one could hold that *whatever must be accepted in order to reach a 'rationally required' goal is true.* This might seem to be what Barry Stroud calls 'the familiar but hollow arrogance of idealism: things must be a certain way because we thinkers must think things are that way.'[37] Importantly, however, if the claims we must *accept* in order to reach the goal are also claims that must be *true* if the goal is to be reached, then their truth follows from the claim that the relevant goals can be achieved: that the world is such to enable us to reach the relevant goal. Such 'idealism'—also known as 'optimism'—does not obviously require any claims of mind-dependence or the metaphysical priority of the mental or the evaluative over anything else. Moreover: to affirm that *cognitive* goals can be achieved is to affirm that the world meets a certain standard of intelligibility. Such a claim, then, is closely related to a version of the Principle of Sufficient Reason.[38]

[35]Enoch and Schechter 2008, footnote 25 and p551.

[36]In a little more detail: they hold that we are justified in accepting claims that result from any 'belief-forming method' which 'is such that it is possible to successfully engage in a rationally required project by employing it, and such that it is impossible to successfully engage in the project if the method is ineffective'; this yields justification because the project being rationally required ensures that 'employing such a method [is] epistemically responsible.' (Enoch and Schechter 2008, 554-555.)

[37]Stroud 2011, 143.

[38]For recent defences of such a principle which do not involve such claims of dependence or priority, see

The second main claim of the critical method is that we can gain justification by epistemically transmuting arguments. The obvious concern here is that epistemically relevant features and statuses are always transmitted *from* whatever we accept in making an argument, *to* the conclusions of those arguments, which would preclude acquiring justification through arguments that require us to accept things without justification.[39]

The reply is to deny the relevant claim about the transmission of epistemically relevant features and statuses. In fact, the apparently straightforward ideas about transmission that underlie this objection are problematic: as Crispin Wright points out, the idea 'that a recognized valid deduction...always *transmits*...epistemic credentials to the conclusion...is generally recognized as open to counterexamples.'[40] This is enough to deal with what seems *obviously* problematic. The correct transmission principles may yet rule out epistemically transmuting arguments, but the discussions surrounding these counterexamples are complex and ongoing. (Wright, incidentally, questions these transmission principles on behalf of a form of epistemically transmuting argument: because he denies that 'any acquired warrant is no stronger than the weakest of one's independently acquired sets of grounds for each of its (ancestral) presuppositions,' he commits to 'jettisoning...forms of closure principle' that imply otherwise.)[41]

Della Rocca 2010 and Dasgupta 2016. Some philosophers central to the Neo-Kantian tradition did embrace the relevant dependence claims, while others were reluctant to do so. Lotze, for example, claims to be 'certain of being on the right track, when I seek in that which should be the ground of that which is.' (Lotze 1879, Volume III, Conclusion.) For contrast, see the chapter on Kuno Fischer in Beiser 2014.

[39]See, e.g., McGlynn 2017 (91): 'How can we hope to build a stable edifice...on foundations which seem to be less...secure than what they're tasked to support?' This 'leaching' problem is that if arguments *merely* preserve epistemic features, a lack of justification in the assumptions implies a lack of justification the conclusion. The idea that arguments *always* preserve epistemic status also leads to an 'alchemy' problem: 'if...one [can] acquire justification for...beliefs founded on [an unjustified claim], it seems to thereby enable one to very easily acquire justification for [that claim] itself [since] some of one's justified beliefs will clearly entail [it]...But...it should not be nearly this easy.' (McGlynn 2014, 174).

[40]Wright 2012, (451). The examples discussed in Dretske 1970 are also often cited in this connection, and Wright cites 'Moore's Proof, McKinsey's paradox, and Putnam's Proof that we are not Brains-in-a-Vat.'. For some of the back-and-forth about transmission principles from Wright's perspective, see Wright 2004a, 207-209, Wright 2012, and section 4 of Wright 2014.

[41]Wright 2004a, 191; Wright 2014, 229. In a little more detail: Wright aims to defend (against regress worries) the 'conservative' claim that to gain (or at least to claim) justification by basic logical inference, we must have 'warrant' for accepting 'the soundness of the relevant [logical] principles of inference.' (Wright 2014, 215-216.) He introduces a notion of 'entitlement' such that 'both entitlements and...justifications [are] types of warrant,' allowing the conservative to 'concede...that we may indeed have no...*justified belief* in [the relevant claims]...but counter[] that we may nevertheless be rationally entitled to accept' them. (Wright 2004a, 204, 177.) This allows for 'a form of conservatism that holds...that...warrant for [the claims] is...required...[and] it is conferred by epistemic entitlement'. (Wright 2014, 222). The arguments employing such inferences would be epistemically transmuting: they justify us in accepting their conclusions, even though their doing so requires us to accept unjustified claims. Proponents of the critical method presum-

It is, then, not unreasonable to think that logical axioms could be justified via the critical method, and so not uncharitable to suppose that any philosopher thinks they can. With that in mind, it is time to return to Frege.

4 Frege's Critical Arguments for Logical Axioms

Here is the story so far. First, we saw that it is not obvious why Frege gives arguments for axioms. Then, we saw that what Frege says about the subject implies that he sees these arguments as epistemically transmuting: they justify us in accepting the axioms, though we are not justified in accepting their premises. Substantial and historical concerns arose: would it make philosophical and historical sense for Frege to see his arguments this way? We began to address these questions by seeing that if a mainstream, philosophically informed reader at the time would expect anything at all from a philosophically responsible logician putting forward an axiom, it would be a particular kind of argument: one which would serve to connect the axiom with a cognitive goal. Though the premises of any such argument could only be unjustified presuppositions, such *critical arguments* were, not unreasonably, thought to be epistemically transmuting, yielding justification for accepting the axioms. So if Frege's arguments fit this description, this will fully address both the substantial and historical concerns. In what remains, I argue that they do.

Normativity and the Logic of the *Basic Laws*

A glance at Frege's arguments makes it clear that they do not mention any cognitive goals. This might seem to establish that he is not doing what the critical method requires. To see why this would be a mistake, it helps to recall something central to Frege's conception of logic.

A glance at the logical laws that appear in Frege's book makes it clear that they do not mention judgements, acts, or thinkers. This might seem to reveal Frege's answer to the question, much debated at the time, whether or not logic is fundamentally *normative*, its task to prescribe how we *ought to think*.[42] A logic book full of laws which mention

ably agree that their unjustified premises have *some* status like Wright's 'entitlement,' because of their relation to a goal. Wright's view, in fact, comes very close indeed to the critical method. For one thing, he thinks that such entitlements can solve the problem of how we attain basic logical knowledge, since the logical inferences which produce justification can underwrite rule-circular arguments to basic logical truths. (Wright 2004b, §VIII.) For another, he holds that cognitive goals are a source of entitlement, when 'a cognitive project is indispensable, or anyway sufficiently valuable.' (Wright 2004a, 192.) One difference is that Wright takes entitlement to extend to assumptions that underlie inferential transitions, rather than for the premises of these inferences themselves. It is a good question whether this difference is significant.

[42] See the first chapter of Carl 1994 for a discussion of the state of the debate at the time.

neither thinking nor normativity certainly suggests allegiance to the non-normative side in the dispute.

But the Foreword to the *Basic Laws* explains that it is quite otherwise. Declaring the way in which 'the logical laws are conceived' to be 'decisive for the treatment of this science,' Frege affirms that logical laws are indeed 'laws of thought...the most general laws, prescribing how to think wherever there is thinking at all.' This is consistent with putting forward the laws Frege does, because 'Every law stating what is the case can be conceived as prescriptive, one should think in accordance with it, and in that sense it is accordingly a law of thought.' Though all laws stating what is the case prescribe that *some* thinking be in conformity with them, what is special about logical laws, distinguishing them from non-logical ones, is that they prescribe that *all* thinking be in conformity with them. It is in terms of the range of thinking for which these laws prescribe that we are to 'conceive' of logical laws.[43] Though this occupies only a few lines of the Foreword to the *Basic Laws*, other work re-affirms that 'the task we assign logic is...that of saying what holds with the utmost generality for all thinking, whatever its subject matter,' describes the resulting 'affinity with ethics', and says more to help us understand what he has in mind.[44]

Here is the moral for us. The *Basic Laws* is primarily a work of logic, not the philosophy of logic: its point is to develop a logic that respects philosophical considerations which are only briefly indicated in that book, if at all. We must not immediately conclude from the fact that Frege's logical laws do not mention oughts or thoughts that he does not share the normative conception of logic: though his laws are not *about* how we ought to think, he nonetheless puts them forward as logical partly *because* they have the normative role for thinking that is characteristic of logical laws. Similarly, we must not immediately conclude from the fact that his arguments for axioms do not mention goals that they are not critical arguments. The question is *why* he puts those claims forward.

With that in mind: I will argue that Frege accepts and puts forward the premises of these arguments because he thinks their truth would enable us to better achieve the cognitive goal of having a logical system which is *simple*, in a sense to be explained. Each argument, then, serves to connect the relevant axiom to this goal, showing that *if* the relevant goal is achievable to the relevant degree, the axioms are true. This connection to the goal fits with the general idea of the critical method described above.[45] Some of

[43]This discussion is at Frege 1893a, xiv-xv. See Taschek 2008, Steinberger 2017, MacFarlane 2002, and Hutchinson 2020 for readings of Frege which give full weight to these passages.

[44]See, e.g., Frege 1879–1891a, Frege 1880–1881a, and Frege 1897, 128.

[45]On this reading, the precise connection established between axiom and goal is in some ways closer to Windelband's variation of the critical method, and in some closer to Lotze's. (See note 34 above.) Like Lotze, Frege requires only that the goal be *better* reachable, rather than reachable at all; also like Lotze, what connects in this way to the goal is the axiom *being true*, not our *recognizing* its truth. Like Windelband,

the evidence for this reading will come from outside the *Basic Laws*, but in the light of the above, this should not surprise us.

What are the premises of Frege's arguments?

Let us look again at Frege's argument for Basic Law I:

> $\Gamma\rightarrow(\Delta\rightarrow\Gamma)$ would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; therefore a $\rightarrow$ (b $\rightarrow$ a).[46]

In this argument, Frege uses (not mentions) special symbols alongside words of natural language. We can understand such vocabulary-mixing sentences the same way we do when someone throws French words into English sentences: as long as we know what the words and symbols mean, we understand what is being said *sans difficulté*.[47]

Frege's claims about what is and is not the True and the False are not just odd ways of saying that something—a sentence, proposition, or whatever—is true or false. Frege thinks that the True and the False are objects—called 'truth-values'—and when he says that something 'is the True,' he means that it is (identical with) that object. The relationship between sentences and these truth-values is that of reference: a true sentence refers to the True. The argument also involves claims about the conditional concept to which '$\rightarrow$' refers. This concept was introduced a few sections before as a two-place function that yields the True unless the first input is the True and the second input is not the True.[48] For the truth-values as inputs, this function gives the outputs we expect: it yields the True when the first input is the False or the second input is the True.

To make these claims about truth values and concepts, Frege uses letters like 'a' and 'Δ.' As Frege later explains, 'letters do not, as a rule, have a reference; they do not designate anything, but only indicate in order to lend generality to the thought.'[49] Frege uses

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however, he shows that the goal's being reachable to the relevant degree implies the truth of the axiom, rather than the truth of the axiom implying that the goal can be reached to the relevant degree.

[46] For ease of reading, I replace Frege's notation with the usual symbols for the corresponding logical functions: '$\rightarrow$' for the conditional, '$\neg$' for negation, and '$\forall$' for the scope-indicating generality symbol.

[47] As Burge and others point out, we can also translate these claims into a single language: 'All of the arguments could be carried out within the language of the logic of Begriffsschrift (by avoiding the modal terminology).' (Burge 1998, 330.) Frege makes the usual suggestions for translating natural-language connectives like 'and' and quantifiers using *Begriffsschrift's* conditional, negation, and generality-sign. To say that something 'is the False' or 'is impossible' is to apply the negation function to it—Frege claims that the only thing a modal term does is offer a 'hint' about the kinds of grounds one has for the claim. (Frege 1879b, §4.)

[48] From §12: 'I introduce the function with two arguments $\zeta\rightarrow\xi$ by means of the specification that its value shall be the False if the True is taken as the ζ-argument, while any object that is not the True is taken as ξ-argument; that in all other cases the value of the function shall be the True.'

[49] Frege 1903b, 274 footnote.

the 'Roman' letters ('a' and 'b') without a scope-symbol, stipulating that they always have maximum scope.[50] (By contrast, 'Gothic' letters, which do not appear in this argument, are used together with another symbol to indicate scope, much as we use 'x' and 'y' while indicating scope with '∀'.) Though Greek letters do not belong to the *Begriffsschrift,* Frege's general statement that 'letters' invoke the concept of generality applies to them too.[51] The claims that appear in the argument for Basic Law I, then, are general, object-level claims concerning truth-values and logical functions—and so too are the claims that make up the arguments for the other Basic Laws.

But which claims are they in particular? The first claim in the argument for Basic Law I is a claim about the conditions under which the conditional concept yields the True and the False. Frege precedes that claim with 'according to §12...', referring back to the introduction of that concept in terms of those conditions. Really, then, this 'first claim' is a sub-conclusion, arrived at via the premise stating the truth and falsity conditions of this concept. The same holds of most claims in these arguments: they are consequences of the statements about truth and falsity conditions with which concepts were introduced.[52]

Those claims which are not simply applications of these conditions rely also on the self-identity of the truth-values or their non-identity with each other. The second premise of the argument for Basic Law I is like this: it is impossible for Γ to be the True and also something other than True because the True is self-identical.[53] According to Frege's well-known doctrine regarding criteria of identity, assertions of self-identity and difference are involved in taking any things to be distinct objects in the first place. (Related ideas were widespread at the time: Lotze had argued that to think of something as an object, it must

[50] 'The *scope* of a *Roman letter* is to include everything that occurs in the proposition apart from the judgement-stroke.' (Frege 1893a, §17.)

[51] The only exceptions to the general claim about letters that Frege mentions are mathematical constants like 'π.' I agree with Weiner 2005, then, that 'The generalization in the statements in which [the Greek letters] appear is generalization over all objects.' (334) Though the evidence is strong for this reading, good questions remain about Frege's use of letters. (For example: why is it important to have Greek letters, which are not part of the *Begriffsschrift*, as well as Roman letters, which are?) These questions require an extended discussion of Frege's treatment of generality and the relation between inference and notation.

[52] Greimann 2008 has pointed this out. For example, a statement of the conditions under which the primitive concept of generality yields the True are a premise in the argument for Basic Law IIa: '∀xΦx is the True only if the value of the corresponding function $\Phi(\xi)$ is the True for every argument.' (Frege 1893a, §20.) A premise in the argument for Basic Law IV (Frege 1893a, §18) notes that under any conditions, the value of the horizontal concept—a one-place function whose value is the True if its argument is the True and the False otherwise—is always either the True *or* the False: '—Γ is...always a truth-value'.

[53] Another example is in the argument for Basic Law IV (Frege 1893a, §18), which includes the premise: '—Δ and ¬Δ are always different...' This depends not only on the conditions under which the horizontal and the negation function have the True and the False as values, but on the fact that the True is not the False.

be 'thought as identical with itself' and 'as different from others.'[54])

In general, then: the claims that make up these arguments concern Frege's logical functions and objects, and follow from the ways they are introduced, to which Frege explicitly refers back. The real premises of the arguments, then, are those introductions. Our question becomes: why does Frege claim that the truth-values are objects, and introduce concepts with truth-and falsity conditions in the ways he does?

A Cognitive Goal

Frege explicitly brings goal-based thinking into these starting points. In the Foreword, for example, he explains that the truth-values appear as objects, 'which at first sight might admittedly appear strange' because of 'how much simpler and more precise everything is made' when they do.[55] He explains the introduction of particular logical concepts in his system by reference to the goal of simplicity as well. He defends including both negation and the conditional in his logic, for example, because of the way concepts with the relevant truth-and-falsity conditions make it possible to have fewer primitive laws—fewer axioms—than would be needed if he included other concepts instead.[56] It is this same kind of simplicity—the minimization of primitive truths—that he has in mind when he explains the 'value' of the concept of generality for logic in terms of the way it enables us to make a claim that 'contains many—indeed infinitely many—particular facts as special cases.'[57] He affirms the most basic claims about these concepts and objects, then—those which introduce them to his system, and from which the arguments for axioms begin—because doing so will enable him to better achieve the cognitive goal of having a logical system that is *simple* in the relevant sense.

Independent of our question about the axioms, there are worries that one might have about these appeals to simplicity.

[54]Frege's discussion is at Frege 1884b, §56 and §62; the latter passage echoes §9 of Lotze 1874.

[55]Frege 1893a, x. Of course, Frege would not appeal to simplicity to motivate dealing *in some way* with truth and falsity; it explains why the True and the False appear as *objects.* There is also an argument in Frege 1892b that these objects are the referents of sentences, but this argument is not put forward as anything more than suggestive, leaving open that there are other options.

[56]See Frege 1918–1919b, 384-385 for the claim that Frege's concept of negation enables 'an economy of logical primitives,' and Frege 1880–1881a, 36-37 for the claims that his conditional allows us to have 'the fewest possible primitive signs.' In turn, the latter passage explains that the reason to have have as few primitive logical signs as possible is that 'the more primitive signs are introduced, the more primitive laws will be needed.' 36 Frege makes such arguments throughout his writings, giving no indication that he has changed his mind about this as a result of the sense/reference distinction, Russell's Paradox, or anything else.

[57]Frege 1923–1925, 278.

1. Isn't the goal of simplicity too flimsy a reason for such pivotal and occasionally counter-intuitive decisions about logic?[58]

This worry registers a major disagreement with Frege, who repeatedly and explicitly identifies the relevant kind of simplicity as *of the first importance* for any science. The minimization of the primitive truths, by which we assemble 'many details under a more comprehensive point of view' and bring order to 'a large—possibly unsurveyable—manifold through one or a few sentences,'[59] is a 'commandment,' 'in itself a goal worth striving for,' whose pursuit is a 'general, basic principle of science.'[60] This is because minimizing primitive truths provides, to the highest degree, scientific understanding, or *explanation*: the 'essence of explanation lies precisely in the fact that a wide—possibly unsurveyable—manifold is governed by one or a few sentences,' and the fewer the better: 'the fewer the number of primitive sentences, the more perfect a mastery we can have.'[61]

2. How can Frege compare his logical system on the score of simplicity with a system which does not include the conditional? Isn't the conditional one of the main things that logical laws are about? And so, wouldn't a system that didn't discuss the conditional simply *not be a logical system at all?*

For his comparisons to make sense, Frege indeed must have a way of characterizing a logical system which can be understood independently of which concepts and objects it is about. We have already seen, however, how this is possible: since Frege's ultimate

[58]Dummett 1973, for example, takes the decision to introduce the truth-values in order to achieve 'great simplification' to be a 'ludicrous deviation' and a 'gratuitous blunder.' (183-184).

[59]Frege 1914b, 221; Frege 1880–1881b, 40.

[60]That is, a 'Gebot' (Frege 1893b, §14), an 'allgemeiner Grundsatz der Wissenschaft' (Frege 1880–1881b, 40.), and 'an sich schon ein erstrebenswerthes Ziel.' (Frege 1884a, §2.)

[61]Frege 1880–1881b, 36, 39. Frege is not alone is seeing this goal as central for science, and to explanation in particular: in the Prolegomena to Liebmann 1876, for example, we are told that 'No science can be counted as perfect and completed...until it...forms a logical whole, in which...an absolutely minimized number of...primitive sentences...[a] narrow tip of primitive thoughts...flows into the broad—indeed infinite—group of...details.' (8) Liebmann goes on to discuss the kind of 'explanation' that is thereby produced. One might worry that Frege cannot really assign the importance to simplicity that these passages suggest, since the he admits in his *Basic Laws* that he has failed fully to obey the 'demand of scientific parsimony' and 'reduce the number of...fundamental laws as far as possible,' in one particular way. (Frege 1893a, vi, §14) But he also tells us that it is only 'considerations of practicality' that led him to do so: his book would exhibit 'excessive length' if he had done so; all this shows is just what it says: that Frege does not think of the *Basic Laws* as wholly satisfying the scientific requirement, because extra-scientific constraints (such as the length of book that publishers are willing to publish and readers to read) prevent it. Presumably, if the logic of the *Basic Laws* proves correct in its essentials, these practical obstacles would be removed, and Frege's system could be expanded into a final, perfect formulation of the system of logic. This certainly provides no reason to doubt his explicit appeals to the simplicity goal as motivating his decisions.

characterization of logical laws is in terms of their normative role rather than what they are about, he can consider what it takes to have a maximally simple system of laws with the relevant normative role without yet having said what they are about.[62]

3. Reflections on how certain objects and concepts would allow for a simpler logical system do not provide evidence in any ordinary way that there really *are* such objects and functions, or that the claims with which they are introduced are really true. Observing that they serve simplicity, then, hardly provides any justification for actually accepting the relevant claims. To make those claims anyway, and to draw the axioms as conclusions from them, looks to simply *presuppose* that the goal of having such a simple system can be reached.

That is, of course, the point. Frege's regress argument, after all, implies that he *could not* be putting forward the premises of the arguments because he is already justified in thinking they are true. What he does instead is just what the critical method calls for.[63]

5 Conclusion

With all the pieces in place, let me briefly summarize what Frege is doing with his arguments, why we should read him that way, and why it is interesting.

What is Frege doing? Frege gives arguments for his axioms because he thinks we must have such arguments, if we are to be justified in accepting the axioms. He thinks his arguments provide such justification even though we are not justified in accepting

[62]Frege's discussions of multiple ways of decomposing the very same thought may also help here. According to Frege 1892a, for example, 'one way of analysing a given thought should make it appear as a singular judgement; another as a particular judgement; and a third, as a universal judgement.' On the various analyses, the same thought would turn out to involve different subsentential referents, and hence be about different concepts. This suggests that he could characterize the logical laws at the level of the *thought*, and compare logical systems which differ in the way they carve these same thoughts up.

[63]One might worry that the critical method is only plausible in connection with *certain* cognitive goals. (Above, we saw contemporary defenders of the idea that goals could be a source of justification rely on the fact that some goals are 'rationally required.') Even if we grant Frege the importance of simplicity for science, one might worry that the goal of constructing a simple logical system is *optional*, so that the critical method loses its plausibility in connection with it. But as several authors have pointed out, Frege thinks a logical system is part of *every* scientific system; which means that every science depends on the simplicity of the logical system. (See, e.g., Burge 1992, 301 and May 2018, 128.) Moreover, Frege claims that 'truth in the strictest sense,' is 'truth in the scientific sense': 'that sort of truth which it is the goal of science to discern.' (See Frege 1906, 186, Frege 1914a, 232, and Frege 1918–1919a, 352 for such claims about truth.) This suggests that he sees a connection between the goal of truth—presumably, a non-optional goal—and what might have seemed like a special scientific goal. This may give him the resources to claim that the goal of a simple system of logic is, in an important sense, not optional.

their premises, because of the way they connect the truth of the axioms with a goal: having a simple logical system.

Why read Frege this way? What he says about axioms, understood in the most straightforward way, implies that he sees his arguments as yielding justification for their conclusions even if we are not yet justified in accepting their premises. It makes philosophical and historical sense for Frege to have done so without explaining it, however, only if he were pursuing a reasonable idea that was also popular at the time. The above idea meets these desiderata: the claim that such an argument can justify an axiom appeared in popular works by the dominant philosophical figures of the day, and recent thinking on the subject assures us that it is reasonable. It is likely to be the only idea about arguments for axioms that meets these desiderata.

Why is this interesting? I see two main reasons.

1. The Neo-Kantian critical method itself is interesting as an (as yet unrecognized) precursor to the contemporary approaches to basic justification sketched in section 3 above. The variations that Lotze, Windelband, and Frege provide on the epistemological themes that lie behind these contemporary approaches should help us identify what is essential and plausible in goal-based approaches to basic justification.

2. Frege's use of this method is interesting for the way it affects his place in the history of philosophy and his significance for the general reader. Recently, such a reader has seemed to be faced with an unappealing choice between two options when it comes to Frege.[64] On the one hand, there is a reading on which Frege is the founder of the current age of philosophy, anticipating major developments of the next century and continuing to speak directly to our contemporary philosophical concerns and positive projects. This reading has come to seem anachronistic, requiring us to ignore historical facts and many details of what Frege is actually doing.[65] On the other hand, there is a reading which aims for scrupulous historical and textual accuracy, emphasizing his connections to his contemporaries. But this reading can seem to place Frege *only* in philosophy's past, his thinking mired in the niche debates and weird obsessions of a transitional period in philosophy, his lasting importance found only in technical discoveries which have since been purified of their philosophical baggage and improved upon.[66] We can seem, then, forced to

[64]See section I of Heis 2013 for some discussion of this wide-ranging interpretive dispute.

[65]The classic source for this charge is Sluga 1980, who accuses readers in the analytic tradition of 'failure...to come to grips with the actual, historical Frege'; see also Weiner 1996 for a case along these lines concerning Frege's philosophy of language in particular.

[66]Here is a classic statement from Dummett 1976, 490: 'Sluga's account of Frege's views appears to

choose between a reading with contemporary interest and one that exhibits historical and textual accuracy, unable to have both—in which case we might be better off not reading Frege much at all. Frege's engagement with the critical method shows us, however, that we can pay close attention to the details of Frege's writing, bringing in its historical context where appropriate, *without* thereby depriving what he says of contemporary interest. Quite the opposite: we see that the concerns of Frege and his contemporaries are still our concerns, and their responses to those concerns still worth taking seriously. Reading Frege in this way not only helps us to make progress on our questions, but serves to reassure us that *ours* are not the niche debates and weird obsessions of a transitional period in philosophy.

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Unless otherwise noted, when the reference is to a non-English work, the translation is my own.

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serve the purpose of playing down his originality and placing him as one among many members of the school of "classical German philosophy". Such a purpose...has the unfortunate effect...of rendering Frege's doctrines a confused tangle of incoherent errors. The interpretation of Frege I gave...had at least the merit of presenting him as a great philosopher, who deserves...the great posthumous reputation which he enjoys.'

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