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# 1 Vector spaces

- Vectors must satisfy 10 conditions (for $u, v \in V$)
  1. $u + v \in V$
  2. $u + v = v + u$
  3. $(u + v) + w = u + (v + w)$
  4. There exists a vector 0 with the property that $v + 0 = v$
  5. For every $v \in V$, there exists a unique $-v$ st $v + (-v) = 0$
  6. if $\alpha \in R, v \in V$, then $\alpha v \in V$
  7. $\alpha (u + v) = \alpha u + \alpha v$
  8. $(\alpha + \beta) u = \alpha u + \beta u$
  9. $\alpha (\beta u) = (\alpha\beta) u$
  10. $1u = u$
- Example 1
  - $V = M_{m,n}$
- Example 2
  - $V = \{f(x), 0 \leq x \leq 1, \text{f is continuous}\}$
  - with the definition $(f + g)(x) = f(x) + g(x)$
    - $(\alpha f)(x) = \alpha f(x)$
- Example 3
  - $V = \mathbb{C} = \{x + iy, x, y \in \mathbb{R}\}$ =complex numbers
  - $(V, \mathbb{R})$
  - $(x + iy)(u + iv) = (x + u) + i(y + v)$
  - $\alpha (x + y) = \alpha x + i\alpha y$
- Example 4
  - $V = \mathbb{C}$
  - $(V, \mathbb{C})$
  - $(\alpha + i\beta)(x + iy) = [\alpha x - \beta y] + i[\alpha y + \beta x]$
- $\dim(\mathbb{C}, \mathbb{R}) = 2$

- $\dim(\mathbb{C}, \mathbb{C}) = 1$
- Subspace of $V$
  - $H$ is a sufspace if given
  - $u, v \in H$, $u + v \in H$ for $\alpha u = H$
- The whole space is a subspace (including 0)
- Kerneland range of a linear transform between $(V, R) \to (W, R)$
  - $T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$
  - $W = M_{2,2}$
  - $(T\ f) = \begin{bmatrix} 7f\left(\frac{1}{4}\right) & f\left(\frac{1}{2}\right) \\ f\left(\frac{2}{3}\right) & f\left(\frac{3}{4}\right) \end{bmatrix}$
  - kernel of T $= \{x \in V | Tx = 0 \in W\}$
  - range of T=$\{y \in W | y = Tx \text{ for some } x \in V\}$

## 2 4.4 - Coordinate systems

- Any vector $x \in V$ can be written in a unique way as $x = \alpha_1 b_1 + \alpha_2 b_2 \cdots \alpha_p b_p$ for $\alpha_i \in \mathbb{R}$
- $V$ let $b_1 \cdots b_p$ be a basis for $V$
  - Then we call the vector $\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{bmatrix} \in \mathbb{R}^p$ the coordinates of $x$ with respect to $\begin{bmatrix} b_1 & \cdots & b_p \end{bmatrix}$
- Notice: for every vector $x \in V$, we get a vector $[x]_B$ in $\mathbb{R}^p$
  - This gives you a linear transformation between $V$ and $\mathbb{R}^p$
  - Moreover, this line\eeqar transformationhas kernel $\equiv \{0\}$, so the mapping is called one-to-one because $T(x) = T(y) \implies x = y$, $T(x - y) = T(x) - T(y) = 0$
- Moral: if $V$ has dimension = p (there is a basis that consists of p vectors), then it can be identified with $\mathbb{R}^p$