Some Tips for Tackling Physics Problems

"Let us think the unthinkable, let us do the undoable, let us prepare to grapple with the ineffable itself, and see if we may not eff it after all."

— Douglas Adams, *Dirk Gently's Holistic Detective Agency*

Here are a few suggested frameworks, approaches, and strategies attempting to summarize, formalize, or capture what it is we do when trying to solve a physics problem.

Despite this being something of a laundry list of tips, our major advice is in fact to *avoid* the temptation of randomly throwing some formulae at a problem hoping something sticks, or of a "cookbook" approach, where you memorize by rote a list of "recipes" for a given topic, and then mechanically attempt to pigeonhole in procrustean fashion any example you encounter into one of these cookie-cutter patterns. We seem to be mixing our metaphors a bit here, but hopefully you get the idea.

The goal of our class, and of the problems we assign, is to help develop your knowledge and comprehension of the fundamental ideas, to build facility with the fundamental tools, and more broadly to hone your skills of critical thinking, organization, analysis, and synthesis, rather than to encourage rote memorization or mechanical equation-hunting or number-crunching. We hope that you begin to perceive the overarching unity of physics in its concepts and approaches to problems, rather than seeing but a mishmash of equations and procedures to merely memorize and regurgitate on homework and exams.

The IDEA Idea

Professor Richard Wolfson, from Middlebury College, has suggested an "IDEA" framework in his exposition and examples in his physics textbook. IDEA is an acronym to help you remember important steps of the physics problem-solving process, when your first reaction might be "I have no idea how to approach this problem:"

Identify: *Interpret* the problem and *identify* key physical concepts and involved, and what constitute the important known and unknowns quantities;

Develop: *Develop* a plan for reaching the solution, *determine* appropriate strategies and tactics, often aided by *drawing* a diagram;

Evaluate: *Execute* the plan, *effect* your strategies, and *evaluate* your answers as needed;

Assess: *Assess* the solution and *ask* yourself if it makes sense, so as to validate you work, solidify your understanding and mine additional insights into the physics.

In particular, students often neglect to draw a picture as part of the "Develop" phase, and too often move on to the next problem before taking the time to "Assess" their solution, by which a little reflection can teach us a lot more about a problem than mere mathematical manipulation.

Specific Suggestions for the Phases of Physics Problem-Solving

The following advice is based on my own experience, as well as the insights of Professor Carleton Caves, at the University of New Mexico, and Professor Hal Haggard, an award-winning physics GSI while here at UC Berkeley:

I. *Getting Started*:

- don't panic—physics should be challenging, but also fun…
- practice other problems—the more extra problems you do, the easier the required problems will become;
- try to develop a mental or literal picture—something you can visualize that captures the essence of the problem;
- exploit symmetries, superposition, and conservation laws wherever possible;
- identify the important physical scales—i.e., key quantities with dimensions such as time, length, or energy—before proceeding;
- try reasoning backward from an answer, in order to help supply the concepts and information you will need to proceed;
- try to guess, estimate, or approximate the answer, using any available technique at your disposal—calculations are much easier if you already know where they need to take you;
- if you do not need to know the *exact* answer (and you rarely do in actual science, outside of textbook assignments and exams), think about whether a simpler *approximate* technique can be used; real physics is often the art of the inspired if uncontrolled approximation;
- such approximations are often possible using separation of scales—i.e., large ratios (or sometimes differences) that exist between the relevant physical scales (of a given dimension) in the problem, that may allow certain mathematical simplifications or expansions;
- think about how the problem is similar to, and different from, other problems that you have done or seen;
- try to first tackle a related, but simpler, problem, that might provide isights into the more general problem—try to make things "as simple as possible, but no simpler," as Einstein taught, or perhaps make them too simple and then incrementally add back complexity;
- try a "divide and conquer" approach—attempt to decompose the problem into simpler or more manageable sub-problems, which you can tackle more easily;
- before using advanced or involved techniques, think about whether a problem or part of the problem can be solved using more elementary methods;
- before starting a lengthy calculation, try to think of a more efficient or clever way to approach the problem;
- but do not spend too much time trying to think of a very clever approach that will only save you a little bit of time, if any—sometimes the straightforward approach is best, particularly for a one-off problem;
- sleep on it—sometimes after thinking very hard about a problem, it is best to move on to something else for a while and let ideas percolate in your subconscious, returning to the problem later with new energy and fresh insights;

- seek help—(if not taking an exam), discuss the problem with peers or an instructor—optimally, *after* you have struggled with the problem on your own;
- engage in conversation—sometimes we get too close to a problem, and others can see what we have repeatedly overlooked; or sometimes just in explaining your thinking out loud, you see where you have gone wrong;
- a measure of "trial-and-error" is fine: science progresses by learning from mistakes;
- similarly, "guess-and-check" and solution via *ansatz* are also time-honored techniques;
- but do not endlessly spin your wheels—while it is important to grapple with a challenging problem, sometimes for a long time, do not keep "going in circles" or keep "getting nowhere" for hours on end, without any signs of progress —ask for help, consult with others, or move on to another problem.

II. *Doing the Problem*:

- in general, do NOT substitute in numerical values until the very end of the calculation —plugging in numbers too soon makes errors much more likely, missteps much harder to trace, and grading much more difficult;
- important exceptions to the previous rule include: solution of several (3+) simultaneous equations in several unknowns, where symbolic manipulation is too cumbersome, and numerical methods on digital computers;
- choose variables, axes, and coordinate systems to simplify your algebra, and/or reduce the number of unknowns, based on symmetry or other considerations;
- be careful to distinguish scalars and vectors;
- choose variables names that will not be easily confused or conflated;
- feel free to introduce (with explanation) your own variables. coordinates, or symbols if they simplify intermediate or subsequent calculations, but express final answers in terms of data and variables given in the statement of the problem;
- do not try to combine too many algebraic steps into one line on your page, or it may be hard to track down any mistakes;
- perform dimensional analysis and other sanity checks at intermediate steps to help catch algebra mistakes early, before they propagate;
- use *Mathematica*, *MATLAB* or other computational tools to help with algebra,integration, series expansions, and visualization;
- use a pencil—everyone makes mistakes;
- work carefully, neatly, and deliberately, and give yourself plenty of room on the page for calculations—continually keeping track of signs and powers of two is easier than finding these sorts of errors after the fact.

III. *Verifying/Validating the Solution*:

- check that your answer has the correct dimensions and units—this is a simple but powerful means to check for algebra errors;
- often, dimensional analysis alone can be used to narrow down the possible answers to only a few (or if you are lucky, even one) combination of input parameters, scaled by some pure (dimensionless) number which typically is not too far (in its order-of-magnitude) from unity;

- verify orders-of-magnitude: given the inputs to your problem, do the outputs seem reasonable in size?
- test your answer in any limiting cases, typically involving very large, vanishing, or very small values for some of the variables or parameters—cases where you can independently infer the correct answer from other considerations;
- try to solve the problem in a second, independent way—this is one of the best ways to check an answer in principle, but is not always possible in practice;
- think critically and carefully about your solution, to see if your answer makes sense in the light of other things you know;
- do whatever else you can think of to try to "break" the solution you just came up with —an answer that survives many attempts to prove it wrong is more likely to actually be correct….

IV. *Writing Up the Problem* *(when it is to be read by actual humans)*:

- again, pencil is preferable to pen, because you can correct inevitable mistakes;
- do not cramp your calculations and derivations—use plenty of space on the page, and set off important long equations on separate lines, like in a textbook;
- if your initial scratch work is messy, rewrite it;
- the solution should consist of a logical progression of steps, presented in sequence;
- all *important* steps should be included AND briefly explained unless self-evident —only work that is presented, and presented in an understandable fashion, can be evaluated and graded;
- that being said, in a final draft, some steps can easily be combined—you can clump, conjoin, or jump over some straightforward algebraic manipulations as long as it clear to the reader where you are going;
- explanations are as important as equations: solving physics problems is not just about getting the right answer, but about doing the right things for the right reasons;
- whenever you are asked an explicit "what," "when," or "where" type question, remember that, implicitly, you are also being asked "how" and "why" —explain *what* you are doing and *why* you are doing it and *how* your conclusions follow;
- explanations do not need to be very long—no Tolstoy novels are needed—just concisely explain the key points so the grader can follow your reasoning;
- do not conflate the physics with the math—remember that in physics, a good *explanation* should ideally usually refer to physical systems and their properties and behaviors, not exclusively to purely mathematical quantities or expressions;
- box or circle your final answers;
- remember the primary objective is to make your assignment clear and well-organized so that: you get in the habit of working in a careful, clear, and organized manner; the grader can understand what you are doing; and, later, you can understand what you did—if you need to bend any of the above rules to better achieve these larger objectives, feel free to do so....

V. *Before Moving On* *(learning more from the problem)*:

- analyze the solution in some depth, thinking about what it is really trying to tell you;
- understand and explicate what the solution says—conceptually, semi-quantitatively and quantitatively. What is the overall behavior of your solution? Does it approach reasonable limiting values? Does it have special points or regimes or features, and why do you think it should have those features? Does it do what you might have anticipated? Does it have any novel or surprising features that would not have been expected? How should we understand those behaviors or features?
- connect the solution to other physics or mathematics you have already learned: does the problem look mathematically similar to other types of problems you have seen? If so, what quantities in the other physics correspond to quantities in the current problem (mathematically or physically speaking)? Is the mathematical relationship purely coincidental or formal, or is there a deeper reason why the mathematics should be similar?
- what further questions are suggested or spurred by this problem? Is there a natural next question that you could ask after finding the solution to this one? Are there elements of your solution that you could try and generalize, or apply to different contexts?
- think about what you find most interesting about the problem;
- contemplate the bigger picture: why was this particular problem assigned? What did you learn from it? What do you know, or what can you do, after struggling with the problem, that you did not know, or could not do, before?

Textbook Problems Versus Real-World Problems

In contrast to textbook problems, *real-life* problems in science or medicine or engineering—or elsewhere for that matter—are often particularly hard because: they are poorly or ambiguously formulated, they may be surrounded and suffused with messy complications, and we cannot look in the back of a textbook for the accepted answer. The psychological value of simply knowing that a problem has been solved by someone else cannot be underestimated, but in research, or life, we often do not know if a question can be answered at all, or how much time and effort it will require to answer it.

Knowing how to ask a good question then becomes at least as important as knowing how to answer it. "A problem well stated is half solved," quipped Charles Ketterling. In trying to ask and answer a useful question, it invariably becomes essential to make various approximations, idealizations, and simplifications, and always to examine critically any answers we do obtain. Intellectual honesty concerning what we have tried to solve and how, and critical thinking concerning tentative progress achieved, are the foundations of scientific progress and scientific integrity. So we shall try to get you to practice these skills and principles in class as well.

Problem-Solving: General Cognitive Features and Obstacles

In some form or another, problem-solving occupies much of our conscious (and sometimes unconscious) time. Despite this, we know all too little about how the human mind goes about high-level problem-solving. But much of what we do know applies in some sense to the specific sorts of problem-solving we pursue in physics. Conversely, physics provides some of the best formal and practical training in logical and conceptual problem-solving more broadly.

In cognitive or gestalt psychology, problem-solving refers to mental processes that people go through to recognize, analyze, evaluate, and solve problems. This involves various stages in the process, including recognition of the problem, a decision to tackle an issue, understanding the scope of a problem, exploring available options, predicting consequences, and taking actions to achieve goals.

There are a number of different mental process at work during problem-solving, including: perceptually recognizing a problem; identifying different aspects of the problem; representing the problem in memory; labeling and describing the problem; judging the relevance of information; processing relevant information that applies to the current problem; generating hypotheses; modeling possible behavior and outcomes; predicting consequences and performing what-if reasoning; and undoubtedly many more.

Understanding and Defining the Problem

Before problem-solving *per se* can actually unfold, it is important to understand the exact nature of the problem. If one's understanding of the issue is inaccurate or incomplete, one's attempts to resolve it may also be incorrect or flawed. Again, a problem well-posed is a problem half-solved. Take time to plan and organize at the beginning before rushing into a problem.

Problem-Solving Strategies

General sorts of problem-solving strategies include:

Algorithms: an *algorithm* is a more-or-less mechanical, step-by-step logical, mathematical, or computational procedure that will produce a correct solution when certain premises are met. While an algorithm may guarantees an accurate answer when correctly applied to a suitable problem, it may not always be the most efficient approach to a particular problem. However, through training, the brain does tend to try to automate certain tasks so that it can attend to more novel problems. And a major goal of science and mathematics is one of "meta-problem-solving," finding new or better algorithms for solving certain classes of problems that we face repeatedly.

Heuristics: a *heuristic* is a mental rule-of-thumb or less formal strategy that may or may not work in a given situation. Unlike algorithms, heuristics do not necessarily guarantee a correct solution, but they can still sometimes lead to very good solutions. Using heuristic problem-solving strategies may allow people to simplify complex problems and/or reduce the total number of possible solutions to a more manageable set.

Trial-and-Error: a *trial-and-error* approach to problem-solving involves generating and evaluating a number of different solutions and ruling out those that do not work. This approach can be a good option if you cannot think of anything else, or have a very limited number of options available. If there are many different choices, you may be better off narrowing down the possible options using another problem-solving technique before attempting trial-and-error.

Insight: in some cases, the solution to a problem can emerge as a sudden *insight*. Insight sometimes occurs analogically, because you suddenly realize that a problem is similar to something that you have dealt with in the past, but in other cases an idea or new stratagem just suddenly emerges, as if mental clouds have parted. The underlying mental processes that lead to insight often happen outside of conscious awareness and are not well understood, just appreciated when they work. If you have actively and intently thought about a problem for some time, often if you "sleep on it" or attend to another task, your subconscious can continue to work on the problem, and may generate insights that you can then follow up consciously.

Common Problems, Obstacles, and Hindrances in Problem-Solving

Problem-solving, particularly in novel domains or situations, requires creativity and is of course not always a smooth or mechanical process. Researchers have identified a number of common obstacles that can interfere with our ability to solve a problem effectively or efficiently:

Irrelevant or Misleading Information: when trying to solve a problem, it is important to try to distinguish between information that is relevant to the issue and distracting data that can lead to inefficient problem solving or even faulty solutions. When a problem is complex, it becomes easier to focus on misleading or irrelevant information. Beware red herrings and false leads!

Bad Assumptions: when confronting a problem, people often make unjustified assumptions about the nature of problem constraints that prevent them from seeing possible solutions. Certainly use insight and even guesses, but be careful when leaping to conclusions or assuming too much.

Functional Fixedness: people tend to view problems and problem-solving tools in a habitual manner. Specifically, they tend to only think of the traditional, official, habitual, customary, established, or prototypical uses for a physical or mental tool or object. Functional fixedness prevents people from perceiving the novelty of certain problems, things, or techniques, or of fully seeing all of the different options that might be available to find a solution—a mental block against using a tool, framework, object, or idea in a new way that might be effective in solving a certain problem. Learning from experience and reasoning by analogy with previous problems is of course essential, but so too is thinking outside of the box.

Mental Set: more broadly, people have a tendency to only use concepts, categories, patterns, and prototypes or stereotypes that have worked in the past, rather than looking for alternative ideas. A mental set leads people to approach problems in a particular way, usually based on past experiences and habits. This can often lead to effective heuristic problem-solving tools, but can also entrench inflexibility, making it more difficult to find novel solutions. Again, definitely learn from experience, and do not re-invent the wheel needlessly, but be open to new approaches, do not get struck in a rut or too set in your ways, and be willing to learn new tricks.

Confirmation Bias: it is usually more satisfying to be right than wrong, and we tend to seek or recall information that confirms our assumptions, decisions, and preexisting beliefs, and to downplay or ignore anomalies and discrepancies. But in science, you should often try to "break" your solution or falsify your hypotheses. Beware wishful thinking and unquestioned assumptions!

Some Additional Hard-Won General Advice

These tips are based on the wit and wisdom of Dr. Charles Wohl, an instructor here at UC Berkeley, and member of the Particle Data Group at LBNL:

• Imagine you have just waded through a few pages and realize you understand very little of what you have read. Do not despair—that is perfectly normal. Learning new physics is like learning to play a new piece of music—for a while there is no music. Read a few pages, take a break, read the pages again, read a few more pages, try a problem, re-read some more, rinse and repeat. Textbooks edit out all of the inevitable missteps and false leads and conceptual fog that gradually made way for tentative understanding on the part of real-world scientists. Do not fall into the psychic trap of thinking physics is pursued only by those to whom it comes easy. At some level, physics is challenging for everyone;

• Read the problems early on. Textbooks are often written backwards. In real life, like on *Jeopardy*, questions usually precede answers. Questions drive our learning and tell us what we are expected to be able to do with what we are learning. With the problems in mind, you will be better able to snag relevant information from the reading and lectures. And sometimes you will find that your subconscious worked on a problem while you thought about something else;

• Do lots of problems. Then do some more. You cannot learn to do physics simply by reading and listening any more than you could learn to play music simply by reading a score or listening to a recording. You learn "up your arm" as much as through your eyes and ears;

• Unless you think you see all the way through a problem, start out on scrap paper. Then when you have found your way to the solution, clean it up. Where did you go wrong? Do you now see how you could have gone more directly from start to finish? Write that up;

• Find someone to talk with about the physics. Verbalizing can force you to try to have complete thoughts, and often will clarify matters all by itself. Plus talking about physics can be fun and interesting;

• Another good way to see if you understand something is to try to recreate it, away from the books, maybe on paper napkins over a cup of coffee. And do not be afraid to talk things through to yourself. The person muttering over in the corner of the cafe might be deranged, or might just be a physicist (or both);

• Do not fight the math. That is like a carpenter or mechanic fighting her tools. Try to get comfortable. If you are more interested in eventually building equipment, doing experiments, or analyzing data, that is great, but some mathematical fluency will still be indispensable;

• But try to see the physics from the math. Listen to what the math is trying to tell you. The usual process includes: formulation of a physical problem, a swim in mathematical waters (sometimes a near-drowning experience), and emergence with a solution to be interpreted physically. There is nothing especially "quantum mechanical" about the needed mathematics — most of it was developed in the nineteenth century to solve classical problems. And once in a while, stop to marvel that all those mathematical manipulations have anything to do with the real world.

Philosophy and Psychology of Learning: Some Facts and Fallacies

Finally, we indulge in a bit more of the educational psychology literature. By now, some of these observations (pioneered by Prof. Carol Dweck and other researchers) have become almost cliched, but still it is useful to contrast two extremes in characteristic student attitudes and approaches which can definitely affect how much physics students learn and retain, and how rewarding the process can be along the way:

The following sorts of assumptions, collectively known as a ***Fixed Mindset***, are common but unhelpful misconceptions amongst students that hinder learning: try to avoid these negative and counterproductive attitudes:

- intelligence and ability are doled out at birth, and little can be done to change static aptitudes and innate traits;
- success can be attributed primarily to this innate aptitude or ability: either you are good at physics or not;
- the more you need to exert yourself in studying physics, the less suited you are for it;
- applying effort might reveal lack of natural ability, so it is best to at least appear competent without trying;
- effort is unpleasant and should be minimized wherever and whenever possible;
- asking questions just reveals what you do not know to the instructor and peers;
- outcomes are everything; failure indicates that your effort was wasted;
- difficulties are evidence that you are not suited to a task;
- when facing challenges, by definition success is uncertain, so it is better to stick with what you know rather than risk failing;
- effort-avoiding behaviors (e.g., studying less, copying from others) or effort-delaying behaviors (i.e., procrastination) may be used to avoid failure or setbacks which could otherwise reveal weaknesses;
- criticism or negative feedback of your actions is tantamount to criticism of you.

Instead, try to foster positive and proactive beliefs about learning associated with what is referred to as a ***Growth Mindset***:

- intelligence and ability are malleable — the brain can be developed and trained;
- education and hard work can improve, build upon, or leverage natural traits;
- learning is a strong function of motivation, effort, perseverance, and effective study habits;
- expertise is built from practice and experience;
- achievements are due primarily to ones actions;
- effort and practice and determination are necessary to master new skills;
- mistakes should be valued as opportunities to improve;
- effort-engaging behaviors (studying harder, attempting different strategies, etc.) are necessary to face and embrace challenges so that learning can happen;
- process can be more important than outcomes;
- challenges and obstacles are necessary for learning;
- failure and frustration offer useful feedback;
- criticism can provide useful information;
- setbacks should prompt more effort because the intrinsic rewards of learning reveal personal strengths and increase personal capacity.

<u>Endeavor</u> to become what is known as a *Self-Aware* and *Self-Regulated Learner*:

- try to maintain a growth mindset;
- explore your passions, but do not expect things to always come easy;
- do not rest on your laurels; seek out new ideas and push into the unfamiliar;
- take responsibility for your own learning and set your own educational goals;
- assess successes or failures in terms factors under your control (e.g., effort expended on a task, effective use of strategies);
- do not be afraid to make mistakes, ask questions, admit perplexity;
- remember that the primary job of a student is to ask questions, not give answers;
- engage habitually in self-observation, self-reflection, and self-evaluation of your learning;
- seek out and make effective use of feedback;
- reflect on mistakes, missteps, and setbacks, as well as successes;
- Identify your own academic strengths and areas for improvement;
- assess and leverage how you learn best: for instance, are you primarily a visual, verbal, aural, logical, or kinesthetic learner?
- cultivate and practice a repertoire of problem-solving strategies and tactics;
- work on physics consistently and regularly: as with physical exercise, it is hard to get in mental shape but easy to get out of shape — do not wait for the night before the problem set is due, but instead try to do at least a little physics every day or at least every other day;
- rehearse and refine knowledge and skills in order to deepen your understanding;
- try focusing on a small number of aspects of quality in your work at any one time;
- monitor and reflect critically upon what you know or do not know; question what you know; share what you know;
- seek our challenging tasks, and accept inevitable obstacles and stumbles;
- remember that doing physics right requires struggle, and, yes, even frustration and failure;
- relish the rewards — understanding more and more about how the universe works!

Problems worthy of attack
prove their worth by fighting back.

The road to wisdom?
— Well, it's plain
and simple to express:
Err
and err
and err again
but less
and less
and less.

—Piet Hein