Compressive Quantum Tomography

Kunal Marwaha

Backstory

Quantum information is interdisciplinary

EECS algorithm, Physics student, Chemistry professor

“We’re onto something,
but **we don’t know enough**”

Compressive Sensing

Efficiently reconstruct complex signal

Key condition:

Sparsity

1 **Undersample**

A camera or other device captures only a small, randomly chosen fraction of the pixels that normally comprise a particular image. This saves time and space.

2 **Fill in the dots**

An algorithm called l_1 minimization starts by arbitrarily picking one of the effectively infinite number of ways to fill in all the missing pixels.

3 **Add shapes**

The algorithm then begins to modify the picture in stages by laying colored shapes over the randomly selected image. The goal is to seek what's called sparsity, a measure of image simplicity.

4 **Add smaller shapes**

The algorithm inserts the smallest number of shapes, of the simplest kind, that match the original pixels. If it sees four adjacent green pixels, it may add a green rectangle there.

5 **Achieve clarity**

Iteration after iteration, the algorithm adds smaller and smaller shapes, always seeking sparsity. Eventually it creates an image that will almost certainly be a near-perfect facsimile of a hi-res one.

Photos: Obama: Corbis; Image Simulation: Jarvis Haupt/Robert Nowak

Sparsity

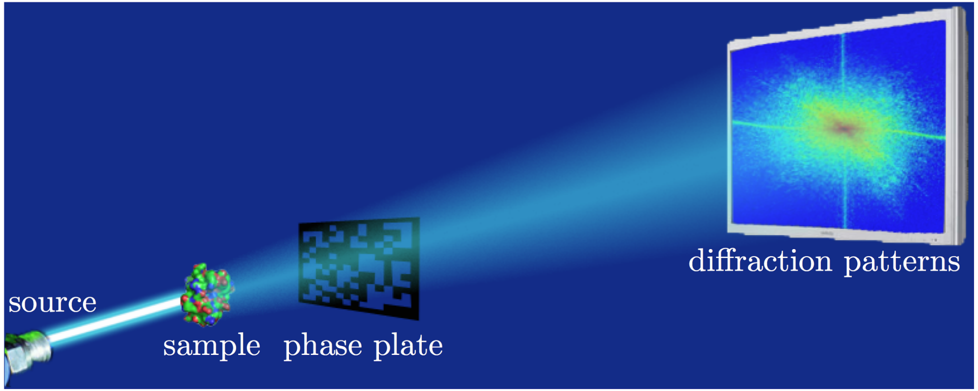
In some domain, signal is **k-sparse**

e.g. a **k**-sparse vector has **k** non-zero elements

$$\begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

What makes this tricky

Need phase to reconstruct signal...



But can only get magnitudes!

$$\vec{y_i} = |\langle \vec{a_i}, \vec{x} \rangle|$$

Compressive Sensing Procedure

1. Carefully design measurement vectors $\vec{a}_i$
2. For each $\vec{a}_i$, measure signal; send to decoder

k-sparse, unknown vector $\vec{x}$

1 measurement per $\vec{a}_i$, send result $\vec{y}_i$

measurement decoder

output $\vec{x}$

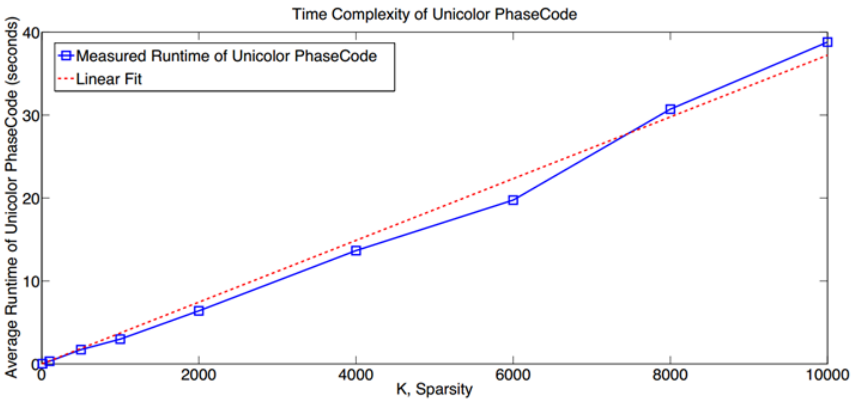
Motivation for Approach: PhaseCode

UC Berkeley EECS: Prof Ramchandran

compressive sensing made for light detection

14K measurements

O(K) decoding time



Pedarsani, Lee, Ramchandran 2014 arXiv 1408.0034

Porting to QM: Requirements

-reconstruct some **sparse** vector $\vec{x}$

-can measure vector numerous times, with arbitrary $\vec{a_i}$ (as decided by the compressive sensing algorithm)

-retrieve only real results $\vec{y_i} \in \mathbb{R}$

Compressive Sensing ⇒ QM?

signal ⇒ **state vector**

sparsity ⇒ **most collapsed states impossible**

$$?|\mathbf{000}\rangle + ?|\mathbf{001}\rangle + ?|\mathbf{010}\rangle + 0|011\rangle + ?|\mathbf{100}\rangle + 0|101\rangle + 0|110\rangle + 0|111\rangle$$

Use cases:

Circuit Verification: Only entangling k qubits

Error/Interference: Finding localized noise

more?

State reconstruction in QM

Determine $|\psi>$ from discrete set of possibilities

- Quantum Hypothesis Testing
- Unambiguous state discrimination

Repeated measurement to estimate $|\psi>$

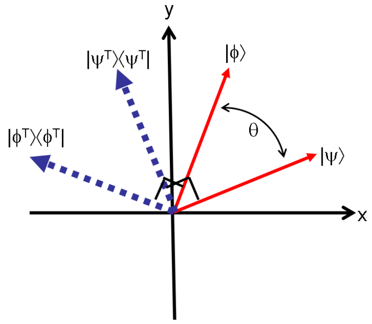
- **Quantum Tomography**
- **Quantum Process Tomography**

Chefles 2000 arXiv quant-ph/0010114

Applications of state reconstruction

Quantum Circuit Design

circuit verification



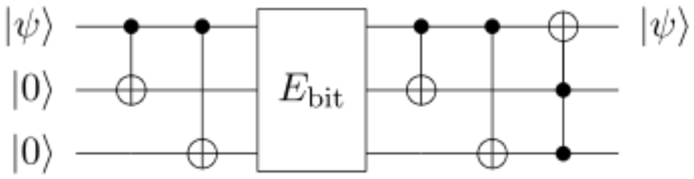
Error Correction

random noise (i.e. stray B-fields)

Interference

adversarial noise

bit-flip codes, parity checks



Example

1000-qubit operation

$$n = 2^{1000} \approx 10^{300}$$

Goal: Determine systematic noise
(alters at most 10 qubits)

$$k < 10^{24}$$

State vector is sparse!

Good candidate for compressive sensing

QM Challenges

Quantum Collapse

measuring the state disturbs the state!

No Cloning Theorem

can't copy state, have to recreate

Operators

must sum to identity

$$\sum_k M_k^\dagger M_k = I$$

New setting: qubits

Modified PhaseCode pipeline

- Converted classical measurement vectors $\vec{a}_i$ into quantum measurement operators A_i
- **1 QM measurement**, repeated sampling to obtain $\vec{y}_i$: each operator occurs w.p. $|A\vec{x}|^2$

Maintains order-optimal decoding time O(K)

Modified Pipeline

1. Prepare many state vectors $\vec{x}$: measure each with M
2. From probability distribution, estimate $|A\vec{x}|^2$, then $\vec{y}_i$

prepare many k-sparse, reproducible vectors $\vec{x}$

Repeated sampling with quantum measurement $M = \{A_i\}$

$\vec{y}_i$ **calculated from probability distribution** $|A\vec{x}|^2$

measurement decoder

output $\vec{x}$

Analysis

Operators: sum to identity $\sum_i A_i^\dagger A_i = I$

Normalized appropriately $A_i = \alpha_i a_i a_i^\intercal$

Proven: This is always possible!

Prepared samples: used to estimate $\vec{y_i}$

more samples ⇒ better estimation

Extendable

any robust compressive sensing algorithms can be used

could trade runtime for ease of implementation

Challenges & Further Discussion

Practical considerations

how do we build measurement operators A_i ?

which qubit construction processes could use this?

optimizing algorithm for low-entanglement operators & estimation error

New domains

other useful settings for QM + compressive sensing?

mixed-state algorithms

References

(1) Shabani 2009 arXiv 0910.5498

(2) Flammia 2012 arXiv 1205.2300

(3) Mirhosseini 2014 arXiv 1404.2680

(4) Pedarsani, Lee, Ramchandran 2014 arXiv 1408.0034

(5) Clarke 2000 arXiv quant-ph/0007063v1

(6) Keyes 2005 http://www.unm.edu/~roy/usd/usd_review.pdf

(7) Kimura 2008 arXiv 0808.3844

(8) Kaniowski 2014 Springer 10.2478/s12175-013-0199-x

(9) Chefles 2000 arXiv quant-ph/0010114

Thanks to Pedarsani, Lee, Ramchandran for the Campanile image!