Tiles of Tantrix

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Summer 2020

1 Context

I recently learned about the mind game Tantrix. Each hexagonal game tile shows three colored paths. See Figure 1 for an image. All paths terminate on different sides of the hexagon. Every tile in Tantrix is unique. How many unique Tantrix-style tiles are there?

2 Naming the paths

Consider a hexagon. A Tantrix path connects two different sides of the hexagon. These sides can be adjacent, two edges apart, or on opposite ends of the hexagon. Let's call them 1-step, 2-step, and 3-step paths, respectively. See Figure 2 for an illustration.

3 Fitting paths on a hexagon

Three paths lie on the Tantrix tile, ending at all six sides of the hexagon. Only a few configurations are possible. They are enumerated below and illustrated in Figure 3.

3.1 Numbering the sides of the hexagon

Let's number the sides of the hexagon from 1 to 6 to describe the different configurations. This will make it easier to describe the paths on the hexagon. Remember that the location of "side 1" is arbitrary; if you choose side 1 to be the top and I choose side 1 to be the bottom, we will get the same configurations, rotated.

Figure 1: Tantrix tiles. Licensed CC BY 3.0 from Wikimedia.

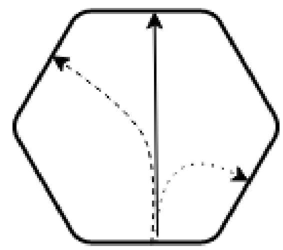

Figure 2: This image shows a 1-step path (dotted), 2-step path (dashed), and 3-step path (solid). Note that any paths between two hexagon edges will be one of these types. Created with an an online diagram maker tool.

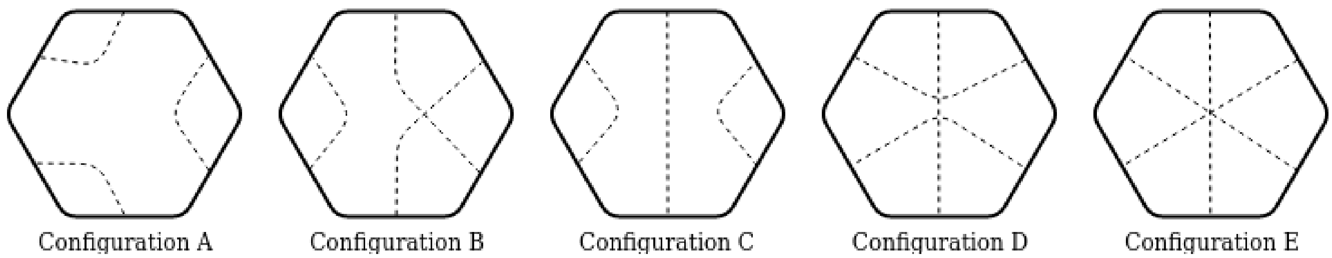

Figure 3: These are the five possible Tantrix tile configurations. Each path starts and ends from a unique side of the hexagon. Created with an an online diagram maker tool.

3.2 With 1-step paths

Suppose the Tantrix tile has a 1-step path. Without loss of generality, let's pair side 1 with side 2. (If we chose a different pair, we'll find the same configurations, rotated.) The other sides can be paired up in the following ways:

3 with 4, and 5 with 6. This uses 3 1-step paths. Let's call it Configuration A.

3 with 5, and 4 with 6. This uses 1 1-step path and 2 2-step paths. Let's call it Configuration B.

3 with 6, and 4 with 5. This uses 2 1-step paths and 1 3-step path. Let's call it Configuration C.

3.3 With 2-step paths

Suppose the Tantrix tile has a 2-step path, this time pairing side 1 with side 3. The remaining sides can be paired up as follows:

2 with 4, and 5 with 6. This is a rotation of Configuration B.

2 with 5, and 4 with 6. This uses 2 2-step paths and 1 3-step path. Let's call it Configuration D.

2 with 6, and 3 with 4. This is another rotation of Configuration B.

3.4 With 3-step paths

Suppose the Tantrix tile has a 3-step path, this time pairing side 1 with side 4. The remaining sides can be paired up as follows:

2 with 3, and 5 with 6. This is a rotation of Configuration C.

2 with 5, and 3 with 6. This uses 3 3-step paths. Let's call it Configuration E.

2 with 6, and 3 with 5. This is a rotation of Configuration D.

4 Coloring paths on a hexagon

Each path on a Tantrix tile has a different color. Even with 3 colors (let's call them X Y and Z), there are multiple ways to uniquely color the same configuration.

4.1 Notation

Here's a short way to describe the coloring on a Tantrix tile. I will use a three-letter combination, like "XYZ", that means color X for the first path, color Y for the second path, and color Z for the third path on the Tantrix tile. We can order the paths any way we like, as long as we keep it the same throughout the discussion. The six possible ways to color a configuration are XYZ, XZY, YXZ, YZX, ZXY, ZYX. So, each configuration has up to 6 different colorings. Some colorings might be rotations of the same tile! Let's find out using a symmetry argument.

4.2 What do you mean, symmetry?

Broadly speaking, symmetry is about making a change to an object and it still "looking the same". With Tantrix, I am referring to rotational symmetry. Some tiles have a special rotation angle: If you rotate by this angle, the (uncolored) tile will look the same. The exact rotation angle divides evenly into 360°. See Figure 4 for an image.

4.3 Configurations A and E

Both of these configurations have 3 copies of one type of path. They share rotational symmetry with angle 120°. You can do this three times before you have rotated all the way around: $3 \quad 120° = 360°$. For coloring, that means that XYZ, YZX, and ZXY are rotations of the same tile. (Similarly, XZY, ZYX, and YXZ refer to the same tile.) So, there are only two unique colorings for these configurations.

4.4 Configurations C and D

Both of these configurations have 1 3-step path through the middle of the hexagon. They share rotational symmetry with angle 180°. If you rotate the tile halfway around, the configuration will "look the same". For coloring, suppose that the first path is the 3-step path. Then XYZ and XZY are rotations of the same tile, as are YXZ with YZX, and ZXY with ZYX. So, there are three unique colorings for these configurations.

4.5 Configuration B

Configuration B has no rotational symmetry smaller than 360°. So, it can be uniquely colored six different ways.

4.6 Summary table

Here's a summarized view of all Tantrix configurations and colorings.

Configuration	# 1-step paths	# 2-step paths	# 3-step paths	Symmetry angle (°)	Unique colorings
A	3	0	0	120	2
B	1	2	0	-	6
C	2	0	1	180	3
D	0	2	1	180	3
E	0	0	3	120	2

5 Counting unique Tantrix tiles

5.1 Unique 3-color tiles

In the table listed in 4.6, the sum of the last column is 16. So, there are 16 unique Tantrix-style tiles with 3 colors.