ENHANCING FEEDBACK PERIMETER CONTROLLERS FOR URBAN NETWORKS
BY USE OF ONLINE LEARNING AND DATA-DRIVEN ADAPTIVE OPTIMIZATION

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ABSTRACT

An adaptive scheme for perimeter control of heterogeneous transportation networks is presented. The proposed methodology utilizes the concept of the Macroscopic Fundamental Diagram (MFD) integrated with an adaptive optimization technique. First, a new MFD-based macroscopic model is introduced to describe the dynamics of heterogeneous urban networks that can be partitioned in a small number of homogeneous regions. The non-linear model describes the evolution of the multi-region system over time assuming the existence of well-defined MFDs. Many linear approximations of the model (for different set-points) are used for designing optimal multivariable integral feedback regulators. Since the resulting regulators are derived from approximations of the non-linear model, they are further enhanced in real-time based on performance measurements and online learning/adaptive optimization. The recently proposed Adaptive Fine-Tuning (AFT), an iterative data-driven algorithm is used for that purpose and its objective is to optimize the gain matrices and set-points of the multivariable perimeter controller based on real-time observations. The derived control scheme is tested in micro-simulation and different evaluation criteria are studied. The urban network of Barcelona, Spain is partitioned in four homogeneous regions and perimeter flow control is applied in the common boundaries between regions. The simulation results show that the total delay in the network decreases significantly by only controlling a small number of intersections. It is worth noting, that since the boundaries of the network are not controlled (only internal intersections are considered) the controller achieves a better distribution of congestion between the regions, thus preventing the network degradation and improving total outflow.

Keywords: Real-time urban perimeter control; macroscopic fundamental diagram; linear feedback regulators; online learning; adaptive optimization.
INTRODUCTION

Traffic congestion in urban environments and modern metropolitan areas is persistently growing over the years, resulting in significant degradation of the infrastructure and excessive commuting delays during the peak hours. Real-time traffic management is deemed to be an efficient and cost effective way to ameliorate traffic conditions and prevent gridlock phenomena in cities. Although many methodologies have been developed for real-time signal control over the last decades (see e.g. (1) for a good review), the design of efficient control strategies for heterogeneous large-scale urban networks that can deal with oversaturated conditions (where queues spill back to upstream links) remains a significant challenge. Strategies that are widely used around the world like SCOOT (2) and SCATS (3) are based on heuristic optimization techniques and are not efficient when the network faces propagation phenomena and queue spillbacks. Other traffic responsive strategies (4, 5, 6) use complex optimization methods which make their online application to large-scale urban networks difficult due to high computational requirements. TUC (7, 8) (see also (9, 10)) is a practicable network-wide control strategy which tries to deal with oversaturated conditions by minimizing and balancing the relative occupancies of the network links. Another decentralized approach that was recently proposed is the max-pressure controller (11, 12), which acts locally in coupled intersections and has been proven (under certain conditions) to stabilize the queues of the network. However, in the case of heterogeneous networks with multiple pockets of congestion and heavily directional demand flows this type of control (i.e. TUC, max-pressure) may not be optimal.

An alternative approach for real-time network-wide control for heterogeneous urban networks that has recently gained a lot of interest is the perimeter control. The basic concept of such an approach is to partition the heterogeneous network into a small number of homogeneous regions and apply perimeter control to the inter-transferring flows along the boundaries between regions. The input flows to a region (which are also output flows for the neighbouring regions) can be controlled at the intersections located at the borders of the region, so as to maximize the total throughput of the system. Perimeter control (or gating) policies have been introduced for single-region homogeneous networks (13, 14) and multi-region heterogeneous networks (15, 16) using different control methodologies. The key modelling tool that is used by all the aforementioned strategies is the Macroscopic Fundamental Diagram (MFD), which provides a concave, low-scatter relationship between network vehicle accumulations [veh] or density [veh/km] and network circulating flow [veh/h]. The concept of a network MFD was firstly introduced in (17), but the empirical verification of its existence with dynamic features is recent (18). The stability of the MFD shape faces two main challenges, which are (a) the hysteresis phenomena that appear at the onset or offset of congestion and (b) the heterogeneity of traffic in urban networks. Heterogeneous networks do not have a well-defined MFD, especially in the congested regime. Partitioning such a network into homogeneous regions (i.e. areas with compact shape that have small variance of link densities) can result in well-defined MFD as shown in (19). The MFD concept is a useful tool for designing control policies, as it provides aggregated relationships between traffic variables and reduces the complexity of traffic flow dynamics (there is no need for tracking the state of each individual link of the network).

There are a couple of works (15, 16) that deal with perimeter control for multi-region systems with MFD-based modelling. However, none of this works deals with parameter uncertainties in the model or short-term and long-term variations in the dynamics of the system. In (15, 20) a...
model predictive control approach is proposed and a nonlinear MFD-based model is used to
describe the dynamics of the system. Although the controller is tested for different errors in the MFDs
and the demand profiles, perfect knowledge of the model parameters is assumed. In (16) a Linear-
Quadratic state feedback Regulator (LQR) and two versions of the optimization problem (with and
without integral action) are studied. The LQR/LQI gain matrices are designed by linearizing the
nominal nonlinear traffic dynamics around the set-points. Note that such nominal optimal control
laws do no guarantee the robustness properties with respect to uncertainties. Some more recent
works (21, 22) try to deal with adaptive schemes in order to improve the performance of the con-
troller. Finally, the stability and robustness of MFD-based systems is studied in (23, 24, 25) under
different adaptive approaches.

In this work a new generic MFD-based model is introduced to describe the aggregated dy-
namics of multi-region systems. Many linear approximations of the model (for different set-points)
are employed to derive optimal multivariable proportional integral (PI) feedback regulators. These
regulators are applied to the multi-region system and provide an initial set of observations. Then,
the gain matrices and set-points of the PI controller are updated in real-time based on performance
measurements by an adaptive optimization algorithm. The initial set of observations that are ob-
tained by linearizing the model are used to enhance the online learning of the algorithm. The over-
all control scheme is tested in micro-simulation for the urban network of Barcelona, Spain, which
includes more than 600 intersections and the impact of the applied perimeter control is evaluated
via the corresponding MFDs and other performance measures. Note, that most of the perimeter
or gating control strategies utilizing MFD modeling and tested in micro-simulation environment,
apply control in the external boundary of the network and as a result queued vehicles create point
queues that do not interact or constrain other movements. In reality, movements outside the pro-
tected zones might be influenced by these queues. In this work, the protected zones/boundaries
are internal to the network and interactions are taken into consideration. This approach signifi-
cantly challenges the performance of feedback controllers, as the disturbance in the system due to
uncontrolled inflow is higher than in systems that control the external boundaries of the network.

The specification of set-points for monocentric networks with well-defined destination at-
tractions is straightforward as the objective is to operate the protected regions at the critical ac-
cumulation that maximizes flow. Nevertheless, heterogeneous networks with multiple regions of
attraction would require a non-trivial choice of set-points (which are related to the level of conges-
tion in each region). Physically speaking, if a control approach can keep all regions below or close
to the critical accumulation of each MFD, then the problem is well resolved (see for example (16)).
A challenge, which is investigated here, is the optimal choice of set-points that can lead hetero-
geneous systems in desired states with minimum congestion. While model predictive approaches
(see for example (20)) can identify close-to-optimal control policies, unreliable predictions might
harden the procedure. In this work we try to overcome these difficulties by identifying values for
set-points (and gains) through an automatic fine-tuning algorithm (building on (26)). AFT (Adap-
tive Fine-Tuning) is an iterative data-driven algorithm that receives a scalar performance index
(e.g. total delay) for different sets of controller parameters (gain matrices ans set-points) and tries
to learn how these parameters affect the regulator performance. In each iteration AFT updates the
values of the parameters aiming at better performance. The control variables consist of the ratios of
inter-transferring flows between neighbourhood regions and the actuators correspond to the traffic
lights of these areas (e.g. boundaries between regions). The performance of a fixed-time policy is
compared to the final controller that is obtained after the convergence of AFT.
The remainder of the paper is organized as follows: next section presents the modeling of aggregated dynamics for an urban network partitioned in $N$ regions. Then, a linear optimal control methodology is applied to different linear approximations of the model. The designed control is enhanced in real-time by a data-driven adaptive optimization algorithm, which is described in details. Finally, the integrated control scheme is applied to the network of Barcelona in micro-simulation and the obtained results are presented.

**AGGREGATED DYNAMICS FOR MULTI-REGION SYSTEM BASED ON MFDS**

Consider an urban network partitioned in $N$ homogeneous regions (Figure 1(a)). The index $i \in N = \{1, 2, \ldots, N\}$ denotes the region of the system and $n_i(t)$ the total accumulation (number of vehicles) in region $i$ at a given time $t$. Let $N_i$ be the set of all regions that are directly reachable from the borders of region $i$, i.e. adjacent regions to region $i$ and $q_{i,\text{in}}(t)$, $q_{i,\text{out}}(t)$ the inflow and outflow of region $i$ at time $t$, respectively. Also, let $d_i(t)$ denote the total uncontrolled traffic demand (disturbance) in region $i$ at time $t$. Note that $d_i(t)$ includes both the internal generated demand (vehicles entering the network from on-street and off-street parking areas) and the external uncontrollable inflows. The conservation equation for each region $i$ of the system reads

$$\frac{dn_i(t)}{dt} = q_{i,\text{in}}(t) - q_{i,\text{out}}(t) + d_i(t)$$

For every region $i$ it is assumed that there exists a production MFD relating the accumulation $n_i(t)$ to the total production $P_i(n_i(t))$ and describes the performance of the sub-system in an aggregated way. This MFD can be easily estimated using measurements from loop detectors and/or GPS trajectories. The total outflow $O_i$ of region $i$ (number of vehicles exiting the region per unit time) either because they finished their trip or because they move to a neighbouring region) can be estimated by $O_i(n_i(t)) = P_i(n_i(t))/L_i$, where $L_i$ is the average trip length for region $i$, which is assumed to be independent of time and destination, internal or external, in $i$. Furthermore, let $M_{ij}(n_i(t))$, $(i \neq j)$ denote the total transfer flow from region $i$ to region $j$ at time $t$. This variable can also be related to the accumulation $n_i(t)$ by using an MFD (as demonstrated later) and estimated from the measurements of all the detectors located in the borders between regions $i$ and $j$ (Figure 1(b)). Finally, $M_{ii}(n_i(t))$ denotes the internal trip completion rate in region $i$ (vehicles finishing their trip inside the region) and is given by

$$M_{ii}(n_i(t)) = O_i(n_i(t)) - \sum_{j \in N_i} M_{ij}(n_i(t))$$

Previous works with model predictive control (15, 20) estimate the flows $M_{ij}$ by utilizing more detailed description of the system state, i.e. $M_{ij}(n_i(t)) = n_{ij}(t)/n_i(t) \cdot O_i(n_i(t))$, where $n_{ij}$ describes the number of vehicles in region $i$ with $j$ as the next destination. As the current work mainly utilizes data from loop detectors to estimate the system states, $n_{ij}$ variables are difficult to be estimated without probe vehicle information. The model still provides a decent description of system dynamics even under adaptive control conditions. To this end, the inflow to region $i$ is the summation of the transferring flows from all its neighbouring regions and is given by

$$q_{i,\text{in}}(t) = \sum_{j \in N_i} u_{ji}(t)M_{ji}(n_j(t))$$
where the control variables \( u_{ij}(t), \forall i \in \mathcal{N}, h \in \mathcal{N}_i \) denote the fraction of the flow that is allowed to transfer from region \( j \) to region \( i \) at time \( t \), to be calculated by the perimeter controller. Equivalently, the outflow of region \( i \) is the summation of the transferring flows to all its neighbouring regions plus the trip completion rate in region \( i \) and is given by

\[
q_{i,\text{out}}(t) = M_{ii}(n_i(t)) + \sum_{j \in \mathcal{N}_i} u_{ji}(t)M_{ji}(n_i(t))
\]

The values of the control variables \( u_{ij} \) are constrained by physical or operational constraints as follows

\[
0 < u_{ij,\text{min}} \leq u_{ij}(t) \leq u_{ij,\text{max}} < 1, \quad \forall i \in \mathcal{N}, j \in \mathcal{N}_i
\]

where \( u_{ij,\text{min}}, u_{ij,\text{max}} \) are the minimum and maximum permissible transferring rates of flows, respectively. Also, each region \( i \) has a maximum accumulation \( n_{i,\text{max}} \)

\[
0 \leq n_i(t) \leq n_{i,\text{max}}, \quad \forall i \in \mathcal{N}
\]

and if \( n_i(t) = n_{i,\text{max}} \) then the region reaches gridlock and all the inflows along the periphery are restricted. Invoking (3)-(4) and (1) the following non-linear state equation is obtained

\[
\frac{dn_i(t)}{dt} = \sum_{j \in \mathcal{N}_i} u_{ji}(t)M_{ji}(n_j(t)) - M_{ii}(n_i(t)) - \sum_{j \in \mathcal{N}_i} u_{ij}(t)M_{ij}(n_i(t)) + d_i(t)
\]

This non-linear model may be linearized around some set-point \( (\hat{n}_i, \hat{n}_j, \hat{u}_{ij}, \hat{u}_{ji}, \hat{d}_i), j \in \mathcal{N}_i \).

The selection of \( \hat{n}_i \) is closely related to the existence of MFDs \( O_i(n_i(t)), M_{ij}(n_i(t)) \), which are approximated by third degree polynomial functions of \( n_i(t) \) and provide a critical accumulation at which the performance of the region is optimized. The desired set-point should satisfy the steady-state version of (7) which reads

\[
0 = \sum_{j \in \mathcal{N}_i} \hat{u}_{ji}(t)M_{ji}(\hat{n}_j(t)) - M_{ii}(\hat{n}_i(t)) - \sum_{j \in \mathcal{N}_i} \hat{u}_{ij}(t)M_{ij}(\hat{n}_i(t)) + \hat{d}_i(t)
\]
By denoting $\Delta x = x - \hat{x}$ analogously for all variables the linearization of (7) around the selected set-point yields

$$\Delta \dot{n}_i(t) = \sum_{j \in \mathcal{N}_i} \Delta u_{ij}(t) M_{ji}(\Delta \hat{n}_j(t)) + \sum_{j \in \mathcal{N}_i} \Delta \dot{n}_j(t) M'_{ji}(\Delta \hat{n}_j(t)) - \Delta n_i(t) M'_{ii}(\Delta \hat{n}_i(t))$$

$$- \sum_{j \in \mathcal{N}_i} \Delta u_{ij}(t) M_{ij}(\Delta \hat{n}_i(t)) - \sum_{j \in \mathcal{N}_i} \Delta u_{ij}(t) \Delta n_i(t)(t) M'_{ij}(\Delta \hat{n}_i(t)) + \Delta d_i(t)$$  \hspace{1cm} \text{(9)}

Applying the linear equation (9) to an urban network partitioned in $N$ regions the following state equation (in vector compact form) describes the evolution of the system in time

$$\Delta \hat{n}(t) = \tilde{A} \Delta \hat{n} + \tilde{B} \Delta u + \tilde{C} \Delta d$$  \hspace{1cm} \text{(10)}

where $\Delta \hat{n} \in \mathbb{R}^N$ is the state deviations vector $\Delta n_i = n_i - \hat{n}_i, \forall i \in \mathcal{N}$; $\Delta u \in \mathbb{R}^M$ is the control deviations vector $\Delta u_{ij} = u_{ij} - \hat{u}_{ij}, \forall i \in \mathcal{N}, j \in \mathcal{N}_i$; $\Delta d \in \mathbb{R}^N$ is the demand deviations vector $\Delta d_i = d_i - \hat{d}_i, \forall i \in \mathcal{N}$; $\tilde{A} \in \mathbb{R}^{N \times N}$, $\tilde{B} \in \mathbb{R}^{N \times M}$ are the appropriate state and control matrices, that are derived by application of (9), $\forall i \in \mathcal{N}, j \in \mathcal{N}_i$; $\tilde{C} = I_{N \times N}$ is the identity matrix.

The continuous time linear state system (10) of the multi-region system may be directly translated in discrete time (with sample time $T$) by use of standard formulas (e.g. zero-order hold method of Matlab Simulink). The resulting discrete time system in vector form reads

$$\Delta \hat{n}(k+1) = A \Delta \hat{n}(k) + B \Delta u(k) + \Delta d(k)$$  \hspace{1cm} \text{(11)}

where $k = 0, 1, \ldots, K - 1$ is the discrete time index and $A \in \mathbb{R}^{N \times N}$, $B \in \mathbb{R}^{N \times M}$ are the corresponding discrete time state and control matrices.

**DESIGN OF MULTIVARIABLE PI FEEDBACK REGULATORS**

The discrete time linear system (11) approximates the original non-linear system around the set-point and can be used for application of efficient methodologies from linear MIMO optimal control theory. The approach of Linear-Quadratic-Integral (LQI) control is employed here. In this approach the state of the system (11) is augmented by additional state variables that integrate the error signal $\Delta \hat{n}$, which is then used as a feedback term to provide zero steady-state error. The new state variables are given by

$$\Delta y(k+1) = \Delta y(k) + C \Delta \hat{n}(k)$$  \hspace{1cm} \text{(12)}

where $\Delta y \in \mathbb{R}^Z$ is the integral vector and $C \in \mathbb{R}^{Z \times N}$ is a matrix that typically consists of 0 and 1 such that $Z$ components (or linear combinations of components) of the system state are integrated in (11). Note that $N + Z \leq M$ must hold in order for the system to be fully controllable. The augmented discrete time system (11)–(12) can be written in compact form as

$$\Delta \hat{n}(k+1) = \tilde{A} \Delta \hat{n}(k) + \tilde{B} \Delta u(k) + \tilde{C} \Delta d(k)$$  \hspace{1cm} \text{(13)}

where $\hat{n}(k) = [\Delta \hat{n}(k) \Delta y(k)]^T$ is the augmented state vector and $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ are the augmented state, control, and demand matrices, respectively, which are given by

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C & I \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C \\ 0 \end{bmatrix}$$  \hspace{1cm} \text{(14)}
Finally, for formulating the LQI optimal control problem the following quadratic objective criterion is defined

$$\min_u J(u) = \frac{1}{2} \sum_{k=0}^{K} \left( \Delta n^T(k) Q \Delta n(k) + \Delta u^T(k) R \Delta u(k) + \Delta y^T(k) S \Delta y(k) \right)$$  \hspace{1cm} (15)$$

where $Q \in \mathbb{R}^{N \times N}$, $R \in \mathbb{R}^{M \times M}$ and $S \in \mathbb{R}^{Z \times Z}$ are user-defined diagonal weighting matrices that can influence the magnitude of each term of the objective criterion and are usually defined via trial-and-error. The optimal closed-loop solution of (15) subject to (13) for an infinite time horizon (i.e. $K = \infty$), and assuming $\Delta d(k) = 0$, leads to the following multivariable PI feedback regulator (see (27) for details)

$$u(k) = u(k-1) - K_p [n(k) - n(k-1)] - K_i [n(k) - \hat{n}]$$  \hspace{1cm} (16)$$

where $K_p, K_i \in \mathbb{R}^{M \times N}$ are the proportional and integral gains of the regulator, which are computed by the solution of the corresponding discrete-time Riccati equation and depend only on the matrices $\hat{A}, \hat{B}, Q, R$ and $S$ defined above. In case that future demand flow predictions are available (i.e. $\Delta d(k) \neq 0$, in (13)), simple feedforward control techniques can be used to integrate the disturbance predictions to the problem solution (27). It should be noted that the number of control variables $M$ depends on the network partition and the sets $N_i, i \in N$, albeit for any arbitrary formation of the $N$-region system $N \leq M$ always holds. Finally, it should be emphasized that a well-known property of the PI regulator (16) is that it provides zero steady-state error (due to the existence of the integral term), i.e. $n(k) = \hat{n}$ under stationary conditions.

The state feedback regulator (16) is activated in real-time at each control interval $T$ and only within specific time windows based on the current accumulations $n(k)$ (i.e. by use of two thresholds $n_{i,start}$ and $n_{i,stop}$ and real-time measurements). The required real-time information of the vehicle accumulations $n(k)$ can be directly estimated via loop detector time-occupancy measurements. Different approaches to estimate MFD related state variables with real data are described in (28). Equation (16) calculates the fraction of flows $u(k)$ to be allowed to transfer between neighborhood regions. In case the ordered values $u_{ij}(k)$ violate the constraints (5) they should be adjusted to become feasible, i.e. truncated to $[u_{ij,min}, u_{ij,max}]$. Moreover, the values of $u(k-1)$ used on the right-hand side of (16), should be the bounded values of the previous time step (i.e. after the application of the constraints) in order to avoid possible wind-up phenomena in the PI regulator. The obtained $u_{ij}(k)$ values are then used to derive the green time durations for the stages of the signalized intersections located at the boundaries of neighborhood regions. The ordered transferring flows are equally distributed to the corresponding intersections and converted to a transfer link green stage duration, with respect to the saturation flow of the link and the cycle time of the intersections.

The structure of the controller (16) is similar to the one used in (16) although derived by a more accurate model. Moreover, here there are no control variables at the external borders of the network but only at the borders between regions. As a consequence, there are no vehicles kept outside of the network in order to protect the congestion of the regions (which is also the case in (14)) and all the (gating) queues created by the controllers are internal to the network and thus affecting other movements. Furthermore, the gain matrices $K_p, K_i$ and set-points $\hat{n}$ of the controller are optimized in real-time by the use of a learning/adaptive algorithm and based on real performance measurements. The closed-loop adaptive optimization scheme that constantly
updates the parameters of the controller is presented in the next section. Our analysis shows that
due to the additional complexities related to uncontrolled external boundaries, previous PI-type
strategies are not capable to perform at the desired states.

ADAPTIVE OPTIMIZATION ALGORITHM DESCRIPTION

The parameters $K_P$, $K_I$, $\hat{n}$ of the regulator (16) are updated in real-time based on measure-
ments of an objective function, so as to optimize the performance of the controller. The Adaptive
Fine-Tuning (AFT) algorithm is used for that purpose. AFT is a recently developed algorithm (see
(26, 29) for details) for tuning the parameters (in the specific case the gain matrices and set-points)
of controllers in an optimal way. It is an iterative algorithm that is based on machine learning
techniques and adaptive optimization principles and adjusts the control gains and set-points to the
variations of the process under control. The working principle of the integrated closed-loop system
(PI regulator and AFT) is presented in Figure 2 and may be summarized as follows:

- The $N$-region MFDs system is controlled in real-time by the multivariable PI regulator ((16))
  which includes a number of tunable parameters $\theta \triangleq \text{vec} (K_P, K_I, \hat{n})$, where $\theta$
  is a vector with size $2 \times (M \times N) + N$ and its elements are the entries of $K_P, K_I, \hat{n}$ taken row-wise.

- At the end of appropriately defined periods $T_c$ (e.g. at the end of each day), AFT algorithm
  receives the value of the real (measured) performance index $J$ (e.g. total delay of the system),
  as well as the values of the most significant measurable external disturbances $x$ (e.g. aggre-
gated demand). Note that the scalar performance index $J(\theta, x)$ is a (generally unknown)
  function of the external factors $x$ and the tunable parameters $\theta$.

- Using the measured quantities (the samples of which increase iteration by iteration), AFT
  calculates new tunable parameter values to be applied at the next period (e.g. the next day)
in an attempt to improve the system performance.

- This (iterative) procedure is continued over many periods (e.g. days) until the algorithm
  converges and an optimal performance is reached; then, AFT algorithm may remain active
  for continuous adaptation or can be switched off and re-activated at a later stage.

The main component of the employed algorithm is a universal approximator $\hat{J}(\theta, x)$ (e.g.,
a polynomial-like approximator or a neural network) that is used in order to obtain an approx-
imation of the nonlinear mapping $J(\theta, x)$, based on all previous samples. At each algorithm
iteration $k_c$, the algorithm uses all the collected data for the sets of parameters applied at iterations
$1, 2, \ldots, k_c$ and performs the steps described in Algorithm 1 to determine the new set of param-
eters for the next period (e.g. next day). A shortcoming of AFT approach (as most data-driven
learning algorithms) is that the first iterations might create controllers that are much worse than
the no control case, due to the lack of information. While the designed controller will be improved
after some iterations, real life implementation would be problematic as the first trials might create
very congested conditions in the system. In our approach, we choose the first iterations based on
the multivariable PI regulator obtained by solving the Riccati equation (see problem formulation
(13)–(15)), for different desired states. This allows for good quality initial solutions that overcome
the discrepancies of first AFT iterations.
APPLICATION OF THE PERIMETER CONTROL SCHEME TO MICRO-SIMULATION

The efficiency of the adaptive flow control scheme described in the previous sections is tested in microsimulation experiments. The Aimsun microscopic environment is used and the real-time implementation of the control scheme is replicated through the simulator API. Only loop detectors data is utilized to estimate the state of the system, highlighting the feasibility and applicability of the developed framework in a real life conditions.

Network description

The urban network of Barcelona, Spain is used as the test site, which is modeled and calibrated in AIMSUN (Figure 3(a)). The network covers an area of 15 square kilometers with about 600 intersections and 1500 links of various lengths. The number of lanes for through traffic varies from 2 to 5 and the free flow speed is 45 kilometers per hour. Traffic lights at signalized intersections are operating on multi-phase fixed-time plans with constant (but not equal) cycle lengths. For the simulation experiments, typical loop-detectors have been installed around the middle of each network link. The OD-based demand that is used for the simulations consist of 123 origin centroids and 132 destination centroids and provides a good replication of real life conditions as it generates realistic traffic congestion patterns in the network. The duration of the simulation is 6 hours including a 15 minutes warm-up period. In the no control case (where the real fixed-
time plans are applied to the intersections) the network faces some serious congestion problems, with queues spilling back to upstream intersections. Note, that the real-time Dynamic Traffic Assignment (DTA) module of the simulator (C-Logit route choice model) is activated every 3 minutes, therefore the drivers adapt to the traffic conditions and the distribution of demand into the
FIGURE 3 The test site of Barcelona, Spain: (a) simulation model with four regions; (b) results of the clustering algorithm and controlled intersections. Blue circles correspond to intersections belonging to $u_{14}$, red to $u_{24}$, green to $u_{34}$ and black to $u_{4j}$, $j = 1, 2, 3$. 

Traffic congestion in the city of Barcelona is unevenly distributed, creating multiple pockets of congestion in different areas of the network. As MFD depends on the distribution of link densities (occupancies, speeds), partitioning heterogeneously loaded cities with uneven distribution of congestion into homogeneous regions is a possible solution to take advantage of well-defined MFDs. In fact, the outflow of the network is a function of both average and variance of link densities. Since traffic conditions are spatially correlated in adjacent roads and congestion propagates network is more realistic. Previous works (30, 31, 32) have shown that driver adaptivity increases the performance of large-scale networks. Note that the developed controller does not utilize any information related to OD, only loop detectors data.
from adjacent links, describing the main pockets of congestion in a city with a small number of clusters without the need for detailed information in every link of the network is conceivable. By partitioning, we aim to group spatially-connected links with close density values within a cluster, which increases the network flow for the same average density. Spatial connectivity is a necessary condition that makes feasible the application of perimeter control strategies. The partitioning algorithm used in this study is an optimization framework called “Snake” (33), which considers heterogeneity index as a main objective function and contiguity is forced explicitly by imposing constraints. This approach needs desired number of clusters as a predefined input and it obtains optimal number of clusters by evaluating heterogeneity metric for different number of clusters. By use of this algorithm, the network of Barcelona is partitioned into 4 homogeneous regions that are shown in Figure 3(b).

The simulation is first executed with the fixed-time signal plans of the city to obtain the data needed for the control design. Figure 4(a) presents the production MFD for the whole network for the first two hours of the simulation (onset of congestion). Each point corresponds to the aggregated measurements of all the detectors and the time interval is 90 sec (equal to the control interval $T$). The network MFD has low scatter and reaches the congested regime (production reduces from 4500 to 2500 veh-km/$T$). The production of each region separately (again for the first two hours) is displayed in Figure 4(b). Note that region 2 is the only region that does not get congested, whereas the rest of the regions get states with increased accumulations and decreased productions. Region 4 is the only one that has common boundaries with all other regions. Figures 4(c) and 4(d) demonstrate the MFDs for the transfer flows $M_{41}, M_{42}, M_{43}$ and the trip completion rate $M_{44}$ as third degree polynomial functions of the region accumulation $n_4$. These functions are assumed not to depend on the control decisions (assumption that has been verified by simulation data not presented here) and are utilized by the model described earlier in order to derive the LQI multivariable regulator.

Simulation set-up and offline design of LQI

The network partitioning presented in the previous section derives $M = 6$ control and $N = 4$ state variables (i.e. $u = [u_{14} \; u_{24} \; u_{34} \; u_{41} \; u_{42} \; u_{43}]^T$ and $n = [n_1 \; n_2 \; n_3 \; n_4]^T$). The duration of the simulation is 6 hours, where the first two hours represent the onset of congestion and the rest 4 hours (offset of congestion) are used to make sure that the network is empty of vehicles and the evaluation metrics are comparable. The simulation step is set to 0.5 sec and the multivariable PI regulator is applied every $T = 90$ sec. The control decisions (after modified to satisfy the operational constraints) are forwarded for application to 28 signalized intersections (out of 600 in the network) which are all across the boundaries of region 4. As shown in Figure 3(b), there are 8 intersections for applying $u_{14}$ (blue circles), 4 for $u_{24}$ (red circles), 5 for $u_{34}$ (green circles), 5 for $u_{41}$, 3 for $u_{42}$ and 3 for $u_{43}$. All the intersections that control the outflow of region 4 are indicated in Figure 3(b) with black circles although they correspond to different control variables. Their location at the borders of region 4 indicates the control variable in which they belong. The signal plans of the aforementioned intersections, and more precisely their operational constraints (e.g. pedestrian phases), determine the corresponding minimum and maximum permissible rates for the control variables $u$ (constraint (5)). The derived $u_{\text{min}}, u_{\text{max}}$ applied at the simulation are given by $u_{\text{min}} = [0.069 \; 0.077 \; 0.061 \; 0.068 \; 0.077 \; 0.077]^T$ and $u_{\text{max}} = [0.827 \; 0.695 \; 0.604 \; 0.824 \; 0.81 \; 0.76]^T$, which are computed by using the minimum
and maximum green phase durations of the 28 intersections.

For the LQI methodology different linearization set-points \( \hat{n} \) are investigated, which lead to different gain matrices that influence the performance of the regulator. The selection of appropriate set-points \( \hat{n} \) for a multi-region system is not straightforward, as at a given time the states of neighbouring regions may be at different regimes of the MFD (congested or uncongested). Here, many different vectors are used for \( \hat{n} \) and provide observations that can be used as initial samples for the learning procedure of AFT algorithm. The matrix \( C \) that provides the state errors and contributes to the integral part of the regulator is set equal to \( C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \), which means that only errors measured for regions 1 and 4 are considered. Since these two regions are more important (masters) the objective criterion minimizes the integral of their state error \( n_i(k) - \hat{n}_i \). The regions 2 and 3 are not included in the integral part, which means that this design does not guarantee zero steady-state error for these regions (slaves). Finally, the diagonal weighting matrices \( Q \in \mathbb{R}^{4 \times 4} \), \( R \in \mathbb{R}^{6 \times 6} \) and \( S \in \mathbb{R}^{2 \times 2} \) are chosen after trial-and-error experiments by studying the behavior of the controller. Physically speaking, these weights depend upon the order of magnitude of each variable and also the weight of each term on the objective criterion. To this end, many different sets were tested until achieving a satisfactory control behavior. Specifically, the diagonal elements of \( Q \) are...
chosen according to the maximum accumulation of each region (albeit with different weights), i.e. $Q_{ii} = 1/n_{i,\text{max}}$ for $i = 1, 4$ and $Q_{ii} = 5/n_{i,\text{max}}$ for $i = 2, 3$; the diagonal elements of $R, S$ are chosen $R_{ii} = 500, S_{ii} = 10^{-6}$ for $i = 1, 2, 3, 4$.

**Preliminary simulation results**

This section presents the results obtained by the simulation experiments. First, AFT is applied online without any prior knowledge of the system and tries to optimize the regulator parameters. AFT runs for 100 iterations starting from an initial point where $K_P = K_I = 0_{6 \times 4}$. For these values the regulator (16) operates as a fixed-time policy and this point is equivalent to the no control (NC) case (i.e. the actual fixed-time plans of the city are applied). The initial values for the set points $\hat{n}$ are obtained from the production MFDs of the NC case (Figure 4(b)) and are equal to $\hat{n} = [4400, 1300, 1800, 5600]^T$. The performance index of AFT (i.e. the objective function $J$ that tries to minimize) is selected to be the total delay of the system, which is available after the end of the simulation. In each iteration the whole simulation of 6 hours is run with the same parameters and the multivariable regulator is activated/deactivated according to the predefined thresholds $n_{i,\text{start}}$ and $n_{i,\text{stop}}$. At the end of the simulation AFT is called to calculate the new values of $K_P, K_I, \hat{n}$ to be used in the next iteration.

Figure 5(a) presents the evolution of the average system delay (measured in sec/km) over the iterations of the algorithm with blue solid line and the initial point with red dashed line (this point corresponds to the fixed-time plan with all the transferring flows distributed equally to the controlled intersections). For the first iterations the system performance is extremely deficient, leading to values of delay three times higher than the NC case. This “spiky” behavior of the algorithm (also reported in (34)) occurs because of the fact that in the first iterations there are not many samples (no knowledge of system performance for different controllers) and the approximator cannot learn from the previous experiments. As the number of iterations increases the learning process becomes better and the objective function exhibits a convergent behavior. To overcome this “spiky” behavior, AFT is combined with the regulators that are derived by applying the LQI methodology to the linearized model. More specifically, different LQI regulators are generated (for different linearization set-points) and are applied to the network for the first iterations. Then, AFT is applied for online optimization, assimilating the knowledge of all the conducted simulations (samples of performance $J$ for different values of $K_P, K_I, \hat{n}$). The results are presented in Figure 5(b). In the first 13 iterations different multivariable PI regulators are applied that are all obtained by the solution of LQI. The figure displays the average value of the performance index (delay) for these 13 iterations with a magenta dashed line. The blue solid line presents the evolution of delay over AFT iterations, while the red dashed line indicates the delay of NC case. The algorithm applies different perturbations of the parameters until it converges. Note by comparing Figures 5(a) and 5(b) that the high values of delays in the first iterations are avoided in 5(b), which allows for a potential real life implementation of the approach (compared to the first problematic iterations of the classical AFT). Note also, that applying AFT with some good initial iterations (coming from LQI methodology) facilitates the convergence of the system to better and smoother solutions (compare Figures 5(a) and 5(b) after iteration 70).

The qualitative characteristics of the best controller (BC) that is obtained after AFT convergence are further investigated in an attempt to interpret its behavior. Figure 5(c) illustrates the time series of accumulations for all regions and for NC (solid lines), BC (dashed lines), respectively.
Figure 5 (a) Evolution of network delay for the 100 iterations of AFT; (b) AFT convergence after running the algorithm with 13 initial points from LQI; (c) time series of regions accumulations for NC and BC; (d) control decisions over simulation time for the BC case.

The controller achieves to maintain the system in better states, i.e. the accumulations of regions 1 and 4 are significantly improved while regions 2 and 3 are slightly deteriorated, as they try to support regions 1 and 4 that have a higher attraction of trips. The BC serves the same number of vehicles in a shorter time than NC (the network empties earlier). The integral of the areas between the solid and dashed lines corresponds to the improvement/deterioration of the total delay. Figure 5(d) displays the trajectories of the control variables for the BC case. The controller is activated after $t = 30$ minutes and stays active for 2 hours and 12 minutes ($t = 162$ minutes). It is clear from the figure that the antagonistic control variables $u_{14}$ and $u_{41}$ or $u_{34}$ and $u_{43}$ exhibit some kind of inverse variation (i.e. when the one increases the other decreases and vice versa) when region 4 gets congested. This also happens for variables $u_{24}$ and $u_{42}$ but to a smaller extend.

Table 1 presents some quantitative results of the simulation experiments. In Table 1(a) the performance index (delay) of the different approaches is reported. Taking NC as the base case, the delay decreases by 7% when the LQI methodology is applied. It should be emphasized that this is the average delay of the 13 points obtained for different linearization set-points and this approach deserves further investigation. When AFT is applied without any prior knowledge the improvement is about 11% (for the best run); however this approach has the “spiky” behavior during the first...
TABLE 1 (a) Performance of different approaches; (b) evaluation criteria for NC and BC simulations.

<table>
<thead>
<tr>
<th></th>
<th>NC</th>
<th>LQI</th>
<th>AFT</th>
<th>LQI/AFT</th>
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<tbody>
<tr>
<td>Delay (sec/km)</td>
<td>425.57</td>
<td>395.79</td>
<td>378.01</td>
<td>331.64</td>
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<tr>
<td>Improvement</td>
<td>–</td>
<td>7%</td>
<td>11.18%</td>
<td>22.07%</td>
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<table>
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<th>Evaluation criteria</th>
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<th>BC</th>
<th>(%)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay</td>
<td>425.57</td>
<td>331.64</td>
<td>-22.07</td>
<td>sec/km</td>
</tr>
<tr>
<td>Space-mean Speed</td>
<td>7.34</td>
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<td>23.84</td>
<td>km/h</td>
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<tr>
<td>Stop Time</td>
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<td>260.36</td>
<td>-24.69</td>
<td>sec/km</td>
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<tr>
<td>Total Travel Time</td>
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<td>43328.37</td>
<td>-23.64</td>
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<tr>
<td>Total Travelled Distance</td>
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<td>391076.29</td>
<td>-3.26</td>
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</tr>
<tr>
<td>Vehicles Served</td>
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<td>202428</td>
<td>0</td>
<td>veh</td>
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</table>

iterations and also even after convergence the performance for different runs is not stable (see Figure 5(a)). By using the LQI points for the first iterations and then applying AFT we obtain a stable convergent behavior and an improvement of some 22%. Table 1(b) compares NC and BC for different metrics obtained by the simulator. BC outperforms NC in all evaluation criteria (22–25% improvement) as the applied controller distributes congestion in a better way. Note that in both cases all the vehicles are served (as the network is empty at the end of the simulation), albeit the total travelled distance (production) of NC case is slightly higher. This happens because of the applied DTA which makes the vehicles choose longer routes (but with shorter travel times) since the network is heavily congested.

CONCLUSIONS

A new macroscopic MFD-based model that describes the aggregated dynamics of multi-region systems is introduced. Linear approximations of the model are used to derive optimal multivariable PI feedback regulators (LQI) for perimeter flow control. Furthermore, the performance of the regulator is enhanced in real-time by an online adaptive optimization algorithm (AFT). The efficiency of the integrated adaptive control scheme was tested in micro-simulation. The studied problem is quite difficult from a control point of view, since the boundaries of the network are not controlled and all the inflows coming from the boundaries are considered disturbance for the system. As a consequence, it is difficult to regulate the system around the set-point by only controlling the internal transferring flows between the regions (because of the high disturbances). Nevertheless, the simulation results indicate that the integrated control scheme (LQI/AFT) can significantly improve the network performance compared to fixed-time signal plans and previous adaptive type controllers.

As illustrated here, the online application of AFT algorithm without any prior observations of the system performance for different control parameters is cumbersome, because of its “spiky” behavior during the first iterations (e.g. days) of application. To overcome this difficulty, measurements of the system performance can be collected for different controllers that are obtained by
applying the LQI methodology. Then, AFT algorithm is applied to fine-tune the parameters of the multivariable controller (gain matrices and set-points) so as to achieve a desirable performance. The proposed methodology is applicable in real life as it is computationally efficient and it only requires loop detectors real-time measurements. Future research directions will deal with investigations about the linearization points of the model (set-points) as well as the activation time of the controllers. This is a mutual problem that can be possibly solved in real-time.

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