Albert Nikolaevich Shiryaev: On the Shoulder of a Giant >>>

Interview of Albert Nikolaevich Shiryaev by Y.K. Leong

Albert Nikolaevich Shiryaev is well-known for his important contributions to probability theory, mathematical and applied statistics and financial mathematics, and in particular, to statistical sequential analysis and optimal stochastic control. He has published more than 160 main scientific papers and is the author or co-author of numerous definitive books and monographs in those fields.

He has received numerous prizes and awards such as the Markov Prize, Kolmogorov Prize, Humboldt Research Award, Honorary Fellow of Royal Statistical Society and honorary doctorates from Freiburg University and Amsterdam University. He has served and continues to serve on the editorial boards of many leading journals in probability theory, statistics and mathematical finance. He had been president of the Bernoulli Society, the Actuarial Society of Russia and the Bachelier Finance Society.

Shiryaev has a long and illustrious career at Moscow State University (as professor since 1970, head of the Probability Department since 1996 and Distinguished Professor since 2003) and at the Steklov Mathematical Institute (head of the Laboratory "Statistics of Stochastic Processes" from 1986 to 2002). Even at the present age of 71, retirement is not on his cards.

When Shiryaev was at the Institute to give keynote lectures at the Institute's workshop on computational finance, Y.K. Leong interviewed him for *Imprints* on 26 August 2005. The following is an edited transcript of an unusual interview which gives us some insights into the scientific legacy of the legendary A.N. Kolmogorov (1903 – 1987), probably the last great universalist of the 20th century. Shiryaev may be considered to be the successor and upholder of the Russian tradition in probability theory established by Kolmogorov.

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chemistry is heavily involved in the mind's function. On the other hand, if you look at it positively, what it suggests is that if we understand our own chemistry, there is a possibility that we can control it, hopefully for the good. This line of research raises some deep ethical issues. If one learns how to manipulate the brain and manipulate taste, it raises the ethical issue of whether one should do that or want to do that.

Albert Nikolaevich Shiryaev

begin? How did you choose the topic for your doctoral dissertation?

A.N. Shiryaev: Before I explain how I chose probability as my specialization, maybe I should say how I became a mathematician. When I was in secondary school, I had many interests. I had a strong interest in sports – I played soccer, I did figure skating and for some years I took lessons in ballet. Twice I danced with the ballet group in the Bolshoi Theatre – my instructress worked in the Bolshoi Theatre. At the same time, because of my relatives, I had an interest in rocketry. I lived in Moscow near the famous rocket center and my uncle's father and many others worked in this place. Also I had a strong interest in a diplomatic career and many times I visited the Moscow Institute of International Relations, but finally, of course, I decided to become a mathematician. I participated in different kinds of Olympiad and finally when I got a medal in secondary school, I entered Moscow State University without examination – they accepted me after an interview with the professors.

As a student at the department of mathematics and mechanics, I did not spend much time in mathematics. In some sense, I began to work in mathematics only after five years of university. The reason was very simple. At that time, the ski coach of Moscow University invited me to be a member of the downhill ski team. I was in very good physical condition and very quickly, after three years, I was the champion in Moscow, and in 1957 I even participated in the Second International Winter Universiade in Grenoble. There were 42 participants and I was placed number 4 and number 7 in slalom and giant slalom. For the Russian people, it was quite good because at that time our country was not very well-known in that sport. So, for three years, I spent a lot of time skiing instead of attending lectures. But at the end of the last (fifth) year, I wrote a diploma paper and it was a good piece of work. Then after many conversations,

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A.N. Kolmogorov finally said to me, "I want to take you as a member of my department at the Steklov Mathematical Institute, but you must select either sports or science."

I was already 23 and not very young for sports; so I decided to stop my active sports life and work in Kolmogorov's department. Kolmogorov gave me many problems and after one year of work I wrote my first paper with my friend Victor Leonov on the technique of calculation of cumulants. Very soon Kolmogorov directed me to some applied problems. As a result, I wrote several papers on the quickest detection problem. The first paper was entitled "The quickest detection problem of the spontaneous effects" and this paper became very popular and many people used it and referred to it. D. Siegmund and B. Yakir, who are here, wrote a lot of papers on problems of that type, very often referring to my paper. After two or three years, Kolmogorov said to me, "You already have all you need for your dissertation." So shortly, I wrote my dissertation, and after that, I took the examinations. It was a little bit of an "inverted" situation. Usually you prepare for and take the examinations in mathematics, languages, philosophy and so on before you do the dissertation, but mine was an "inverse" story.

So I defended my (candidate) dissertation, and in my work for the dissertation, I solved some optimal stopping problems in the Markovian setting. It turned out that stochastic calculus was very important for these problems and I began to work very actively in this direction. I organized special seminars in the Steklov Institute and they were very popular for many years. We published our proceedings and as a result more than 50 of my students defended their theses effectively. They are not "PhD" theses in your sense. In Russia, we have two dissertations – the "PhD" dissertation and then the "Doctor of Science" dissertation. Generally, out of 10 who wrote the first dissertation, only one will go on to write the Doctor's dissertation.

As a result, I published a book on optimal stopping rules – two editions in Russian, and one translation into English for Springer. I also wrote with my pupil Robert Liptser books on stochastic processes and our main interest was in non-linear filtering. At that time we realized the importance of the theory of martingales, and we worked actively in this area. As a result, we also wrote a little book on the theory of martingales and with my French co-author Jean Jacod the book *Limit Theorems for Stochastic Processes*. I began to work in the Steklov Institute in 1957 and I am still its member.

I: Were you also a member of the mathematics department of Moscow University?

S: Yes. Kolmogorov attracted me to Moscow University simply for his lectures on probability theory. He was in both places – Moscow University and the Steklov Institute. He was chair of the department of probability in Moscow University. After him the chair was B. Gnedenko. Now I am the chair of that department. It is a very big department. Every year we accept more than 50 students for specializing in probability, and we have two groups of students – one for probability theory and the other for actuarial and financial mathematics. In 1994, I began to work in financial mathematics and was probably the first to give lectures on financial mathematics in Moscow University. I wrote a big book, published here in Singapore, on *Essentials of Stochastic Finance*. This was reprinted five times and is popular. Recently, the second Russian edition has been published, and World Scientific has asked me about the second English edition, but I have no time because I am writing several books, one book with my colleague from Denmark Goran Peskir on optimal stopping free-boundary problems, another book with Ole E. Barndorff-Nielsen on change of time and change of measures, which will be published by World Scientific.

I: Was your Russian book on financial mathematics the first book published on the subject in Russia?

S: It was first published in English, and it was the first book on financial mathematics published in Russia. Even the great Russian newspaper *Izvestia* published good reviews of the book, and they said that it was important for the development of the Russian economy. At the same time that I was working on the book, I was involved with several publications and books about Kolmogorov. Before the death of Kolmogorov in 1987, we published three volumes on the selected works of Kolmogorov. I was involved in it and I felt that I had to do it. Now we plan to publish six volumes of his selected works: the first volume embraces the papers by Kolmogorov in mathematics and mechanics, the second on probability theory and mathematical statistics, the third on the theory of information and the theory of algorithms, the fourth about mathematics and mathematicians, and we have plans to publish also volumes 5 and 6.

I: Are the complete works of Kolmogorov published?

S: Not all. Of course, not. In fact, the archives of Kolmogorov belong to me in some sense. Kolmogorov's widow wrote in a testimony that all the archives of Kolmogorov's work belong to me. I must say the following. Two years ago, in 2003, we had a very big conference dedicated to the centennial of the birth of Kolmogorov. I was essentially involved in its organization, and we published before this conference three books related to Kolmogorov. The first volume consists of two parts: the first part is the biography of Kolmogorov written by me (more than 200 pages) and the second part is a bibliography of his work (mathematical papers, papers in encyclopedias, textbooks and papers for secondary schools and so forth). The second volume is a big volume on the

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correspondence between Kolmogorov and Paul Alexandrov, the creator of general topology. They were friends and had a very interesting correspondence with a lot of mathematical visions. Finally, the third volume is very interesting – it's the diary of Kolmogorov. Practically nobody had seen it before. I found the diary in Kolmogorov's country house. So we published it and now I want to propose to World Scientific to publish an English translation. The diary is very interesting and unusual – he began to write the diary when he was 40. At the beginning, Kolmogorov wrote the following: that he dedicated his diary to his 80th anniversary in the hope that he will understand what he wrote in his forties. In this book, you will find a lot of interesting pages. One page is of the following type – "what I must do to be a great person". Of course, he wrote this ironically. I want to show you an interesting page – his future mathematical plans.

I: How many years did the diary cover?

S: Not too much – two to three years. Here it is very detailed. But later he wrote, not very periodically – his plans on what he should do from 44 to 53, from 54 to 63, etc.

I: Did he follow his plans?

S: Yes, surprisingly, surprisingly. For example, everybody was surprised why he practically stopped working in mathematics after 60 when he started working for secondary schools. But he wrote this down and he planned it. He "predicted" that he should work in this area.

I: Did he stop working completely in mathematics after 60?

S: Of course, he worked in mathematics, but he spent a lot of time writing textbooks on algebra and geometry for secondary schools. He organized a special school and a journal for students of 15-17 years of age and with special abilities in mathematics and physics. He worked and gave classes like the usual teachers.

I: Was it in Moscow or all over Russia?

S: It was in Moscow but the boys and girls were from different cities in Russia. Even students from Moscow may not be selected. They were specially selected by local or All-Union Olympiads.

I: Was Kolmogorov a good lecturer in schools?

S: It's a bit difficult to say. In fact, his manner of speaking was very fast. Very often he jumped and omitted intermediate steps. It was his usual style, and many people said that it was very difficult to follow him.

I: It must be very interesting working with Kolmogorov.

S: It was, of course, very interesting but it was not simple. It was definitely clear that if you did not work very successfully, he would lose interest in you. In some sense, you should be at a good level and have initiative and as a result we had to spend a lot of time mathematically when we were young.

I: Were you very close to Kolmogorov personally?

S: Of course, I know Kolmogorov personally very well. When I began to work at the Steklov Mathematical Institute together with my friend Victor Leonov, Kolmogorov asked us to be his informal secretaries. We attended his lectures and wrote lecture notes for students. As a result, we lived practically two days every week in his country house. We skied with him, and later, I had a car and we visited many Russian cities because Kolmogorov had a vast knowledge of Russian icon art. He knew a lot about Russian churches and the details about their construction and so forth. Communication with him was not very simple because often you have the feeling that there was some kind of screen between you and him. You always feel that in front of you is a brain which works continuously and it was amazing that at the same time he has the ability to think about different topics.

I: You must have felt tense.

S: Yes, at all times you would feel the tension. He was so "non-trivial" that you could not say some trivial thing. He knew music very well – also literature, poetry, archaeology, history, geography. He had a fantastic memory, especially for geography, history and so on. You know that his beginning was very unusual – his mother died two hours after he was born. His father was killed during the civil war, and essentially, his mother's sister brought him up. When he was 5 years old, he made a lot of unusual observations. He discovered, for example, that $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$ and so forth. I asked him how he understood those things. It turned out that his solution was purely geometric. He also solved the following problem when he was only 5 years old: Suppose you have a button. You can fix it to the coat if the thread goes through at least two button holes. The question is: in how many ways? He arrived at an absolutely correct answer. He was only 5 years old. From the beginning, he had a strong mathematical ability.

I: Nobody taught him?

S: No, he did it all by himself. When he was 12 or 14, he studied mathematics at a very high level, reading mathematical texts in encyclopedias and trying to reconstruct the proofs. So he began very, very early to do mathematics.

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I: Nowadays mathematics is so wide and highly specialized and yet Kolmogorov did so many fields.

S: Two years ago, we had a conference with the title of *Kolmogorov* and *Contemporary Mathematics*. We had six sections in which Kolmogorov worked: dynamical systems and ergodic theory, theory of functions and functional analysis, theory of probability and mathematical statistics, mathematical logic and complexity, turbulence and hydrodynamics, geometry and topology. He wrote a lot of papers in all these areas and he essentially created many fields. He is the father of modern probability theory. Even the topological notions in cohomology were introduced by him. In turbulence, there is a famous law called the "two-thirds law" of the type of Newton's laws, and it is his contribution. He introduced the notion of complexity which gives the possibility of applying probability even to non-probabilistic objects. The notion of complexity is the crucial clue. I remember that before the organization of the Kolmogorov conference, I was thinking about getting money for the organization. At one point, I asked Microsoft for money and they gave the money, saying – "Yes, Kolmogorov! It's very important in complexity, mathematical logic, computing…" I also asked money from Boeing and they also gave us money (because of turbulence of Kolmogorov).

I: Do you think that in the future somebody can be like Kolmogorov, versatile in so many fields and with so much impact?

S: It's hard to say. In some sense, it is difficult to predict that we will have a person of the following type. Let's look at the "encyclopedic mathematicians" [universalists] – Poincaré, Hilbert, von Neumann, Kolmogorov. It's practically very difficult to give another name.

I: What about Norbert Wiener?

S: He was broad, but I think Kolmogorov worked in many, many different areas. We know Wiener's work in filtration, interpolation but Kolmogorov did that before him. Wiener wrote in his books I am a mathematician and Ex-prodigy that Kolmogorov discovered it a little bit earlier. Wiener, of course, did something in probability – he introduced Wiener measures and the properties of the so-called Wiener trajectories, but in some sense they were particular cases. Kolmogorov was very wide and he was great in the creation of new concepts like complexity in mathematics. Probability space, conditional probability and expectation belong to him. I wrote that if we take a Russian mathematical encyclopedia, we find Kolmogorov axioms, K duality, K integral, K criterion, K inequality, K space, K equation, K-Smirnov criterion, K-Chapman equations. If you take any encyclopedia on probability and mathematical statistics, you will find Kolmogorov axiomatization, K self-similarity, K law of two-thirds, K criterion, K matrix, K model, K distribution, K statistic, K law of five-thirds, K self-similarity, K spectral theory.

I: Did Kolmogorov ever meet von Neumann?

S: Yes, von Neumann gave the opening talk at the Amsterdam Mathematical Congress and Kolmogorov gave the last talk. They had a very short conversation. As I understood, it was not a very long discussion. As for Wiener, Kolmogorov met him in Moscow, but with Wiener it was a very strange story. When Wiener arrived he made a call to Kolmogorov and said, "I want to meet and have a talk with you." Kolmogorov said, "Please come tomorrow at six o'clock." And Wiener came but at six o'clock in the morning. [Laughs] For us it is clear that it must be 6.00 pm.

I: Could the Kolmogorov "phenomenon" be a result of the Russian system and environment?

S: It is difficult to say. In some sense, he was a genius from the beginning. When he was a student of Moscow University, the mathematical school of Lusin flowered and many well-known mathematicians appeared at that time – Lusin, Khinchin, Kolmogorov, Novikov, Petrovskii and many others. It was a special period when the Moscow mathematical school understood that the methods of the theory of functions are very important. In some sense, Kolmogorov said that his success in the creation of probability theory was based on the understanding that the theory of functions and theory of measures play a very important role. Kolmogorov was not a member of the Communist Party but the high-ranking leaders of the Party realized how great Kolmogorov was.

I: The Soviet government appreciated and understood the value of Kolmogorov …

S: Yes, exactly. In the Soviet Union, the highest decoration for anyone is the Order of Lenin. It was awarded to Kolmogorov seven times for his contributions and work in mathematics. There is a famous international story: in 1940 when we had a person called Lysenko who wanted to close genetics down. But at that time Kolmogorov wrote a paper about the brilliant confirmation of Mendel's laws. Politically, it was very dangerous but nobody arrested him. In the beginning of the Second World War, Stalin asked Kolmogorov to begin work – not military work, but work related to the defense force. By the way, two years ago, there was a conference entitled Mathematics and war. They invited me and I wrote a paper about the defense work of Kolmogorov during the Second World War.

I: It was not classified work? The Russian government allowed it to be published?

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S: It was a mathematical problem. In the beginning of the war, we had in Russia many light small planes. Suppose we use these planes for bombing and it is necessary to predict where the bombs will land. It depends on the velocity and so on. It was necessary to create ballistic tables for bombing. Kolmogorov did it by himself and he discovered the following interesting phenomenon. Suppose you have a bridge and want to destroy the bridge. Usually we would want to hit the center of the bridge. But Kolmogorov discovered that, in fact, it was necessary to create an "artificial" bombing. You would try to aim at one point, but he said, "No, it's not correct. Sometimes it is necessary to try to hit here or try to hit there." In other words, it is necessary to create an "artificial deviation". It was the beginning of much work of this type and he created the tools for this work.

I: It seems that the Russian system of education is very successful in developing problem-solving skills. What is it due to?

S: In some sense, it is true. The basic reason is that Russian mathematics has a very good tradition in the secondary school education and in the universities. We had a lot of great mathematicians who created many different schools of mathematics. Kolmogorov created the school of probability theory, Petrovskii created the school of differential equations, Novikov and Markov created the school of algebra and mathematical logic, Pontryagin created the school of continuous group theory and after that, he worked in the theory of optimization (the Pontryagin maximum principle). And simply, we have great people who created scientific schools and they were related to the education in the universities.

It reminds me of the following. At the end of the Second World War, Lysenko or Stalin (I don't know exactly who) said that science is the enemy of randomness in the sense that science is trying to make order in everything. But the representative of the philosophy school began to attack probability theory, saying that probability theory investigated the notion of independence, but everything in the world is related, and so the notion of independence is nonsense. They then said that Kolmogorov's department of probability is dealing in idealism. Kolmogorov was invited to the conference, and there was a discussion about independence and randomness. And Kolmorogov told them. "Let's take the government lottery. The randomness that you will win is guaranteed by government. Suppose that it is not true. Then it means that the government creates unfair lotteries."

I: Your faculty is called the "Faculty of Mechanics and Mathematics". For us, mechanics and mathematics seem to be a strange combination …

S: It's theoretical mechanics. We have in this faculty two sections. One section is in mathematics, and it is clear what they are doing. The other section is in mechanics; they are doing turbulence, hydrodynamics, elasticity … – in some sense, partial differential equations with applied aspects. They investigate what the form of the airplane should be, how it depends on the velocity and so on, but by using mathematical methods. They have some part of the engineering aspects, but it's mainly the theory.

I: Do you consider yourself to be an applied probabilist now?

S: No, of course, not. I remember that at a banquet after defending my doctor's dissertation, several people proposed toasts. One person said that Shiryaev was a probabilist, Another said that he was a statistician, another said he worked in applied probability. But Kolmogorov said, "We are mathematicians, and if you are a good mathematician, you should be able to solve any problem – theoretical problems, applied problems and so on." Now I am doing financial mathematics, but I don't work directly in the bank or for the banks. Simply, we understood that financial mathematics and financial engineering give rise to a lot of new theoretical problems, and we are trying to solve them. In the coming conference on Monday and Tuesday, the talks will be exactly about this.

I: But now you are more interested in financial mathematics.

S: Not exactly. I think it will be very bad if I will concentrate only in financial mathematics because, first of all, around me there are a lot of students, and as head of the department of probability, I should have a good orientation in many different theoretical aspects. I cannot give them only problems in financial mathematics because I must think about developing probability theory as a science and developing mathematical statistics as a science. From some point of view, financial mathematics is now very attractive because of its many new problems and more job opportunities. It is also necessary to mention not only financial mathematics but also actuarial or insurance science. In Russia, I was the president of the Actuarial Society for four years, and so we began to work in this direction. It's true that our students were able to get good salaries after university. That is, of course, important, but in some sense it is a pity that a lot of our good students leave Russia to continue their education mainly in the United States and England. Many of them got jobs in the United States and other countries.

I: Is your department doing anything to retain your talents and encourage them to stay in Russia?

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S: It's a very difficult question. I know that several people have returned to Russia, but look, you have young people going to the United States for their dissertation and it is a period when they begin to have families, children, houses, and life is life. So they continue to stay in those places. But I know several cases, not young people, who retired in Russia. But now, it is not easy to get good positions in Russia. For example, our Steklov Mathematical Institute is very small. It's a good institute; in some sense, it's like the Institute for Advanced Study in Princeton. We are doing theoretical work and we are proud to be members of this Institute.

I: How many members are there in the Steklov Institute?

S: We have 12 departments in the institute and about 120 research members. They are permanent members. We have practically no visitors. I worked in the Steklov Institute all my life and I am very happy. The Institute belongs to the Academy of Science and if we ask for a new position for a good young person, usually we get it.

I: What do you think of the future of mathematics in Russia?

S: Of course, we want to continue the good Russian tradition in mathematics. I want to say that our administration of the Academy of Science, for example, is trying to do it. Who is the president of the Russian Academy of Science? Academician Yu. Osipov who is a mathematician. Who is director of the Steklov Mathematical Institute? Academician V. Kozlov who is vice-president of the Academy of Science. Who is the rector of Moscow State University? Academician V. Sadovnichy who is also a mathematician. They have a lot of power and they are trying to preserve the tradition, not only for mathematics but in Russian science. As a result, we have a good administrative group for mathematics. Of course, they are doing a lot of things for many other fields, but I think that it is a positive point for preserving the good Russian tradition in mathematics. Also, many academic people paid a lot of attention to education in secondary school and in university. This is to keep the good Russian tradition. That is why, for example, in the 40s and 50s, mathematics in Russia was good. For example, Kolmogorov worked purely in research in the Academy of Science and at the same time worked for education in Moscow University. Good scientists in research at the same time gave lectures and seminars at the universities. As a result, students get a good opportunity in understanding in which direction it is necessary to work. There is consolidation and interplay between the Academy of Science and education. This is very important and, in some sense, it increases the possibility of keeping our good Russian tradition in mathematics.