Mathematical Conversations

Alice Chang : Analyst in Conformal Land >>>

Alice Chang

Interview of Alice Chang by Y.K. Leong

Sun-Yung Alice Chang is well-known for her many important contributions to real harmonic analysis, geometric analysis, nonlinear partial differential equations and applications of analysis to problems in differential geometry. In 1995 the American Mathematical Society awarded her the Ruth Lyttle Satter Prize in Mathematics (awarded every two years to a woman for outstanding research in mathematics) for her deep contributions to the study of partial differential equations on Riemannian manifolds.

Born in Xian, China, she grew up in Taiwan and had her undergraduate education at the Taiwan National University in Taipei and her PhD at the University of California at Berkeley. She has taught at the State University of New York at Buffalo, University of Maryland and University of California at Los Angeles before moving, in 1998, to Princeton University where she is a full professor.

She has given invited addresses at major mathematics meetings and conferences, including a 45-minute talk at the International Congress of Mathematicians (ICM) at Berkeley in 1986, a one-hour plenary talk at the ICM at Beijing in 2002 and an AMS Colloquium talk in 2004. In 2001 she gave the Emmy Noether Lecture of the U.S.-based Association for Women in Mathematics. She has served as editor of several leading mathematical journals and was Vice-President of the American Mathematical Society from 1989 to 1991. In 1988, she received the Outstanding Woman of Science Award from University of California at Los Angeles. Her life and work is a fine example of what women are capable of achieving in mathematics and has set an inspiring role model for women pursuing careers in the scientific field. Her husband Paul Yang is also her long-term collaborator in mathematical research, and they have a son and a daughter.

The Editor of *Imprints* interviewed Alice Chang at the Institute on 12 June 2004 during her visit to give invited lectures at the program on "Geometric Partial Differential Equations". The following is a vetted account of the interview in which she talked about her school years, her fascination with and devotion to mathematical research, and her views about the need to encourage women of talent in mathematics.

Imprints: Was mathematics your first career choice when you were at university?

Alice Chang: Yes, it was. In Taiwan there is an entrance examination for college, but I was one of the small percentages of students who did not need to take the entrance exam. I was one of the "pao-song" (literally, "guarantee send" in Chinese), the people in each high school who can choose which college to attend without taking the entrance exam. The positions are allocated according to class standing. I was ranked first in my high school. I had my college education in Taiwan National University in Taipei. In high school, I liked both Chinese literature and mathematics. In college, I decided to major in mathematics.

I: You started as an analyst but you are now interested in problems about geometry. Do you consider yourself to be an analyst first and then a geometer, or the other way around?

C: I consider myself to be an analyst first and a geometer second because of my background and the way I think about mathematics. So basically I am an analyst and now I am working on problems which are very geometric in nature. Fortunately, I have other co-authors who are more of a geometer than an analyst. So we cooperate with each other. I always think of myself as an analyst.

I: There have been many recent developments at the interface between geometry and partial differential equations. When and how did the interaction start?

C: It has a long history. The interaction between geometry and partial differential equations (or between geometry and analysis in general) is most natural. I would say it started even in the nineteenth century. Geometers like Poincaré already used the analytic approach to study problems in geometry in the late 1890s. There are also the geometers of the previous generation, like S. S. Chern, Atiyah and Singer, who laid the foundations for approaching problems in geometry using analytic methods. There has also been pioneering work by contemporary people like Nirenberg, Uhlenbeck, Schoen and Yau.

Continued on page 9

Continued from page 8

I: Partial differential equations are analytic in nature.

C: Yes. You first study the problem in the plane domain and then you study the problem in Euclidean 3-space and it is most natural to study it in higher dimensional space. There the curvature and the geometry come in.

I: It seems that geometry is becoming very analytical in nature. Is there any intrinsic geometry that is involved or is it just a matter of using the language of geometry? Does it involve any geometric intuition per se?

C: I wouldn't put it that way. I think that analysis is a tool in studying geometric problems. Geometry provides concrete examples for studying some problems at the interface between geometry and analysis. The geometric objects are concrete examples for problems in analysis. For example, you want to know what happens on the sphere – that is a concrete model. You have the analysis which is abstract analysis – convergence, weak convergence. To apply the abstract theory you need concrete examples. Geometric objects provide those examples. Of course, it is intrinsic geometry that is involved. And geometric intuition plays an important role in the approach. Analysis is sometimes a tool.

I: Geometric intuition is not something that everyone has. Is that so?

C: That is true, but on the other hand, I think that everybody has some type of geometric intuition. A problem would not be natural without the geometric intuition.

I: There are some famous mathematicians who are able to look at geometric problems and see the results even before they can prove them. This must have involved a lot of geometric intuition.

C: Yeah, yeah. If you are talking about 2-dimensional problems, maybe some people have more intuition than other people. It could also be a way of training, from their background, from the way they see things. It is true that some people have more intuition than others.

I: Is it possible to develop such geometric intuition?

C: I think so. For example, a lot of analysis problems need a lot of intuition. It's not just geometric problems that need intuition.

I: But analysis is more axiomatic.

C: Yes, it is more structured and more systematic. You are trying to derive formulas to solve a problem. However in the direction of approach to the problem - in most times, you also need intuition. You need to have some picture in your mind in both geometry and analysis. Maybe more so in geometry.

I: Have you applied your mathematical ideas to problems in physics or other scientific areas?

C: I hope to do so in the future. At the moment, no. Some of the problems I am working on are related to problems in mathematical physics. Sometimes I do read the literature in mathematical physics and see the interaction between the problems I'm working on and developments in mathematical physics. But so far, I have not applied my results to problems in that direction.

I: With the advent of computers of increasing power, and since computers operate in an essentially discrete domain, do you think that it will be necessary in the future to "discretize" geometry in order to make full use of the power of the computer?

C: I think it's the other way around. Let me explain. I'm saying that geometry has always been developed from approximation. This concept of a discrete approach to geometry has been there at the beginning. People took that approach not because of computers. For example, even in the old times people already think of the circle in terms of approximations of polygons as the number of sides gets larger and larger. The discrete approach to geometry was there before the computer. But now using computers, it's easier to take large data sets and test them.

I: Has there been much influence of the computer on the theoretical development of geometry? Is there such a thing as "computational geometry"?

C: Of course, there has been influence of computers on the theoretical development of geometry. For example, you can now construct minimal surfaces using computational methods to generate approximations and more examples. So it has a lot of influence on many areas of mathematics. But on the other hand, this computational method will never replace abstract thinking or imagination. You first need to have the idea of something that happens, and then you use the computer to test the intuition. I always think of the computer as a tool and it cannot replace the abstract thinking and intuition.

I: What are your favorite pastimes when you are not doing research?

C: I like to take walks and ride my bicycle. I like to read novels and I enjoy classical music. When I was young, I played a little bit on the piano, but not now.

Continued on page 10

Continued from page 9

I: Mathematics has traditionally been a male-dominated activity. From your experience, did you encounter extra obstacles in your mathematical career? How do you think we could encourage more female students in the university to take up mathematics?

C: It is true that mathematics has traditionally been a male-dominated profession especially at the research level. But on the other hand, I think this is due to reasons which are more – how do I describe it – social issues because women used not to have time to devote to any career. This profession could be a good profession for women. It requires a lot of thinking and you have to be very calm and patient and willing to think through things. I don't think that it should be a male-dominated profession. It's only for historical reasons. From my own experience, I think the main obstacle was that when my children were young, I felt I did not have enough time to do the work I liked to do. This is probably true for any career woman in any profession, not just for mathematics. I think this is a profession quite suitable for women in the long term. Mathematics could be done at any time. You can choose the subject you want to do research in.

I think there should be more women faculty to serve as role models. It's very hard for a woman to think that this is a possible career if the faculty in a department are all men. You do need role models. Also, I think you should encourage women. Let it be known that this profession is suitable for women. This is a problem faced by many departments in the US and around the world. It is very difficult to increase the women faculty. There are no graduate students who are women. How do you increase your women faculty? You have to reach certain standards. But this should be gradually changed with more and more women getting into graduate school and then there would be more and more women in the pipeline for assistant professors. They should be encouraged. It's a complicated issue. First, one has to understand that the intellectual abilities of men and women are the same. There must be confidence to encourage women to get into the profession.

I: What advice would you give to a beginning male or female graduate student in mathematics?

C: First, you have to be really interested in mathematics to be a graduate student in mathematics. It's a long-term commitment. You have to think that this is not temporary – you get your PhD and then … You have to be devoted and really like the subject to be a graduate student in mathematics. The other thing is that, I feel, nowadays there is a lot of information (the world is changing very fast) through the web and conferences and so on. Maybe it's more difficult for young people to really quiet down and think through a subject more deeply - the fundamentals, the basic background. I think young people should not be pushed by fashion and should not be forced to be very quick and have a lot of publications. They should instead think more quietly and think through the foundations of the subject.

I: Of course, mathematics has a lot of competition from more lucrative subjects like economics, computer science, financial mathematics.

C: The competition is always there, of course. First, you must have a real love for the subject, so you really want to understand something. So you are willing to devote your time and take the long-term approach.

I: Also, mathematics is a very demanding subject.

C: I think if you want to do well in any subject, it is demanding. If you want to be a good musician, a good painter, a good economist or to know your subject well, it takes a lot of devotion.

I: Can you tell us something about the latest work you are working on?

C: I'm working in a field called conformal geometry. Recently we are trying to use fully nonlinear partial differential equations to study patterns in geometry; in particular, the latest project I'm involved in is to classify a certain type of 4-manifolds up to diffeomorphism using the analytic approach.

I: Do you think collaboration is very important for mathematical research?

C: It is. You learn much faster by talking to other people. For me, I'm an analyst and, in my project, it's important for me to talk to geometers. In this case, fortunately for me, one of my main collaborators is my husband, Paul Yang, who is a geometer. I think it's very important to collaborate and talk to other people.

I: Papers written by single authors and papers written with other authors are often given different weights. What is your view on this?

C: For papers written by yourself and papers written with others, the weights should be a little different. If you have to do everything by yourself, the weight should be heavier because the speed will be slower. If you have collaborators, you could have more papers. When you work alone, you may have only one paper. If you have collaborators, you may have two or three. Even if you count it as one-half, it's still fair. However, the quality of a person's papers is more important than their quantity.

Continued on page 11

Continued from page 10

I: Knowledge is now so broad that it is quite natural for research to become more like a collective activity.

C: But I also want to say that in the evaluation system, we should leave room for people to work alone. Some of the fundamental work requires deep thinking and a long-term commitment. Some people may want to work on their own project rather than in collaboration with others and be obliged to listen to other people's opinions and be influenced by them. In the mathematical community, we should leave room for people who want to do work in their own way. Mathematical research is not just a scientific approach; the nature of mathematics is sometimes close to that of art. Some people want individual character and an individual way of working things out. They should be appreciated too. There should be room for single research and collaborative research.